Simulation study of optimal cooperative collision avoidance between multiple robots with constraints

Igor Škrjanc, Gregor Klančar
Faculty of Electrical Engineering, University of Ljubljana, Tržaška 25, SI-1000 Ljubljana, Slovenia

Abstract— **In this paper a new cooperative collision-avoidance method for multiple nonholonomic robots with constraints and known start and goal velocities based on Bernstein- Bézier curves is presented. In the simulation example the velocities of the mobile robots are constrained and the start and the goal velocity are defined for each robot. This means that the proposed method can be used as subroutine in a huge path-planning problem in the real time, in a way to split the whole path in smaller partial paths. The reference path of each robot from the start pose to the goal pose, is obtained by minimizing the penalty function, which takes into account the sum of all the paths subjected to the distances between the robots, which should be bigger than the minimal distance defined as the safety distance, and subjected to the velocities which should be lower than the maximal allowed velocities of each robot. When the reference paths are defined one of the trajectory tracking control algorithms should be used to define the control. The simulation results of the path planning algorithm and some future work ideas are discussed.**

I. Introduction

Collision avoidance is one of the main issues in applications for a wide variety of tasks in industry, human-supported activities, and elsewhere. Often, the required tasks cannot be carried out by a single robot, and in such a case multiple robots are used cooperatively. The use of multiple robots may lead to a collision if they are not properly navigated. Collision-avoidance techniques tend to be based on speed adaptation, route deviation by one vehicle only, route deviation by both vehicles, or a combined speed and route adjustment. When searching for the best solution that will prevent a collision many different criteria are considered: time delay, total travel time, planned arrival time, etc. Our optimality criterion will be the minimal total travel time of all mobile robots involved in the task, subject to a minimal safety distance between all the robots and subject to velocity constraints of each mobile robot.

In the literature many different techniques for collision avoidance have been proposed. The first approaches proposed avoidance, when a collision between robots is predicted, by stopping the robots for a fixed period or by changing their directions. The combination of these techniques is proposed in [1] and [2]. The behavior-based motion planning of multiple mobile robots in a narrow passage is presented in [3]. Intelligent learning techniques were incorporated into neural and fuzzy control for mobile-robot navigation to avoid a collision as proposed in [4], [5]. Also, some adaptive navigation techniques for mobile robots navigation appeared, as proposed in [6].

In our case we are dealing with cooperative collision avoidance where all the robots are changing their paths cooperatively to achieve the goal. The control of multiple mobile robots to avoid collisions in a two-dimensional free-space environment is mainly separated into two tasks, the path planning for each individual robot to reach its goal pose as fast as possible and the trajectory tracking control to follow the optimal path. In our paper only the first part will be presented.The second part is well known in the literature and will not be presented here.

The paper is organized as follows. In Section II the problem is stated. The concept of path planning is shown in Section III. The idea of optimal collision avoidance for multiple mobile robots based on Bézier curves is discussed in Section IV. The simulation results of the obtained collision-avoidance control are presented in Section V and the conclusion is given in Section VI.

II. Statement of the problem

The collision-avoidance control problem of multiple nonholonomic mobile robots is proposed in a two-dimensional free-space environment. The simulations are performed for a small two-wheel differentially driven mobile robot of dimension $7.5 \times 7.5 \times 7.5$ cm. The

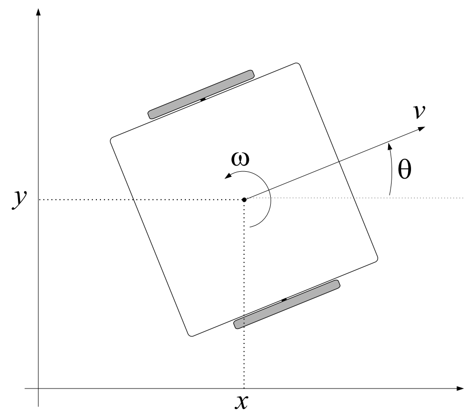

Fig. 1. The generalized coordinates of the mobile robot.

