# REAL TIME CONTROL FOR TIME DELAY SYSTEM

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**KEYWORDS**

delay system, PI controller, stability region, Hermit-Biehler theorem, optimization, genetic algorithm.

**ABSTRACT**

In this paper, we consider the control of time delay system by Proportional-Integral (PI) controller. By Using the Hermite-Biehler theorem, which is applicable to quasi-polynomials, we seek the stability region of the controller and the computation of its optimum parameters. We have used the genetic algorithms to lead the complexity of the optimization problem. An application of the suggested approach for real-time control on a heater $PT-326$ was also considered.

## INTRODUCTION

Systems with delays represent a class within infinite size largely used for the modelling and the analysis of transport and propagation phenomena (matter, energy or information)(Niculescu, 2001; Zhong, 2006). They naturally appear in the modelling of processes found in physics, mechanics, biology, physiology, economy, dynamics of the populations, chemistry, aeronautics and aerospace. In addition, even if the process itself does not contain delay, the sensors, the actuators and the computational time implied in the development of its control law can generate considerable delays (Niculescu, 2001) . The latter have a considerable influence on the behaviour of closed-loop system and can generate oscillations and even instability (Dambrine, 1994).

PID controllers are of high interest thanks to their broad use in industrial circles (Cedric, 2002). Traditional methods of PID parameter tuning are usually used in the case of the systems without delays (Lequin et al, 2003; Liu et al, 2001). An analytical approach was developed in (Silva et al, 2005; Zhong, 2006) and allowed the characterization of the stability region of delayed systems controlled via PID. Indeed, by using the Hermit-Biehler theorem applicable to the quasi-polynomials (Bhattacharyya, 1995; Silva et al, 2005), the authors have developed an analytical characterization of all values of the stabilization gains ($K_p$ ,$K_i$ ,$K_d$ ) of the regulator for the case of first order delay system. The same technique is used to provide a complete characterization of all P and PI controller that stabilize a first order delay system which considered as less complicated than the PID stabilization problem (Silva et al, 2000). In order to have good performances in closed loop, it is necessary to suitably choose the parameters of the regulator in the zone stability.

In this work, we look for optimum regulators under different criteria and we present the results of this approach when applied to the temperature control of a heater air stream $PT-326$.

This paper is structured as follows: in section 2, we present the theorem of Hermit-Biehler applicable to the quasi-polynomials. Section 3 is devoted to the problem formulation for first order delay system controlled via PI controller. In order to obtain optimal regulator in the zone of stability, a description of the genetic algorithms is presented in section 4. Section 5 is reserved for real time example to test the control method.

## PRELIMINARY RESULTS FOR ANALYZING TIME DELAY SYSTEM

Several problems in process control engineering are related to the presence of delays. These delays intervene in dynamic models whose characteristic equations are of the following form (Silva et al, 2002, 2005):

$$\begin{aligned}\delta(s) = d(s) + e^{-L_1 s}n_1(s) + e^{-L_2 s}n_2(s)\\ +... + e^{-L_m s}n_m(s)\end{aligned} \quad (1)$$

Where: $d(s)$ and $n(s)$ are polynomials with real coefficients and $L_i$ represent time delays. These characteristic equations are recognized as quasi-polynomials. Under the following assumptions:

$$\begin{aligned}&(A_1)\ \deg(d(s)) = n\ and\ \deg(n_i(s)) < n\\ &for\, i = 1, 2, ..., m\\ &(A_2)\ L_1 < L_2 < ... < L_m\end{aligned} \quad (2)$$

One can consider the quasi-polynomials $\delta^*(s)$ described by :

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$$\delta^*(s) = e^{sL_m}\delta(s) = e^{sL_m}d(s) + e^{s(L_m-L_1)}n_1(s) + e^{s(L_m-L_2)}n_2(s) + ... + n_m(s) \quad (3)$$

The zeros of $\delta^(s)$ are identical to those of $\delta^*(s)$ since $e^{sL_m}$ does not have any finite zeros in the complex plan. However, the quasi-polynomial $\delta^*(s)$ has a principal term since the coefficient of the term containing the highest powers of $s$ and $e^s$ is nonzero. If $\delta^*(s)$ does not have a principal term, then it has an infinity roots with positive real parts (Bhattacharyya, 1995; Silva et al, 2005).

