COLLABORATIVE GRANULAR MODELING AND SIMULATION

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ABSTRACT

With the remarkably diversified plethora of design methodologies and algorithmic pursuits present today in system modeling including fuzzy modeling, we also witness a surprisingly high level of homogeneity in the sense that the resulting models are predominantly concerned with and built by using a data set coming from a single data source.

In this study, we introduce a concept of collaborative granular modeling. In a nutshell, we are faced with a number of separate sources of data and the resulting individual models formed on their basis. An ultimate objective is to realize modeling at the global basis by invoking effective mechanisms of knowledge sharing and collaboration. In this way, each model is formed not only by relying on a data set that becomes locally available but also is exposed to some general modeling perspective by effectively communicating with other models and sharing and reconciling revealed local sources of knowledge.

Several fundamental modes of collaboration (by varying with respect to the levels of interaction) are investigated along with the concepts of collaboration mechanisms leading to the effective way of knowledge sharing and reconciling or calibrating the individual modeling points of view. The predominant role of information granules with this regard is stressed.

For illustrative purposes, the underlying architecture of granular models investigated in this talk is concerned with rule-based topologies and rules of the form "if R_i then f_i" with R_i being a certain information granule (typically set, fuzzy set or rough set) formed in the input space and f_i denoting any local model realizing a certain mapping confined to the local region of the input space and specified by R_i.

1. INTRODUCTORY COMMENTS

Fuzzy modeling (Angelov et al., 2008; Crtespo and Weber, 2005; Kacprzyk and Zadrozny, 2005; Kilic et al, 2007; Molina et al., 2006; Pedrycz and Gomide, 1998; Pham and Castellani, 2006) exhibits a surprisingly diversity of design methodologies. The concepts and architectures of neurofuzzy systems, evolutionary fuzzy systems are becoming more present in the literature. In spite of this variety, there is one very visible development aspect that cuts across the entire field of fuzzy modeling, that is fuzzy models are built on around a single data set. What becomes more apparent nowadays is a tendency of modeling a variety of distributed systems or phenomena, in which there are separate data sets, quite often quite remote in terms of location or distant in time. The same complex phenomenon could be perceived and modeled using different data sets collected individually and usually not shared. The data might be expressed in different feature spaces as the view at the process could be secured from different perspectives. The models developed individually could be treated as a multitude of sources of knowledge. Along with the individual design of fuzzy models, it could be beneficial to share sources of knowledge (models), reconcile findings, collaborate with intent of forming a model, which might offer a global, unified, comprehensive and holistic view at the underlying phenomenon. Under these

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circumstances an effective way of knowledge sharing and reconciliation through a sound communication platform becomes of paramount relevance, see Figure 1.

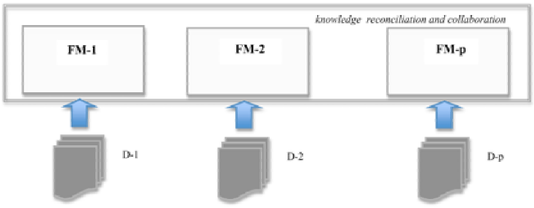


Figure 1: A General Platform of Knowledge Reconciliation and Collaboration in Fuzzy Modeling

A situation portrayed in Figure 1 is shown in a somewhat general way not moving into the details. It is essential to note that the mechanisms of collaboration and reconciliation are realized through passing information granules rather than detailed numeric entities.

The general category of fuzzy models under investigation embrace models described as a family of pairs $<R_i, f_i>$, i=1, 2, …,c. In essence, these pairs can be sought as concise representations of rules with R_i forming the condition part of the i-th rule and f_i standing in the corresponding conclusion part. It is beneficial to emphasize that in such rules, we admit a genuine diversity of the local models formalized by f_i. From the modeling perspective the expression $f_i(\mathbf{x},\mathbf{a}_i)$ could be literally *any* modeling construct, namely

- fuzzy set,
- linear or nonlinear regression function,
- difference or differential equation,
- finite state machine,
- neural network

One can cast the fuzzy models in a certain perspective by noting that by determining a collection of information granules (fuzzy sets) R_i, one establishes a certain view at the system/phenomenon. Subsequently, the conclusion parts (f_i) are implied by the information granules and their detailed determination is realized once R_i have been fixed or further adjusted (refined).