architecture of our robots has a nonintegrable constraint in the form $\dot{x}\sin\theta - \dot{y}\cos\theta = 0$ resulting from the assumption that the robot cannot slip in a lateral direction where $q(t) = [x(t) \;\; y(t) \;\; \theta(t)]^T$ are the generalized

Proceedings 21st European Conference on Modelling and Simulation
Ivan Zelinka, Zuzana Oplatková, Alessandra Orsoni ©ECMS 2007
ISBN 978-0-9553018-2-7 / ISBN 978-0-9553018-3-4 (CD)

coordinates, as defined in Fig. 1. The kinematics model of the mobile robot is

$$\dot{q}(t) = \begin{bmatrix} \cos\theta(t) & 0 \\ \sin\theta(t) & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v(t) \\ \omega(t) \end{bmatrix} \tag{1}$$

where $v(t)$ and $\omega(t)$ are the tangential and angular velocities of the platform. During low-level control the robot's velocities are bounded within the maximal allowed velocities, which prevents the robot from slipping.

The danger of a collision between multiple robots is avoided by determining the strategy of the robots' navigation, where we define the reference path to fulfil certain criteria. The reference path of each robot from the start pose to the goal pose is obtained by minimizing the penalty function, which takes into account the sum of all the absolute maximal times subjected to the distances between the robots, which should be larger than the defined safety distance and maximal allowed velocities of each mobile robot.

III. Path planning based on Bernstein-Bézier curves

Given a set of control points $P_0, P_1, \ldots, P_b$, the corresponding Bernstein-Bézier curve (or Bézier curve) is given by

$$\mathbf{r}(\lambda) = \sum_{i=0}^{b} B_{i,b}(\lambda)\mathbf{p}_i$$

where $B_{i,b}(\lambda)$ is a Bernstein polynomial, λ is a normalized time variable ($\lambda = t/T_{max},\ 0 \leq \lambda \leq 1$) and $\mathbf{p}_i,\ 0 = 1, \ldots, b$ stands for the local vectors of the control point P_i ($\mathbf{p}_i = P_{i_x}\mathbf{e}_x + P_{i_y}\mathbf{e}_y$, where $P_i = \left(P_{i_x}, P_{i_y}\right)$ is the control point with coordinates P_{i_x} and P_{i_y}, and $\mathbf{e}_x$ and $\mathbf{e}_y$ are the corresponding base unity vectors). The absolute maximal time T_{max} is the time needed to pass the path between the start control point and the goal control point. The Bernstein-Bézier polynomials, which are the base functions in the Bézier-curve expansion, are given as follows:

$$B_{i,b}(\lambda) = \binom{b}{i} \lambda^i (1-\lambda)^{b-i},\ i = 0, 1, \ldots, b$$

which have the following properties: $0 \leq B_{i,b}(\lambda) \leq 1,\ 0 \leq (\lambda) \leq 1$ and $\sum_{i=0}^{b} B_{i,b} = 1$.

The Bézier curve always passes through the first and last control point and lies within the convex hull of the control points. The curve is tangent to the vector of the difference $\mathbf{p}_1 - \mathbf{p}_0$ at the start point and to the vector of the difference $\mathbf{p}_b - \mathbf{p}_{b-1}$ at the goal point. A desirable property of these curves is that the curve can be translated and rotated by performing these operations on the control points. The undesirable properties of Bézier curves are their numerical instability for large numbers of control points, and the fact that moving a single control point changes the global shape of the curve. The former is sometimes avoided by smoothly patching together low-order Bézier curves.

