TAYLOR-TYPE RULES AND PERMANENT SHIFTS IN PRODUCTIVITY GROWTH

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ABSTRACT
This paper uses simulation techniques to evaluate variants of the Taylor rule in a model where the underlying trend rate of growth is subject to permanent shifts. We show that the original Taylor rule is not particularly well-formulated for dealing with this type of environment. However, we also find that a Taylor-type rule in which the output gap is replaced by output growth or one that is augmented by an explicit policy response to deviations of the price level from a target path will look much like the optimal policy.

INTRODUCTION
During the 1970s, there was a worldwide slowdown in productivity growth accompanied by a rising inflation trend. In the 1990s, the opposite occurred—an unexpected rise in real growth was accompanied by a surprising decline in the inflation trend. In both episodes, the shift in real productivity growth appeared to be permanent. There is empirical evidence (Kiley 2003) that medium-to-low frequency measures of inflation and productivity growth are negatively correlated in U.S. data. He explains the regularity as an inherent dynamic of the New Keynesian Phillips Curve.

In this paper, we show that the negative correlation is driven by the monetary policy regime. In a New Keynesian model with sticky prices, there would be no correlation between these trends. The optimal policy is probably not feasible. However, we show that there are feasible policies that approximate the optimal policy and would greatly reduce or even reverse the sign of the correlation between these trends. This is true in a model with a New Keynesian Phillips Curve.

The Taylor rule (Taylor, 1993) was originally intended as a purely descriptive device—a way of characterizing what monetary policymakers tended to do in practice. Although not the optimal policy in any particular model, the Taylor rule has been shown to work well in a variety of models and situations where the policymaker is uncertain about the trends and structure of the economy. However, one situation in which it does not work well is when there are permanent shocks to technology growth. In this paper, we use simulation techniques to show how the Taylor Rule fails to deliver good policy. We also show one might modify the Taylor rule in such an environment to improve policy.

The next section describes the model we use and motivates our interest in permanent technology growth shocks. Then, we use the model to explain the macroeconomic dynamics under optimal policy which provides a benchmark for evaluating practical policy options. We evaluate alternative policies that have been recommended to deal with transitory shocks to technology growth and show that they also work well in the presence of permanent shocks.

THE NEW KEYNESIAN MODEL
We modify a simple New Keynesian model used in earlier work (Gavin et al. 2005) to include shocks to the growth of the exogenous technology factor (Pakko 2002). The model features a well-defined demand for money, sticky prices, and a Taylor type interest rate rule for implementing monetary policy.

Households are infinitely lived agents who seek to maximize their expected utility from consumption, \( c_t \), and leisure, \( l_t \),

\[
E_t \left[ \sum_{s=0}^{\infty} \beta^s \left( \ln(c_s) + \frac{l_t - L_s}{1 - \sigma} \right) \right],
\]

subject to the following budget constraint, time constraint, and capital accumulation equation:

\[
P_t \left( c_t + i_t \right) + M_t + B = P_{w,n} + P_{q,k} \dot{k} + D_t + M_{t+1} + R_t \dot{B} + T_t,
\]

\( l_t + n_s + \phi = 1 \), and \( k_{t-s} - \dot{k} = \phi \left( \frac{i_t}{k_t} \right) \chi - \delta k_t \),

where $R_t$ is government bonds, $P_t$ is the price level, $i_t$ is investment, $M_t$ is the nominal money stock, $w_t$ is the real wage rate, $n_t$ is labor, $q_t$ is the rental rate on capital, $k_t$ is the capital stock, $D_t$ is the firms’ profits remitted to the households, $R_t$ is the gross nominal interest rate, $T_t$ is a transfer from the monetary authority, $\delta$ is the depreciation rate, and $s_t$ represents the shopping-time costs of holding money balances: $s_t = \zeta \left( \frac{P_t}{M} \right)$. The parameter $\phi$ represents capital adjustment costs which are given by $i_t - \phi(i_t / k_t)k_t$. In the steady state $\phi(i_t / k_t) = i_t / k_t$ and $\phi'(i_t / k_t) = 1$.