In light of the discussion on knowledge reconciliation and mechanisms of collaboration, it becomes apparent that the interaction can focus on information granules R_i and communication schemes that invoke exchange of granules whereas conclusion parts can be adjusted accordingly once the collaborative development of information granules has been completed.

The main objectives of the study, that is reflected by the organization of the material, is to formulate and discuss a variety of collaborative models of fuzzy models as well as highlight the design principles. The constructs resulting through such collaboration give rise in one way or another to granular constructs of higher order, where the elevated level of granularity is a consequence of reconciliation of knowledge coming from the individual models. The principle of justifiable granularity is presented and shows how granularity emerges as a result of summarization of numeric information (and numeric membership values, in particular). A number of collaborative schemes are discussed where we identify main concepts and present some general ways in which such schemes can be realized. We also show how type-2 fuzzy sets (including interval-valued fuzzy sets) are formed as an immediate result of collaboration.

Throughout this study, we adhere to the standard notation. In particular information granules – fuzzy sets are denoted by capital letters. The notation and terminology is the one being in common usage in the area of fuzzy sets.

2. THE PRINCIPLE OF JUSTIFIABLE GRANULARITY

The essence of the principle of justifiable granularity (Pedrycz, 2005) is that a meaningful representation of a collection of numeric values (real numbers), say $\{x_1, x_2, …, x_N\}$ can be realized as a certain information granule (Bargiela and Pedrycz, 2003, 2008; Zadeh, 1997, 2005) rather than a single numeric entity, no matter how such single individual has been selected. What is being done in statistics is an example of this principle that is realized in the language of *probabilistic* information granules. A sample of numeric data is represented not only by its mean or median (which is a very rough description) but also by the standard deviation. Both the mean and the standard deviation imply a realization of a certain probabilistic information granule, such as e.g., a Gaussian one. The probabilistic information granules are just one of the possibilities to construct an information granule to represent a collection of numeric data.

In case of other formalisms of information granulation, the development of the corresponding granules is guided by a certain optimization criterion. In general, in such criteria, we manage two conflicting requirements. The one is about forming an

information granule of sufficiently high level of experimental evidence accumulated behind it and in this way supporting its existence. The second one is about maintaining high specificity of the resulting information granule.
We discuss several general cases to venture into more algorithmic details of the realization of information granules. We show a construction of interval-based information granules as well as information granules represented as fuzzy sets.
(a) the design of interval-based information granule of numeric data. The data are illustrated in Figure 2.

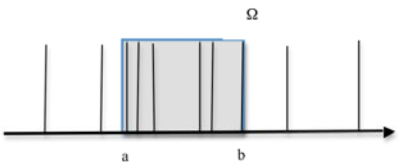


Figure 2: Realization of the Principle of Justifiable Granularity For Numeric Data and Interval Form of Information Granules

We span the numeric interval Ω (=[a, b]) in such a way that (i) the numeric evidence accumulated within the bounds of Ω is as high as possible. We quantify this requirement by counting the number of data falling within the bounds of Ω, that is card{ $x_k \in \Omega$}, which has to be maximized. At the same time, we require that (ii) the support of Ω is as low as possible, which makes Ω specific (detailed) enough. These two requirements are in conflict. A possible way to combine them into a single criterion is to consider the ratio

$$Q = \frac{\mathrm{card}(x_k \in \Omega)}{\mathrm{supp}(\Omega)} = \frac{\mathrm{card}(x_k \in \Omega)}{|b-a|} \qquad (1)$$

which is maximized with regard to the end-points of the interval, namely max $_{a,b}$ Q. The modified version of (1), which offers more flexibility in the development of the information granule involves a decreasing function of the length of the interval |b-a, say| f(|b-a|) which along with some parameters helps control an impact of granularity of the interval on the maximization of Q. For instance, we consider f(|b-a|) = exp(-α|b-a|) with a being a certain parameter assuming positive values. The Q reads as

$$Q = \mathrm{card}(x_k \in \Omega)\exp(-\alpha\,|b-a|) \qquad (2)$$

(b) here we design the interval information granule considering that the numeric data come with membership values, that is we are concerned with the pairs (x_k, μ_k) where μ_k stands for the k-th membership value. This specific design scenario is included in Figure 3.