The properties of Bézier curves are used in path planning for nonholonomic mobile robots. In particular, the fact of the tangentiality at the start and at the goal points and the fact that moving a single control point changes the global shape of the curve. Let us assume the starting pose of the mobile robot is defined in the generalized coordinates as $\mathbf{q}_s = [x_s, y_s, \theta_s]^T$ and the velocity in the start pose as v_s. The goal pose is defined as $\mathbf{q}_g = [x_g, y_g, \theta_g]^T$ with the velocity in the goal pose as v_g, which means that the robot starts in position $P_s(x_s, y_s)$ with orientation θ_s and velocity v_s and has the goal defined with position $P_g(x_g, y_g)$, the orientation θ_g and the velocity v_g.

Let us define five control points P_s, P_1, P_o, P_2 and P_g which uniformly define the fourth order Bézier curve. The control points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ are added to fulfill the velocity and orientation requirements in the path. The need for flexibility of the global shape and the fact that moving a single control point changes the global shape of the curve imply the introduction of control point denoted as $P_o(x_o, y_o)$. By changing the position of point P_o the global shape of the curve changes. This means that having in mind the flexibility of the global shape of the curve and the start and the goal pose of the mobile robot, the path can be planned by four fixed points and one variable point. The Bézier curve is now defined as a sequence of points P_s, P_1, P_o, P_2 and P_g in Fig 2, where D stands for the distance between the start and the goal control point. The Bernstein polynomials of the fourth order ($B_{i,b},\ i = 0, \ldots, b,\ b = 4$), and the control points define the curve as follows:

$$\mathbf{r}(\lambda) = B_{0,4}\mathbf{p}_s + B_{1,4}\mathbf{p}_1 + B_{2,4}\mathbf{p}_o + B_{3,4}\mathbf{p}_2 + B_{4,4}\mathbf{p}_g \tag{2}$$

or

$$\begin{aligned} \mathbf{r}(\lambda) = (1-\lambda)^4 \left[x_s\ y_s\right]^T + \\ +4\lambda(1-\lambda)^3 \left[x_1\ y_1\right]^T + 6\lambda^2(1-\lambda)^2 \left[x_o\ y_o\right]^T + \\ +4\lambda^3(1-\lambda)\left[x_2\ y_2\right]^T + \lambda^4 \left[x_g\ y_g\right]^T \end{aligned} \tag{3}$$

The control point P_o will be defined using optimization, and the control points P_1 and P_2 are defined from the boundary velocity conditions.

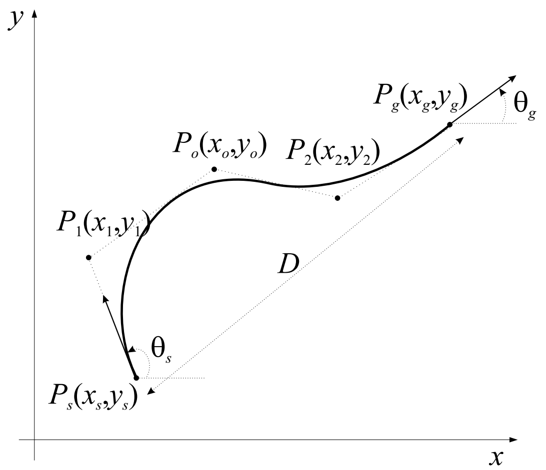

Fig. 2. The Bézier curve.

Let us therefore defined the velocity in the normalized time λ

$$\begin{aligned}\mathbf{v}(\lambda) = \dot{\mathbf{r}}(\lambda) = -4\,(1-\lambda)^3\,\mathbf{p}_s + \\ +4\,(1-\lambda)^2\,(1-4\lambda)\,\mathbf{p}_1 + 12\lambda\,(1-\lambda)\,(1-2\lambda)\,\mathbf{p}_o + \\ +4\lambda^2\,(3-4\lambda)\,\mathbf{p}_2 + 4\lambda^3\mathbf{p}_g \quad (4)\end{aligned}$$

The velocity vectors in the start position ($\lambda = 0$) and in the goal position ($\lambda = 1$) than become:

$$\begin{aligned}\mathbf{v}(0) &= 4\mathbf{p}_1 - 4\mathbf{p}_s \\ \mathbf{v}(1) &= 4\mathbf{p}_g - 4\mathbf{p}_2 \qquad (5)\end{aligned}$$