The stability of the system with characteristic equation (1) is equivalent to the condition that all the zeros of $\delta^*(s)$ must be in the open left half of the complex plan. We said that $\delta^*(s)$ is Hurwitz or is stable. The following theorem gives a necessary and sufficient condition for the stability of $\delta^*(s)$ (Silva et al, 2000, 2001, 2002, 2005).

**theorem 1** Let $\delta^*(s)$ be given by (3), and write:

$$\delta^*(j\omega) = \delta_r(\omega) + j\delta_i(\omega) \quad (4)$$

where $\delta_r(\omega)$ and $\delta_i(\omega)$ represent respectively the real and imaginary parts of $\delta^*(j\omega)$ . Under conditions $(A_1)$ and $(A_2)$, $\delta^*(s)$ is stable if and only if:

1: $\delta_r(\omega)$ and $\delta_i(\omega)$ have only simple, real roots and these interlace,

2: $\delta_i^{'}(\omega_0)\delta_r(\omega_0) - \delta_i(\omega_0)\delta_r^{'}(\omega_0) > 0$ for some $w_0$ in $[-\infty, +\infty]$
where $\delta_r^{'}(\omega)$ and $\delta_i^{'}(\omega)$ denote the first derivative with respect to $w_0$ of $\delta_r(\omega)$ and $\delta_i(\omega)$, respectively.

A crucial stage in the application of the precedent theorem is to verify that and have only real roots. Such a property can be checked while using the following theorem (Silva et al, 2000, 2001, 2002, 2005).

**theorem 2** Let $M$ and $N$ designate the highest powers of $s$ and $e^s$ which appear in $\delta^*(s)$ . Let $\eta$ be an appropriate constant such that the coefficient of terms of highest degree in $\delta_r(\omega)$ and $\delta_i(\omega)$ do not vanish at $\omega = \eta$ . Then a necessary and sufficient condition that $\delta_r(\omega)$ and $\delta_i(\omega)$ have only real roots is that in each of the intervals $-2l\pi + \eta < \omega < 2l\pi + \eta$, $l = l_0, l_0+1, l_0+2...$
$\delta_r(\omega)$ or $\delta_i(\omega)$ have exactly $4lN + M$ real roots for a sufficiently large $l_0$ .

## PI CONTROL FOR FIRST ORDER DELAY SYSTEM

We consider the functional diagram of figure 1, in which the transfer function of delayed system is given by (5)

$$G(s) = \frac{K}{1+Ts}e^{-Ls} \quad (5)$$

Where $K$ , $T$ and $L$ represent respectively the state gain, the constant time and the time delay of the plant. These three parameters are supposed to be positive.

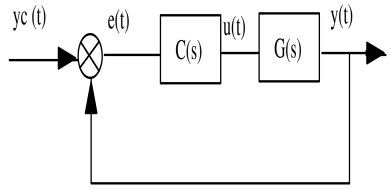


Figure 1: Closed-loop control of a time delay system

The PI Controller is described by the following transfer function:

$$C(s) = K_p + \frac{K_i}{s} \quad (6)$$

Our objective is to analytically determine the region in the $(K_p, K_i)$ parameter space for which the closed-loop system is stable.

**theorem 3**
The range of $K_p$ value, for which a solution to PI stabilization problem for a given stable open-loop plant exists, is given by (Silva et al, 2000, 2001, 2002, 2005):

$$-\frac{1}{K} < K_p < \frac{T}{KL}\sqrt{\alpha_1^2 + \frac{L^2}{T^2}} \quad (7)$$

Where $\alpha_1$ the solution of the equation $tan(\alpha) = -\frac{T}{L}\alpha$ in the interval $[\frac{\pi}{2}, \pi]$