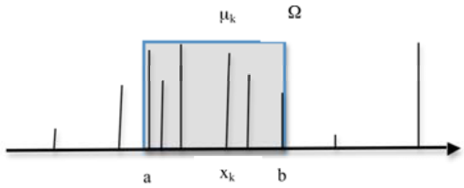


Figure 3: Realization of the Principle of Justifiable Granularity for Numeric Data with Membership -graded (weighted numeric data) and Interval Form of Information Granules

The same design development as discussed in (a) applies here. As each data point comes with the associated membership value, the numeric evidence accumulated within Ω has to be computed in such a way that they are present in the calculations and contribute to the accumulated experimental evidence behind Ω. We determine the σ-sum of the evidence, that is

$\sum_{x_k \in \Omega} \mu_k$ This leads to the maximization of the following performance index

$$Q = \frac{\sum_{x_k \in \Omega} \mu_k}{\mathrm{supp}(\Omega)} \qquad (3)$$

Alternatively, we can focus on the formation of the information granule Ω, which leads to the minimum of changes of the membership grades of the corresponding data. To admit x_k with membership μ_k to Ω, we need to change (elevate) the membership grade and this change is equal to 1-μ_k. Similarly, if we exclude x_k from Ω, the corresponding change (suppress) in membership value is μ_k. Refer to Figure 4. The criterion of interest is that of the sum of all possible changes made to the membership grades. We construct Ω in such a way that the changes in membership values are as low as possible. Formally, the performance index is expressed as

$$Q = \sum_{x_k \in \Omega}(1-\mu_k) + \sum_{x_k \notin \Omega} \mu_k \qquad (4)$$

and its minimization leads to the interval-type of information granule,

$$\mathrm{Min}_{a,b:\, a<b}\, Q \qquad (5)$$

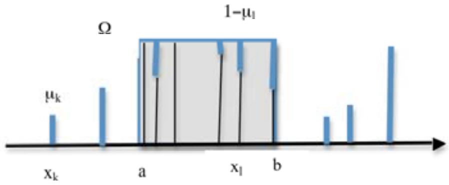


Figure 4: The Design of Interval Information Granule Realized Through the Minimization of the Criterion of Modification of Membership Grades

If the constructed information granule of interest is a fuzzy set rather than the interval, the above considerations are slightly revisited to account for membership degrees of the information granule A. An example of this type of optimization is illustrated in Figure 5.

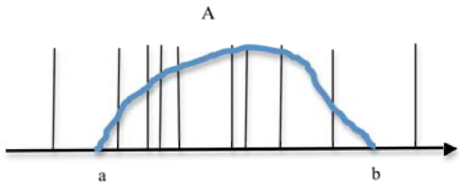


Figure 5: Realization of the Principle of Justifiable Granularity for Numeric Data and Information Granule Represented as Fuzzy Set A

The position of the modal value of A is determined by taking the numeric representative of the data (say, mean or median). Typically, to arrive at semantically meaningful A, we require that the membership function A is unimodal. Given some type of the fuzzy set (say, triangular, parabolic, etc), the optimization of the spreads of the fuzzy set is realized independently for the left- and right-hand spread. The performance index considered here is a slightly modified version of (1), that is

$$Q = \frac{\sum_{k=1}^{N} A(x_k)}{\text{supp}(A)} \quad (6)$$

Considering the fixed form of the membership functions, here are two optimization problems of parametric character: $\text{Min}_a Q$ and $\text{Min}_b Q$.
Some further flexibility can be added to the problem by introducing a parameter-enhanced version of Q, which reads as follows

$$Q = \frac{\sum_{k=1}^{N} A^{\gamma}(x_k)}{\text{supp}(A)} \quad (7)$$

where $\gamma > 0$. For $\gamma \in [0,1]$ there is less emphasis is placed on the membership values in the sense these values are "inflated". Note that if $\gamma \rightarrow 0$ then (6) reduces to the previous interval type of information granule. In a more general setting one can consider any continuous and increasing function g_1 of $\sum_{k=1}^{N} A(x_k)$, that is $g_1(\sum_{k=1}^{N} A(x_k))$ and a decreasing function g_2 of supp(A) that is $g_2(\text{supp}(A))$.
In case, the numeric data are associated with some membership values (μ_k), those are taken into account in the modified version of the performance index, which includes these values

$$Q = \frac{\sum_{k=1}^{N} \mu_k A(x_k)}{\text{supp}(A)} \quad (8)$$

All the algorithms realizing the principle of justifiable granularity produce an information granule (either an interval or a fuzzy set) based on a collection of numeric data. The nature of the numeric data themselves can be quite different. Two situations are worth highlighting here:

(a) The numeric data could result from measuring some variables present in the system. In this case, information granules are treated as non-numeric data, which can be then used in the design of the model and highlight the structure of a large number of numeric data.
(b) The numeric data are membership values of some fuzzy sets reported for a certain point of the universe of discourse. The granular representation resulting from the discussed construct gives rise to the information granule of higher type, fuzzy set of type-2, to be more specific. Depending on the nature of the information granule formed here, we arrive at interval-valued type-2 fuzzy sets or just type-2 fuzzy sets.

The principle of justifiable granularity can be used in case of functions, which are then made granular. Given the pairs of input-output data ($\mathbf{x}_k$, y_k), k=1, 2,…, N and having the best numeric mapping "f" we realize its granular mapping, say interval-like format (f_-, f_+). The level of granularity expressed here as the integral of difference between the bounds. The objective is to make the value of the integral as low as possible while "covering" as many output data as possible, see Figure 6.

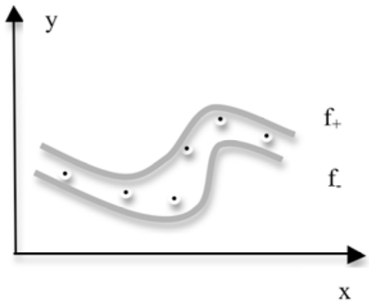


Figure 6: Realization of the Principle of Justifiable Granularity: From Numeric Mapping to its Granular Realization

3. KNOWLEDGE RECONCILIATION: MECHANISMS OF COLLABORATION

The collaboration in the formation of fuzzy models is mostly focused on the collaborative formation of information granules as they form a backbone of fuzzy models. The conclusion parts are mostly realized based on the locally available data and they are constructed once the information granules have been established.

Here there are a number of possible mechanisms of interaction between the individual models when exchanging the findings about the structure of information granules. In contrast to the hierarchical mode of collaboration (to be discussed in Section 4), the mechanisms presented here can be referred to as a one-level collaboration. The form of interaction depends on the level of compatibility considering available spaces of data and spaces of features (inputs) and commonalities among them. Refer to Figure 7.

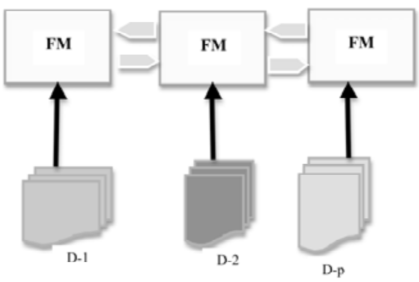
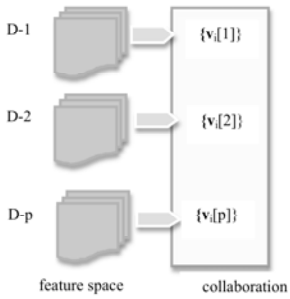


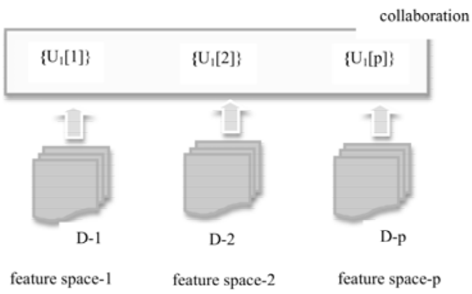
Figure 7: Collaboration Among Fuzzy Models Realized Through Communication At the Level of Information Granules

The findings of the corresponding character are exchanged (communicated among the models) and actively used when carrying out information granulation at the level of the individually available data sets. In what follows, we elaborate on the main aspects of the collaboration modes referring the reader to the literature on their algorithmic details (Pedrycz, 2005; Pedrycz and Rai, 2008). The taxonomy provided here is based on the commonalities encountered throughout the individual data sources. Those could be present in terms of the same feature space or the same data being expressed in different feature spaces.