In the steady state, labor is fixed as the technology factor and consumption growth at rate $g$. The household Euler equation is complicated by the presence of the shopping time specification. For the moment, consider a shopping time function that depends only on real balances and not on consumption. In such a case leisure will be separable in utility and we would have the standard intertemporal Euler equation associated with log utility.

\[
\frac{c_t}{\beta c_{t+1}} = (1 + r_t)
\]

where $r_t$ is the real interest rate on one-period bonds. In a stable, long-run equilibrium, consumption will grow at rate $g$. Equation (1) shows that the interest rate will be:

\[
(1 + r_t) = \frac{(1 + g)}{\beta}.
\]

That is, the real interest rate will be positively related to the underlying growth rate of the economy.

Firms produce heterogeneous goods which are aggregated into a composite consumption good. Sticky prices are generated by a standard Calvo pricing framework in which only a fraction of the firms are permitted to reset their prices each period.

The firms are monopolistically competitive producers of output, $y_n$, according to

\[
y_{n,t} = (k_{t,n})^{\gamma} (Z_{t,n})^{\chi},
\]

where $\gamma$ indicates the number of periods since the firm last adjusted its price, $n_{t,n}$ is firm labor demand, $k_{t,n}$ is firm demand for capital and $Z_t$ is an economy-wide productivity factor.

The most important extension of the model for this paper is a technology process by which the long-run growth trend is subject to variation over time.

The technology shock in this case is a composite of a growth shock ($Z_{t,g}$) and a level shock ($Z_{t,l}$):

\[
\ln(Z_{t}) = \ln(Z_{t,g}) + \ln(Z_{t,l}),
\]

where each of these shocks follows an independent AR(1) process:

\[
\ln(Z_{t,g}) = \rho_Z \ln(Z_{t-1}) + (1 - \rho_Z)\overline{g} + v_{g,t},
\]

\[
\ln(Z_{t,l}) = \rho_Z \ln(Z_{t-1}) + (1 - \rho_Z)\ln(\overline{g}) + v_{l,t},
\]

where $\overline{g}$ is a trend rate of growth used as the baseline for model simulations. We consider a series of experiments using a special case for which the growth shocks are permanent; i.e., $\rho_Z = 1$. The level shocks are assumed to be highly persistent with $\rho_L = 0.95$.

Each period, every firm must make two decisions. First, firms determine the cost minimizing combination of capital and labor given their output level, the wage rate, and the rental rate of capital services. Second, the pricing decisions are made. In particular, the probability a firm can set a new price is $\eta$ and the probability a firm must charge the price that it last set $j$ periods ago is $(1-\eta)$. The composition of output purchased by households is

\[
y_{j,t} = \sum_{j=0}^{\infty} \eta(1-\eta)^j y_{j,j}^{(j-1)/\gamma},
\]

where $y_{j,t} = c_t + i_t$.

Before describing the policy rule, we briefly describe the assumptions used to assign values to the model parameters. In the utility function, the value of $\sigma_i$ is set at 7/9. The steady state labor share is 0.3. That calibration implies a labor supply elasticity of real wages approximately equal to 3. The household discount factor is 0.99. The shopping time parameter, $\gamma$, is set to unity, implying an interest rate elasticity of money demand equal to $-0.5$. The capital share of output is set to 0.3 and the capital stock is assumed to depreciate at 2 percent per quarter. The price elasticity of demand is set equal to 6, implying a steady state markup of 20%. We set the probability of price adjustment equal to 0.25 implying that firms change prices on average once a year. The average and marginal capital adjustment costs around the steady state are zero (i.e., $\phi(i) = i / k$ and $\phi'(i) = 1$). The elasticity of the investment-to-capital ratio with respect to Tobin’s $q$, $\chi = \left[ \left( i / k \right) \phi'(i) / \phi'(i) \right]^{-1}$, is set to 5.

The model is closed with a monetary policy rule. Taylor (1993) showed that the path of the federal funds rate during the period from 1987 through 1992 could be fairly well-described by a simple rule by which the nominal interest rate was
manipulated in response to two gaps: the gap between observed inflation, \( \pi_t \), and the target rate, \( \pi^* \), and gap between output, \( y_t \), and its long-run equilibrium level, \( y^* \). The Taylor rule can be written as:

\[
R_t = \pi_t + \theta_\pi (\pi_t - \pi^*) + \theta_y (y_t - y^*) + u_t.
\]

(2)

In this specification, Taylor treated the real interest rate, \( \pi_t \), and the inflation target, \( \pi^* \), as constants (both set equal to 2.0 in the original work). He set both reaction parameters, \( \theta_\pi \) and \( \theta_y \), equal to 0.5.