This means that the vectors to the control points $\mathbf{p}_1$ and $\mathbf{p}_2$ are defined as follows:

$$\begin{aligned}\mathbf{p}_1 &= \mathbf{p}_s + \frac{1}{4}\mathbf{v}(0) \\ \mathbf{p}_2 &= \mathbf{p}_g - \frac{1}{4}\mathbf{v}(1) \qquad (6)\end{aligned}$$

According to the orientation of the robot in the start and goal positions θ_s and θ_g and given start and required tangential velocities of the robot v_s and v_g, the velocity vector can be written in x and y components as follows:

$$\begin{aligned}\mathbf{v}(0) &= [v_x(0)\ v_y(0)]^T = [v_s\cos\theta_s\ \ v_s\sin\theta_s]^T \\ \mathbf{v}(1) &= [v_x(1)\ v_y(1)]^T = [v_g\cos\theta_g\ \ v_g\sin\theta_g]^T \qquad (7)\end{aligned}$$

Using Eqs. 6 and 7, the control points P_1 and P_2 are uniformly defined. The only unknown control point remains P_o which will be defined by optimization to obtain the optimal path which will be collision safe.

IV. Optimal collision avoidance based on Bernstein-Bézier curves

In this subsection a detailed presentation of cooperative multiple robots collision avoidance based on Bézier curves will be given by taking into account the velocity constraints of the mobile robots. Let as assume the number of robots equals n. The i-th robot is denoted as R_i and has the start position defined as $P_{si}\,(x_{si}, y_{si})$ and the goal position defined as $P_{gi}\,(x_{gi}, y_{gi})$. The normalized time variable of i-th robot is denoted as $\lambda_i = t/T_{max_i}$, where T_{max_i} stands for the absolute maximal time of the i-th robot. The reference path will be denoted with the Bézier curve $\mathbf{r}_i(\lambda_i) = [x_i(\lambda_i), y_i(\lambda_i)]^T$. In Fig. 3 a collision avoidance for $n = 2$ is presented for reasons of simplicity.

The safety margin to avoid a collision between two robots is, in this case, defined as the minimal necessary distance between these two robots. The distance between the robot R_i and R_j is $r_{ij}(t) = \mid \mathbf{r}_i(t) - \mathbf{r}_j(t) \mid,\ i = 1,\ldots,n,\ j = 1,\ldots,n,\ i \neq j$. Defining the minimal necessary safety distance as d_s, the following condition for collision avoidance is obtained $r_{ij} \geq d_s,\ 0 \leq \lambda_i \leq 1,\ i,j$. Fulfilling this criteria means that the robots will never meet in the same region defined by a circle with radius d_s, which is called a non-overlapping criterion. At the same time we would like to minimize the sum of absolute maximal times for all robots. The length of the path at the normalized time $s_i(\lambda_i)$ is defined as $s_i(\lambda_i) = \int_0^{\lambda_i} v_i(\lambda_i)d\lambda_i$, where $v_i(\lambda_i)$ stands for the tangential velocity in the normalized variable λ_i

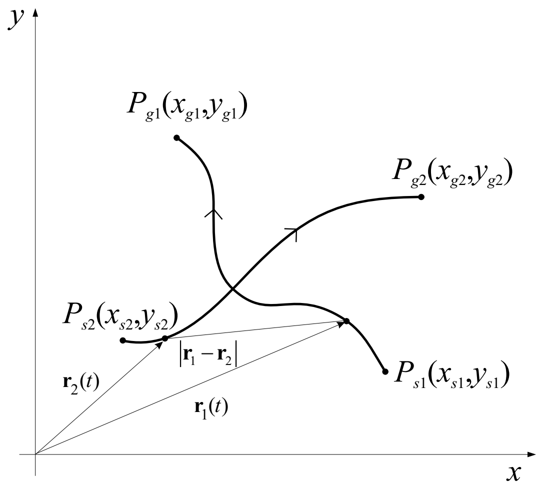

Fig. 3. Collision avoidance based on Bernstein-Bézier curves.