**Proof** The closed loop characteristic equation of the system is given by:

$$\delta(s) = (KK_i + KK_ps)e^{-Ls} + (1+Ts)s \quad (8)$$

we deduce the quasi-polynomials:

$$\delta^*(s) = e^{Ls}\delta(s) = (KK_i + KK_ps) + (1+Ts)se^{Ls} \quad (9)$$

substituting $s = jw$ , we have:
$\delta^*(j\omega) = \delta_r(\omega) + j\delta_i(\omega)$
where:

$$\begin{cases} \delta_r(\omega) = KK_i - \omega\sin(L\omega) - Tw^2\cos(L\omega) \\ \delta_i(\omega) = w\left[KK_p + \cos(L\omega) - Tw\sin(L\omega)\right] \end{cases} \quad (10)$$

Clearly, the parameter $K_i$ only affects the real part of $\delta^*(j\omega)$ whereas the parameter $K_p$ affects the imaginary part. According to the first condition of theorem 1, we should check that the roots of $\delta_i$ and $\delta_r$ are simple. By using the theorem 2, while choosing , $M = 2, N = l = 1$ and $\eta = \frac{\pi}{4}$, we observe that $\delta_i(\omega)$ has simple roots for any $K_p$ checking (7) (Bhattacharyya, 1995; Silva et al, 2005).
the application of the second condition of theorem 2, led us to:
$E(\omega_0) = \delta_i^{'}(\omega_0)\delta_r(\omega_0) - \delta_i(\omega_0)\delta_r^{'}(\omega_0) > 0$
for $\omega_0 = 0$ (a value that annul $\delta_i(\omega)$) we obtain:
$E(\omega_0) = (\frac{KK_p+1}{L})KK_i > 0$ which implies $K_p > \frac{-a_0}{K}$ since $K > 0$ and $K_i > 0$.

This proofs the first inequality given by(2)in Theorem 1. We consider that $z = L\omega$ , we get:

$$\delta_r(z) = K\left[K_i - \frac{z}{KL}(\sin(z) + \frac{T}{L}z\cos(z))\right] \quad (11)$$

It results that $a(z) = \frac{z}{KL}(\sin(z) + \frac{T}{L}z\cos(z))$ then

$$\delta_r(z) = K\left[K_i - a(z)\right] \quad (12)$$

for $z_0 = 0$, we obtain :

$$\delta_r(z_0) = K(K_i - a(0)) = KK_i > 0 \quad (13)$$

for $z_j \neq z_0, j = 1, 2, 3...$, we obtain:

$$\delta_r(z_j) = K(K_i - a(z_j)) \quad (14)$$

Interlacing the roots of $\delta_r(z)$ and $\delta_i(z)$ is equivalent to $\delta_r(z_0) > 0$ (since $K_i > 0$), $\delta_r(z_1) < 0, \delta_r(z_2) > 0$... We can use the interlacing property and the fact that $\delta_i(z)$ has only real roots to establish that $\delta_r(z)$ possess real roots too. From the previous equations we get the following inequalities:

$$\begin{cases} \delta_r(z_0) > 0 \\ \delta_r(z_1) < 0 \\ \delta_r(z_2) > 0 \\ \delta_r(z_3) < 0 \\ \delta_r(z_4) > 0 \\ \vdots \end{cases} \Rightarrow \begin{cases} K_i > 0 \\ K_i < a_1 \\ K_i > a_2 \\ K_i < a_3 \\ K_i > a_4 \\ \vdots \end{cases} \quad (15)$$

where the bounds $a_j$ for $j = 1, 2, 3...$ are expressed by:

$$a_j = a(z_j) \quad (16)$$

Now, according to these inequalities, it is clear that we need only odd bounds (which to say $a_1$,$a_3$ ). It has to be strictly positive to get a feasible range for the controller parameter $K_i$. From (Silva et al, 2000, 2002), $a_j$ is positive (for the odd values of $j$ ) for every $K_p$ verifying (7). Hence, the conditions giving by (15) are reduced to:

$$0 < K_i < \min_{j=1,3,5...}\{a_j\} \quad (17)$$

Once the stability domain is determined, the question is what are the optimum parameters of the PI controller which guarantee the good performance of the closed-loop system? On the following, the genetic algorithm is proposed to answer this need.