Collaboration through exchange of prototypes Here, as shown in Figure 8, the data are described in the same feature space and an interaction is realized through prototypes produced locally.



(a)



(b)

Figure 8: A Schematic View at Collaboration Through Exchange of Prototypes (a) and (b) Partition Matrices

Collaboration through exchange of partition matrices Here the data are described in different feature spaces (they might overlap but are not the same). The data in each data set are the same but described in different feature spaces. The exchange of findings and collaboration is realized through interaction at the level of partition matrices. Note that these matrices abstract from the feature spaces (the spaces do not appear there in an explicit way) but the corresponding rows of the partition matrices have to coincide meaning that we are concerned with the same data).

Formally the underlying optimization problem can be expressed by an augmented objective function, which is composed of two components

$$Q = Q(D - ii) + \alpha \sum_{\substack{jj=1, \\ jj \neq ii}}^{p} \| \mathbf{G}(ii) - \mathbf{G}(jj) \|^2 \qquad (9)$$

The first one, Q(D-ii) is focused on the optimization of the structure based on the locally available data (so the structure one is looking based on D-ii). The second one is concerned with achieving consistency between the granular structure **G**(ii) and the structure revealed based on other data. The positive weight (α) is used to set up a certain balance between these two components of the augmented objective function (local structure and consistency among the local structures). The notation **G**(ii) is used to concisely denote a collection of information granule obtained there, say **G**(ii) ={ G_1[ii], G_2[ii],…, G_c[ii]}. As mentioned, such granules could be represented (described) by their prototypes or partition matrices.

If we consider the FCM-like optimization (Bezdek, 1981), the objective function can be written down in a more explicit fashion as follows

$$Q = \sum_{i=1}^{c} \sum_{\substack{k=1 \\ \mathbf{x}_k \in \mathbf{D}-ii}}^{N} u_{ik}^{m} \| \mathbf{x}_k - \mathbf{v}_i[ii] \|^2 + $$

$$+\alpha \sum_{\substack{jj=1, \\ jj \neq ii}}^{p} \| \mathbf{G}(ii) - \mathbf{G}(jj) \|^2$$

(10)

In case of communication at the level of the prototypes, Figure 8(a), the objective function becomes refined and its term guiding the collaboration effect arises in the form

$$\sum_{\substack{jj=1, \\ jj \neq ii}}^{p} \| \mathbf{G}(ii) - \mathbf{G}(jj) \|^2 = \sum_{i=1}^{c} \sum_{\substack{jj=1, \\ jj \neq ii}}^{p} \| \mathbf{v}_i[ii] - \mathbf{v}_i[jj] \|^2$$

(11)

For the communication with the aid of partition matrices, Figure 8(b) the detailed expression for the objective function reads as follows

$$\sum_{\substack{jj=1, \\ jj \neq ii}}^{p} \| \mathbf{G}(ii) - \mathbf{G}(jj) \|^2 =$$

$$\sum_{i=1}^{c} \sum_{k=1}^{N} \sum_{\substack{jj=1, \\ jj \neq ii}}^{p} (u_{ik}[ii] - u_{ik}[jj])^2 \| \mathbf{v}_i[ii] - \mathbf{v}_j[jj] \|^2$$

(12)

It could be noted that there is a certain direct correspondence between the prototypes and the partition matrix in the sense that each of one could be inferred given that the other one has been provided. More specifically, we envision the following pair of mappings supplying equivalence transformations,

$$\{U, D\} \rightarrow \mathbf{V} = \{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_c\}$$
$$\{\mathbf{V}, D\} \rightarrow U \qquad (13)$$

This transformation can bring a certain unified perspective at the mechanisms of exchange of information granules. For instance, one can convey a collection of the prototypes and they can induce a partition matrix over any data set.

4. KNOWLEDGE RECONCILIATION: A HIERARCHY OF FUZZY MODELS

The overall schematic view of the hierarchical knowledge reconciliation is presented in Figure 9. The knowledge acquired at the level of the models FM-1, FM-2, …, FM-p is concisely arranged in a certain knowledge signature that is a collection of information granules and the associated local models. To emphasize their origin, let us an extra index in squared brackets. For instance, the knowledge signature coming from the ii-th location is denoted as { <R_i[ii], f_i[ii]>}.