$$v_i(\lambda_i) = \mid \dot{\mathbf{r}}(\lambda_i) \mid = \left(\dot{x}_i^2(\lambda_i) + \dot{y}_i^2(\lambda_i)\right)^{\frac{1}{2}}$$

where $\dot{x}_i(\lambda_i)$ stands for $\frac{dx_i(\lambda_i)}{d\lambda_i}$ and $\dot{y}_i(\lambda_i)$ for $\frac{dy_i(\lambda_i)}{d\lambda_i}$.

To define the feasible reference path that will be collision safe and will satisfy the maximal velocity v_{max_i} of the mobile robot, the real time should be introduced. The relation between the tangential velocity in normalized time framework and the real tangential velocity is the following

$$v_i(t) = \frac{1}{T_{max_i}} v_i(\lambda_i)$$

The length of the path of the robot R_i from the start control point to the goal point is now calculated as:

$$s_i = \int_0^1 \left(\left(\dot{x}_i^2(\lambda_i)\right) + \dot{y}_i^2(\lambda_i)\right)\right)^{\frac{1}{2}} d\lambda_i$$

Assuming that the start P_{si}, the goal P_{gi} and P_{1i} and P_{2i} control points are known, the global shape and length of each path can be optimized by changing the flexible control point P_{oi}. The collision-avoidance problem is now defined as an optimization problem as follows:

$$minimize\ \sum_{i=1}^{n} \max\left(T_{max_i}\right)$$
$$subject\ to$$

$$\begin{aligned}d_s - r_{ij}(t) \leq 0,\ \ \forall i,j,\ i \neq j,\ \ 0 \leq t \leq \max_i\left(T_{max_i}\right) \\ v_i(t) - v_{max_i} \leq 0,\ \ \forall i,\ \ 0 \leq t \leq \max_i\left(T_{max_i}\right) \quad (8)\end{aligned}$$

The minimization problem is called an *inequality optimization* problem. Methods using penalty functions transform a constrained problem into an unconstrained problem. The constraints are added to the objective function by penalizing any violation of the constraints. In our case the following penalty function should be used to have the unconstrained optimization problem

$$minimize\ F = \sum_i \max\left(T_{max_i}\right) +$$

$$+c_1 \sum_{ij} \max_{ij} (0, d_s - r_{ij}(t)) +$$
$$+c_2 \sum_{i} \max_{i} (0, v_i(t) - v_{max_i}),$$
$$i, j,\ i \neq j,\ \ 0 \leq t \leq \max_{i} (T_{max_i})$$
$$subject\ to$$
$$\mathbf{P}_o, \mathbf{T}_{max} \qquad (9)$$

where c_1 and c_2 stand for a large scalar to penalize the violation of constraints and the solution of the minimization problem $\min_{\mathbf{P}_o} F$ is a set of n control points $\mathbf{P}_o = \{P_{o1}, \ldots, P_{on}\}$ and $\mathbf{T}_{max}$ a set of n maximal times $\mathbf{T}_{max} = \{T_{max_1}, \ldots, T_{max_n}\}$. Each optimal control point $P_{oi},\ i = 1, \ldots, n$ uniformly defines one optimal path, which ensures collision avoidance in the sense of a safety distance and will be used as a reference trajectory of the i-th robot and will be denoted as $\mathbf{r}_i(t)$. The optimal solution is also subjected to the time, because also the velocities of the robots are taken into account in the penalty function.