## GENETIC ALGORITHMS

The Genetic Algorithms (AGs) are iterative algorithms of global search of which the goal is to optimize a specific function called criterion of performance or cost function (Godelberg, 1991). In order to find the optimal solution of a problem by using AGs, we start by generating a population of individuals in a random way. The evolution from one generation to the following is based on the use of the three operators' selection, crossover and mutation which are applied to all the elements of the population (Godelberg, 1991). Couples of parents are selected according to their functions costs. The crossover operator is applied with a $P_c$ probability and generates couples of children. The mutation operator is applied to the children with a $P_m$ probability and generates mutant individuals who will be inserted in the new population. The reaching of a maximum number of generations is the criterion of stop for our algorithm. Figure 2 shows the basic flow chart of AGs. The principle of regulator parameters optimization by the genetic algorithms is shown by figure 3.

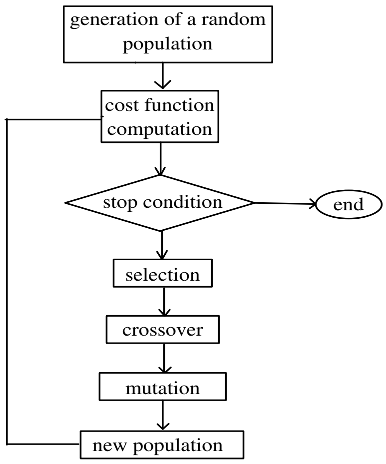


Figure 2: Steps of the genetic algorithm evolution

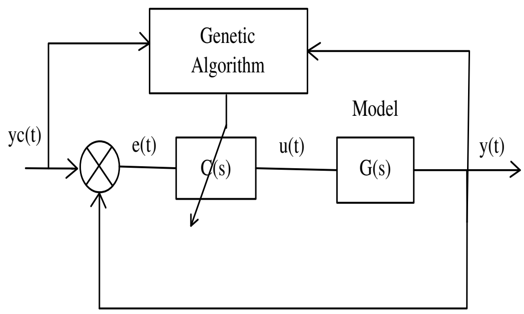


Figure 3: The optimization principle by genetic algorithm

In our case, we are interest to search the optimum controller parameters in the area of stability using one of following criterion $ISE$,$IAE$,$ITAE$ and $ITSE$ described by relation (23):

$$\begin{cases} ISE = \sum_{0}^{t_{\max}} e(t)^2 \\ IAE = \sum_{0}^{t_{\max}} |e(t)| \\ ITAE = \sum_{0}^{t_{\max}} t\,|e(t)| \\ ITSE = \sum_{0}^{t_{\max}} te(t)^2 \end{cases} \quad (18)$$

If we want to minimize the tuning energy, the $ITAE$ and the $IAE$ criteria are considered. In the case where we privilege the rise time, we take the $ITSE$ criterion. In order to guarantee the tuning energetic cost, we choose the $ISE$ criterion ( Villain, 1996).
The calculation steps of the control law are summarized by the following algorithm:

1. Introduction of the following parameters:
   - $max_{pop}$individuals number by population
   - initial population
   - $gen_{max}$ generation number
2. Initialization of the generation counter ($gen = 1$)
3. Initialization of the individual counter ($j = 1$ )
4. For $t = 1s$ to $t = t_{max}$ efficiency evaluation of $j^{th}$ population individual $fitness(J) = \frac{1}{1+J}$
5. Individual counter incrementing ($j = j + 1$ ).
   - If $j < max_{pop}$ , going back to step 4
   - If not: application of the genetic operators (selection, crossover, mutation) for the founding a new population
6. Generation counter incrementing ($gen = gen + 1$ ), If , going back to step 3.
7. taking $K_popt$ and $K_iopt$ which correspond to the best individual in the last population (individual making the best fitness).

On the following, the genetic algorithm characterized by generation number equal to 100, $P_c = 0.8$ , $P_m = 0.08$ and individual number by population equivalent to 20.