In our work we add a weakly persistent interest rate error (autocorrelation coefficient set to 0.3) written as \( u_t = \rho u_{t-1} + \epsilon_t \). The inflation target also follows a persistent AR1 process with the autocorrelation coefficient set equal to 0.95 in our simulations: \( \pi_t = a \pi_t + (1 - a) \pi_t + \epsilon_t \).

We include two policy shocks because we think it is important to make a distinction between shocks to the central bank’s long-run objective and shocks to the liquidity position (or the short term interest rate). Although the two are not independent in the long run—the long-run inflation rate must be consistent with the accumulation of short-run liquidity positions—there is no reason to think that these shocks will be correlated in the short run.

We develop a baseline policy, approximately optimal, with which to compare the Taylor and alternative rules. We say approximately because the distortions due to monopolistic competition and the shopping time friction are small relative to the distortion associated with sticky prices and, therefore, are ignored. Perfect inflation stabilization (an infinite value for \( \theta_\pi \)) would eliminate the sticky price distortion. We approximate the optimal policy by setting \( \theta_\pi = 9999 \) and \( \theta_y = 0 \).

**INTEREST RATE RULE DYNAMICS**

Figure 1 shows the model response of inflation to a 0.1% shock to the quarterly growth rate of the technology factor—this results in a permanent increase in the long-run growth rate from 1.6% to 2.0% at an annual rate. The figures show percent deviations from steady state values for the optimal policy and the Taylor Rule regime. Under the optimal policy, the inflation rate is zero.

As we saw in Equation (1), the higher growth of consumption is associated with a higher real interest rate. Under the optimal policy, the central bank keeps inflation at the steady state target and the nominal interest rate rises to match the increase in the real rate. Under the Taylor Rule the increase in wealth leads to an increase in demand that is split between higher output and higher inflation. The fundamental flaw in the Taylor rule is that it depends in part on the steady state path for real output. When there is a permanent increase in the growth rate, there is an ever increasing rise in the perceived output gap (until the central bank can learn about the trend and adjust the parameters of the rule). The initial rise of inflation causes an increase in price dispersion among firms, markups fall, and output rises above the optimal path. Both the nominal and real rates rise above the optimal path. Households work and save too much under the Taylor Rule policy. The get the real rate higher, the inflation rate must rise. In this case, it rises in the first year and falls thereafter.

To see why inflation must eventually fall in the Taylor Rule case, note that by Equation (2) the deviation between the real rate from \( \pi^* \) is equal to \( \theta_\pi \) times the inflation gap plus \( \theta_y \) times the output gap. Since the real rate remains above the optimal rate for an extended period, an ever growing output gap must be partially offset by a negative inflation gap. This problem does not occur in the flexible price model because the real rate does not have to rise above the optimal path. The flexible price outcome for real variables is essentially the same as in the optimal economy because there is no distortion associated with a dispersion of prices across firms.

The permanent growth shock is an increase in expected lifetime income for the household. Under the optimal policy, the increase in wealth leads people to consume more goods and more leisure immediately. Figure 2 shows the impulse responses of the capital stock to the same shock.
Figure 2. Capital Response to Real Growth Shock

Following the shock people want to smooth consumption of both goods and leisure. Under the optimal policy, with inflation steady, there is no distortion from sticky prices and output does not rise as it does with the Taylor Rule. The desire to work less leads to an initial decline in savings and investment. The capital stock falls below the steady state and does not return to the previous steady state path for about 5 years. Note that after 5 years, there will be a new steady state path for output and the capital stock.

The capital stock does not decline as much (almost not at all in this case) as in the optimal policy because the higher interest rates (and higher wages) encourage people to work more, supplying enough output to support a bit higher consumption and much more investment than in the optimal case.

Note also that we are conducting our experiments under the assumption that the central bank does not recognize the change in the growth rate of technology immediately. The steady state growth rate for all real variables will change. Intuitively, our investigation is to find policies that will work well while the central bank learns about these changes.

Alternative Policies

Even if the policymaker wanted to follow an optimal policy, conditions under which inflation variability can be literally reduced to zero are quite restrictive. It is unlikely that such a policy is feasible in practice. There are two important insights that suggest ways for policymakers to get closer to the optimal policy. The first was made by Athanasios Orphanides (Orphanides 2003) who specifically recommends using the growth rate of output rather than the output gap in the policy rule because there is noise in estimates of potential output. We assume that the path of potential output is smooth so that changes in the output gap are dominated by changes in actual output. Therefore, not much is lost by responding to output growth rather than the output gap. Furthermore, given the appropriate choice of the reaction coefficient on output growth, the policy can prevent any long-deviation of inflation from target because it allows the real rate to grow in proportion to the growth rate. In Equation (1), with log utility specification, the real rate rises one for one with the balanced growth rate. The first row of Table 1 shows the policy parameters we use to compute the impulse responses of inflation, the capital stock and the real interest rate under the output growth rule (labeled as Diff y in the charts).