V. Simulation Results

In this section the simulation results of the optimal cooperative collision avoidance between three mobile robots are shown. The study was made to elaborate the possible use in the case of a real mobile-robot platform. In the real platform we are faced with the limitation of control velocities and accelerations. The simulation study was done for two mobile robots only, because of the transparency. The maximal allowed tangential velocity of the first mobile robot is $v_{max_1} = 0.3m/s$ and the maximal allowed tangential velocity for the second mobile robot is defined as $v_{max_2} = 0.25m/s$.

The starting pose of the first mobile robot R_1 in generalized coordinates is defined as $\mathbf{q}_{s1} = \left[0, 1, -\frac{\pi}{4}\right]^T$ and the goal pose as $\mathbf{q}_{g1} = \left[1, 0.5, -\frac{3\pi}{4}\right]^T$. The boundary velocities of the first mobile robot are the start tangential velocity $v_{s1} = 0.10m/s$ and the goal tangential velocity $v_{g1} = 0.10m/s$. The second robot R_2 starts in $\mathbf{q}_{s2} = \left[1, 0, -\frac{3\pi}{4}\right]^T$ and has the goal pose $\mathbf{q}_{g2} = \left[0.5, 1, \frac{-3\pi}{4}\right]^T$. The boundary velocities of the second mobile robot are the start tangential velocity $v_{s2} = 0.10m/s$ and the goal tangential velocity $v_{g2} = 0.10m/s$. The x and y coordinates are defined in meters. The safety distance is defined as $d_s = 0.40m$.

The optimal set $\mathbf{P}_o$ can be found by using one of the unconstrained optimization methods, but the initial conditions are very important. The optimization should be started with initial parameters which ensure a feasible solution. We are optimizing the total sum of all paths which are subjected to the certain conditions according to the safety distances and velocities of the robots. The velocity condition implies the implementation of the maximal time for each robot into the optimization routine. This implies that the initial set $\mathbf{P}_o$ will be defined as

$$\mathbf{P}_o = \{(x_{o1}, y_{o1}),\ (x_{o2}, y_{o2})\}$$

where x_{oi} and y_{oi} are defined as follows:

$$x_{oi} = \frac{x_{si} + x_{gi}}{2},\ \ y_{oi} = \frac{y_{si} + y_{gi}}{2},\ i = 1, 2$$

The initial maximal times are defined as $T_{max_1} = 10s$ and $T_{max_2} = 20s$ to fulfill the maximal velocities constraints. The penalty parameters are $c_1 = 2$ and $c_2 = 2$. The obtained results of the optimization routine are the following $P_{o1}(1.2505, 0.4996)$, $P_{o2}(0.1076, 0.3161)$ and $T_{max_1} = 7.3004$ and $T_{max_2} = 7.3006$. The minimal value of penalty function F equals to maximal time T_{max_2}.

The real tangential velocities profiles of avoiding robots R_1 and R_2 in normalized time variable are given in Fig. 4. It is shown that the velocities profiles of both robots fulfill the boundary velocities requirements and also fulfill the allowed maximal velocities conditions. The simulated positions of both two robots that are cooperatively avoiding the collision are shown in Fig. 5. In Fig. 6 the distances between the mobile robots are shown. It is also shown that all the distances r_{12} satisfy the safety-distance condition.

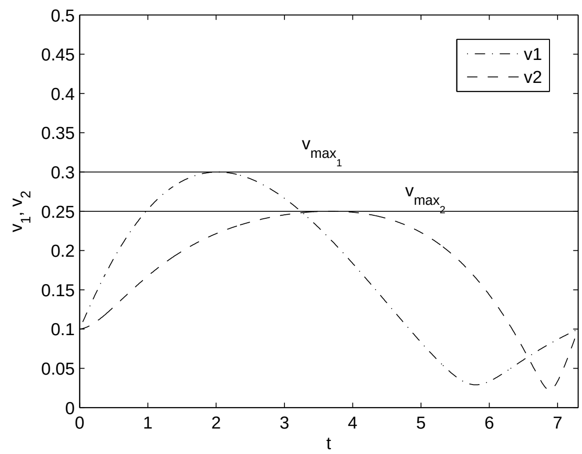

Fig. 4. The real velocities of avoiding robots R_1 and R_2 in normalized time variable.