## APPLICATIONS

This section presents an application example for the temperature control of an air stream heater (process trainer $PT - 326$), figure 4. This type of process is found in many industrial systems such as furnaces, air conditioning, etc. The $PT - 326$ has the basic characteristics of a large plant, with a tube through which atmospheric air is drawn by a centrifugal blower. Before being released into the atmosphere, air becomes heated by passing into a heater grid. Temperature control is realized by varying the electrical power supplied to the heater grid. Air is pushed through a tube by a fan blower and heated at the inlet. The mass flow of air through the duct can be adjusted by setting the opening of the throttle. There is an energized electric resistance inside the tube. Heat, due to the Joule effect, is released by resistance and transmitted by convection

This process can be characterized as a linear time delay system. Time delay depends on the position of the temperature sensor element that can be inserted into the air stream at any one of the three points throughout the tube, spaced at 28, 140 and 280 $mm$ from both the heater and the damper position. The system input, $u(t)$ , is the voltage applied to the power circuit feeding the heating resistance, and the output,$y(t)$ , is the outlet air temperature, expressed by a voltage, between $-10$ and $10V$. the $PT - 326$ heating process is shown in figure 5.

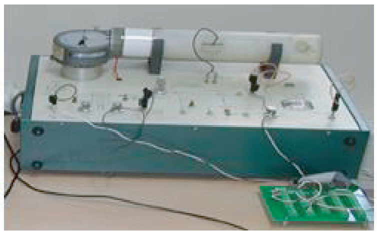

Figure 4: PT-326 heat process trainer

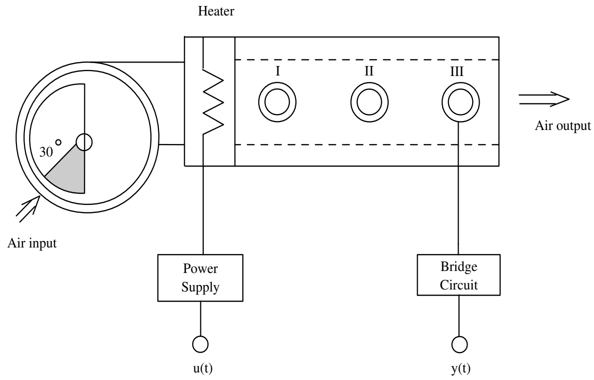


Figure 5: Schematic illustration

The behavior of the $PT - 326$ thermal process is governed by the balance of heat energy. When the air temperature inside the tube is supposed to be uniform, a linear delay system model can be obtained. Thus, the transfer function between the heater input voltage and the sensor output voltage can be obtained as: $\frac{y(s)}{u(s)} = \frac{K}{1+Ts}e^{-Ls}$
For the experiment, the damper position is set to 30, and the temperature sensor is placed in the third position. The measurements acquisition is done by "$PCI - DAS1002$" card's, the sampling time is taken equal to $T_e = 0.03$ second. This choice takes account of the time computing and the constant time of the plant. Firstly data is taken by operating the heater open loop with square input 0V/2V as shown as figure 6. The data is then used to obtain models to represent the plant An enlargement of the first step response up to 1000 iterations is shown in the following figure. We define:
Step1: step response up to 1500 iterations
Step3: step response from 3000 up to 4500 iterations
Step5: step response from 6000 up to 7500 iterations
Step7: step response from 9000 up to 1000 iterations

The transfer's functions corresponding to each step response are estimated by conventional graphical method;

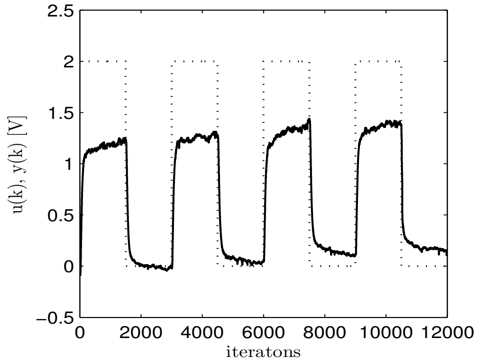


Figure 6: Square input $0V/2V$ and outputs of $PT-326$

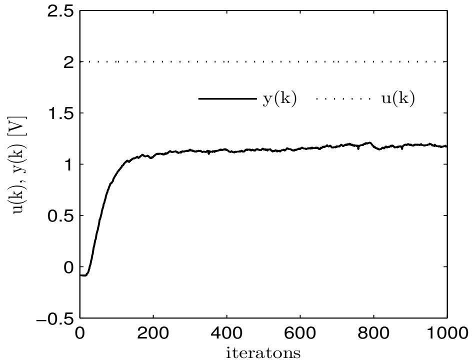


Figure 7: First step response for $PT-326$

and they are represented in table 1. So the $PT-326$

Table 1: system parameter

| | Step 1 | Step 3 | Step 5 | Step 7 | mean |
|---|---|---|---|---|---|
| $L$ | 0.57 | 0.6 | 0.54 | 0.54 | 0.5625 |
| $K$ | 0.587 | 0.586 | 0.5872 | 0.5872 | 0.5869 |
| $T$ | 1.572 | 1.5671 | 1.572 | 1.572 | 1.572 |