The other insight (from Svensson 1999) is that when a central bank cannot commit to an inflation target, it can achieve the commitment outcome by adopting a target for a price level path. Essentially, this is the same as adopting a long-run or multi-period average inflation objective. The central bank must correct past deviations of inflation from target in order to stay on the path. This prevents the inflation from declining over time as we saw in the 1990s following the increase in productivity growth.

Our second rule is a rule for the price level path. We adopt the policy that we used in earlier work and was found to work well in reducing long-run inflation risk (Gavin et al. 2009). This rule reacts one for one to changes in inflation and also to deviations of the price level from a path for the price level that is growing at the target inflation rate. The rule is written as 

\[ R_t^* = r + \theta_p (p_t - p_t^*). \]

The parameters for this price level path rule are shown in the second line of Table 1.

Table 1. Policy Parameters

<table>
<thead>
<tr>
<th>Rule</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diff y</td>
<td>( \theta_{\pi} = 0.5, \theta_{dy} = 1 )</td>
</tr>
<tr>
<td>Price Path</td>
<td>( \theta_{p} = 0, \theta_{\pi} = 0.2 )</td>
</tr>
<tr>
<td>Combination</td>
<td>( \theta_{p} = 0, \theta_{dy} = 1, \theta_{\pi} = 0.2 )</td>
</tr>
</tbody>
</table>

The impulse responses of inflation and the capital stock for each rule are shown in Figures 3 and 4 with the optimal case.
Figures 3 and 4 show that both policies look much more like the optimal policy than does the Taylor Rule. Including the price path in the rule allows some inflation following the growth shock, but it keeps longer-run expectations near the target. A key result, that both policies keep the real interest near the optimal path.

**COMPUTATIONAL EXPERIMENTS**

In this section we examine the performance of each of these rules as well as a combination rule when there is a larger mix of shocks disturbing the economy. The shocks are distributed normally with mean zero. The standard deviations (s.d.) of the shocks are scaled for quarter data. The shock to the technology level (s.d.=.0075) explains about 70 percent of the variance in output. The output growth shock (s.d.=.001) is calibrated to replicate the uncertainty in aggregate output and the variability induced by growth shocks reported by Gorodichenko and Shapiro (2007). The inflation target shock (s.d. = .001) is calibrated to roughly match the range of desired inflation objectives that appear in Federal Reserve official testimony and speeches. The liquidity shock is calibrated to approximately match the error that is induced by the use of 25 basis point discrete changes in the fed funds target. As a consequence, even at the (approximately) optimal policy, there will still be some inflation volatility due to policy shocks.

We examine four policies: the Taylor Rule, the output growth rule, the price path rule, and a combination rule that includes the reaction to output growth in the price path rule. See the bottom line of Table 1 for the parameters in the combination rule.

In each experiment, we simulate two models—the one with the optimal policy response and one of four alternative policies. In each of 10,000 iterations, we start at the steady state and generate 40 quarters of data using random generated innovations for each of the shocks. Then we report the root mean squared deviation of inflation and output growth in the alternative policy case from the optimal policy outcome. The results for inflation and output for the Taylor Rule and our three alternative policies are shown in Figures 5 and 6, respectively.

**Figure 5. Long-Run Uncertainty about Inflation**

**Figure 6. Long-Run Uncertainty in Output Growth**

In Figure 7, we show the correlations between inflation and output growth over different time horizons. At one quarter, average inflation and output growth are positively correlated for all four policy regimes. Over horizons longer than one year, the trends are positively correlated for all regimes, except for the Taylor Rule.

**Figure 4. Capital Response to Growth Shock**
This paper shows that the Taylor Rule performs poorly in the presence of permanent output growth shocks. However, policies that have been suggested to deal with temporary growth shocks also work well in the presence of permanent shocks. The negative correlation between inflation and productivity growth may be explained by monetary policy that pays too much attention to the output gap.

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AUTHOR BIOGRAPHIES

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