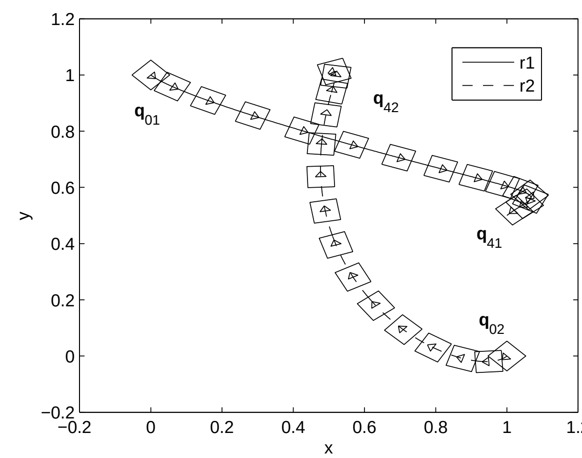

Fig. 5. The paths of collision avoiding robots R_1 and R_2.

They are always bigger than prescribed safety distance d_s.

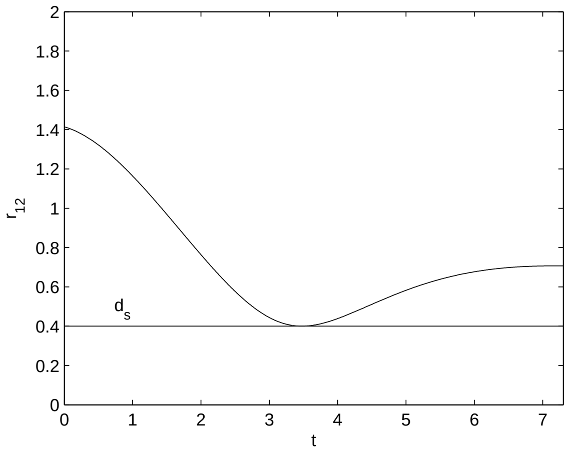

Fig. 6. The distance r_{12} between robots R_1 and R_2.

VI. Conclusion

The optimal cooperative collision-avoidance approach based on Bézier curves allows us to include different criteria in the penalty functions. In our case the reference path of each robot from the start pose to the goal pose is obtained by minimizing the penalty function, which takes into account the sum of all absolute maximal times subjected to the distances between the robots, which should be bigger than the minimal distance defined as the safety distance and the maximal velocities of the robots. The proposed cooperative collision-avoidance method for multiple nonholonomic robots based on Bézier curves shows great potential and in the future will be implemented on a real mobile-robot platform.

References

[1] R.C. Arkin, Cooperation without communication: multiagent schema-based robot navigation, J Robot Syst., vol. 9, pp. 351-364, 1992.

[2] K. Sugihara, I. Suzuki, Distributed algorithms for formation of geometric patterns with many mobile robots, J Robot Syst., vol. 13, pp. 127-139, 1996.

[3] L. Shan, T. Hasegawa, Space reasoning from action observation for motion planning of multiple robots: mutal collision avoidance in a narrow passage, J Robot Soc Japan, vol. 14, pp. 1003-1009, 1996.

[4] C.G. Kim, M.M.Triverdi, A neuro-fuzzy controller for mobile robot navigation and multirobot convoying, IEEE Trans Sys Man Cybernet SMC Part-B, vol. 28, pp. 829-840, 1998.

[5] N. Kubota, T. Morioka, F. Kojima, T. Fukuda, Adaptive behavior of mobile robot based on sensory network, JSME Trans C, vol. 65, pp. 1006-1012, 1999.

[6] A. Fujimori, P.N. Nikiforuk, M.M. Gupta, Adaptive navigation of mobile robots with obsticle avoidance, IEEE Trans Robot Automat, vol. 13, pp. 596-602, 1999.