is described by the following transfer function: $\frac{y(s)}{u(s)} = \frac{0.58}{1+1.57s}e^{-0.56s}$
In order to determine $K_p$ values, we look for $\alpha$ in interval $[0, \pi]$ satisfying $\tan(\alpha) = 2.89\alpha \Rightarrow \alpha = 1.764$. $K_p$ range is given by:$-2 < K_p < 8.25$.The system stability region, obtained in plane is presented in figure 8. From figure 8, our $K_p$ and $K_i$ population's individuals are choosing between $[-2, 8.25]$ and $[0, 7.45]$. PI controller optimum parameters supplied by genetic algorithm are presented by the following table. The following

Table 2: optimum PI parameters

| criterion | $ISE$ | $IAE$ | $ITAE$ | $ITSE$ |
|---|---|---|---|---|
| $K_p opt$ | 3.67 | 3.33 | 2.94 | 3.95 |
| $K_i opt$ | 4.24 | 4.36 | 4.04 | 4.33 |

algorithm describes the real-time implementation of the PI control law:

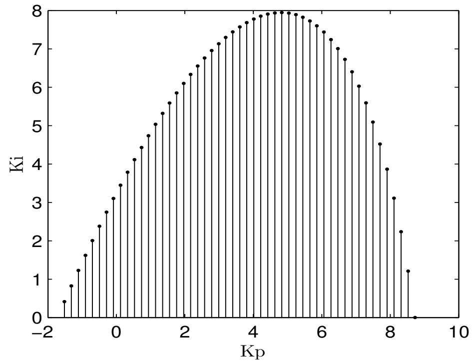


Figure 8: PI controller stability domain for $PT-326$

1. Initialization, $k = 1$
2. Output acquisition $y(k)$
3. Error computation $e(k) = y_c(k) - y(k)$
4. Control law Computation
   $u(k) = u(k-1) + K_p e(k) + (K_i T_e - K_p)e(k-1)$
5. Application of the control law $u(k)$ to the process
6. Exhausting of the sampling time, $k = k+1$ then go to step 2.

The followings figures show the responses of the closed loop systems in the case of PI controller designed by the genetic algorithms as described in table 2.

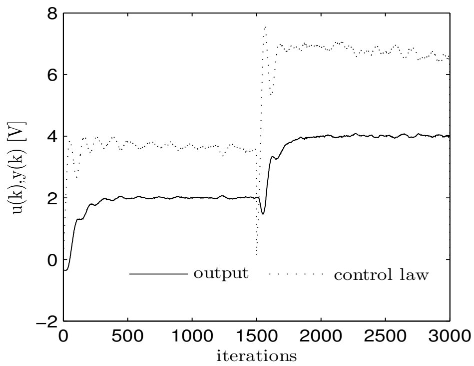


Figure 9: Evolution of the output and the control law (Case: $PI - ISE$)

It is clear from these figures that the closed loop system is stable and the output $y(k)$ tracks the step input signal. From Table 3 we conclude that the minimal variance of control law was generate by the $PI - ISE$ controller.

Table 3: performance comparison

| | $ISE$ | $IAE$ | $ITAE$ | $ITSE$ |
|---|---|---|---|---|
| $var(u(k))$ | 2.6708 | 2.779 | 2.8728 | 2.7234 |

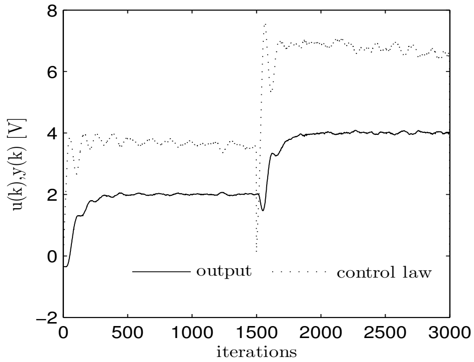


Figure 10: Evolution of the output and the control law (Case: $PI - IAE$)

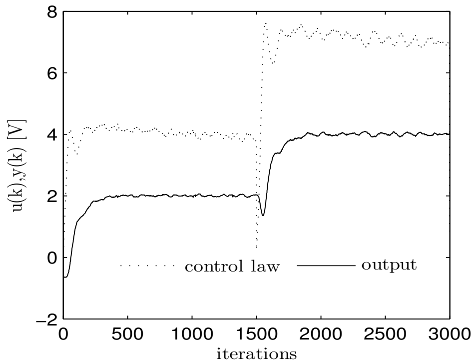


Figure 11: Evolution of the output and the control law (Case: $PI - ITAE$)

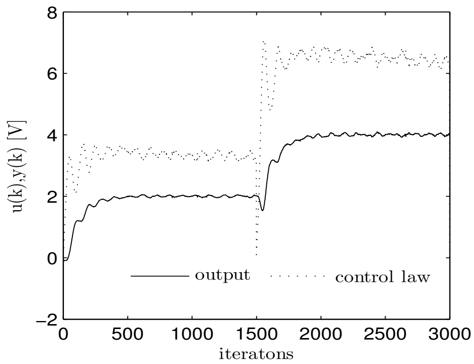


Figure 12: Evolution of the output and the control law (Case: $PI - ITSE$)

## CONCLUSION

In this work, we use the Hermit-Biehler theorem to compute the region stability for first order delay system controlled by PI regulator. Lastly, we were interested in search of optimal PI for a given performance criteria ($ISE$, $IAE$, $ITAE$, $ITSE$), inside the stability region. In regard to the complexity of the optimization problem, we used the genetic algorithms. The validation of these results was tested in real time temperature control.

## REFERENCES


Bhattacharyya, S. P. , Chapellat, H. , and Keel, L. H. (1995). *Robust Control:The Parametric Approach*. Upper Saddle River, NJ: Prentice-Hall. [1, 2]

Cedric, L. , (2002). Pontryagin revisit: implication sur la stabilit des PID retards. *Conference Internationale Francophone d'Automatique CIFA'02*. [1]

Dambrine, M. (1994). *Contribution l'tude de la stabilit des systmes retards*. Thse de doctorat, Universit de Lille. [1]

Godelberg, D.E.(1991). *Genetic Algorithms in search, optimization and machine learning*. Addison-Wesley, Massachusetts. [3]

Lequin, O. , Gevers, M., Mossberg, M., Bosmans, E. and Triest , L. (2003). Iterative feedback tuning of PID parameters: comparison with classical tuning rules. *Control Engineering Practice*,N11, pp. 1023-1033. [1]

Liu, G.P. and Daley, S. (2001). Optimal-tuning PID control for industrial systems. *Control Engineering Practice*,N 9, pp. 1185-1194. [1]

Niculescu, S.-I. (2001). *Delay effects on stability*. Springer, London. [1]

Silva, G. J. , Datta, A. and Bhattacharyya, S. P. (2000). Stabilization of Time Delay Systems. *Proceedings of the American Control Conference*, pp. 963-970. [1, 2, 3]

Silva, G. J. , Datta, A. and Bhattacharyya, S. P. (2001). Stabilization of First-order Systems with Time Delay using the PID controller. *Proceedings of the American Control Conference*, pp. 4650-4655. [2]

Silva, G. J. , Datta, A. and Bhattacharyya, S. P. (2002). New synthesis of PID controller. *IEEE transactions on automatic control*,vol. 47,No. 2. [1, 2, 3]

Silva, G. J. , Datta, A. and Bhattacharyya, S. P. (2005). *PID controllers for time delay systems*. Springer, London. [1, 2]

Villain, M. (1996). *Systmes asservis linaires*. ellipses/ dition marketing S.A. [4]

Zhong, Q.C. (2006). *Robust control of time delay system*. Springer, London. [1]


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