In the hierarchical reconciliation of knowledge, we distinguish two general approaches, which depend on a way in which the knowledge is being utilized. The corresponding visualization of the essence of this mechanism is presented in Figure 9.

Passive approach In this approach, we are provided with the knowledge signature, mainly the information granules Ri[ii], which are reconciled at the higher level of the hierarchy.

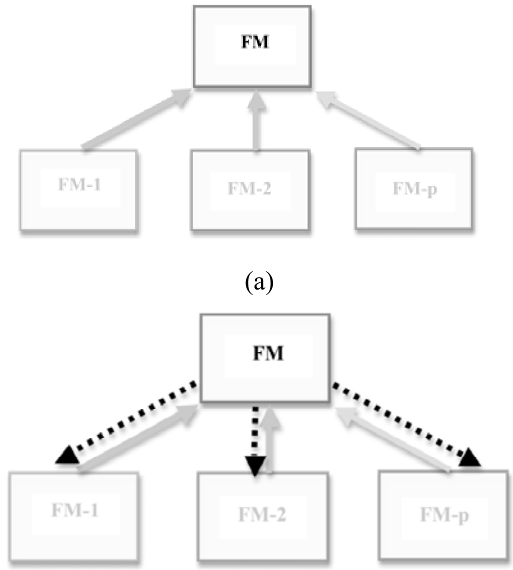


Figure 9: Passive and Active Hierarchical Knowledge Reconciliation Through Interaction Among Fuzzy Models

The prototypes obtained at the lower level are available at the higher level of hierarchy. Here two main directions are sought:

(a) we choose a subset of prototypes, which are the most representative (in terms of a certain criterion) of all $R_i[ii]$s, i=1, 2, …, c_{ii}, ii=1, 2, …, P. Some reconstruction criterion can be involved here using which we express to which extent Ri[ii] are "reconstructed" by the most representative subset of the prototypes. The problem formulated in this way is of combinatorial nature and may invoke the use of methods of evolutionary optimization through which an optimal subset of the prototypes can be established.

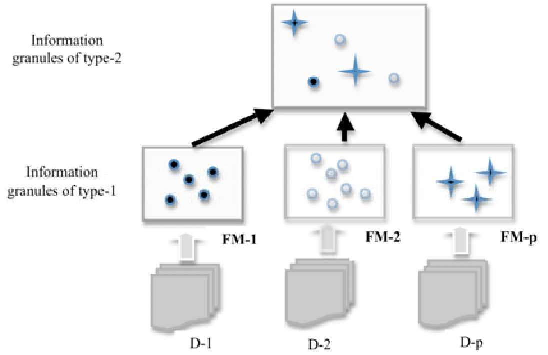
(b) the reconciliation of knowledge is realized through clustering of these prototypes, which results in a family of high-level prototypes (and information granules) over which a fuzzy model is constructed. The associated local models are formed on a basis of some data available at the lower level. The approach is called *passive* as the formation of the model at the higher level does not impact (adjust) in any way the local knowledge (models) available at the lower level.

Active approach In contrast to the passive approach, here we allow for some mechanisms of interaction. The results of clustering of the prototypes realized at the higher level of the hierarchy are used to assess the quality of the information granules of the models present at the lower level. For instance, one determines how well a certain information granule is expressed (reconstructed) by the information granules developed at the higher level of the hierarchy. This feedback signal, shown in Fig 9 (b) by a dotted line, is sent back to the lower level where the formation of information granules is guided by the quality of the clusters quantified by the feedback message. The clustering mechanism applied at the lower level is updated and the clusters, which were deemed the least fit are adjusted to become more in line with the findings produced at the higher level of the hierarchy. To accommodate this feedback, the modification of the clustering method can be realized by incorporating the changes to the objective function. For instance, in the FCM, the objective function is adjusted as follows where γ_i is a certain function quantifying the quality of the i-th cluster and supplied from the higher level of the hierarchy. For instance, if this cluster is evaluated there as being irrelevant from the global perspective, then the low values of γ_i used in the objective function discounts the relevance of the i-th cluster at the lower level.

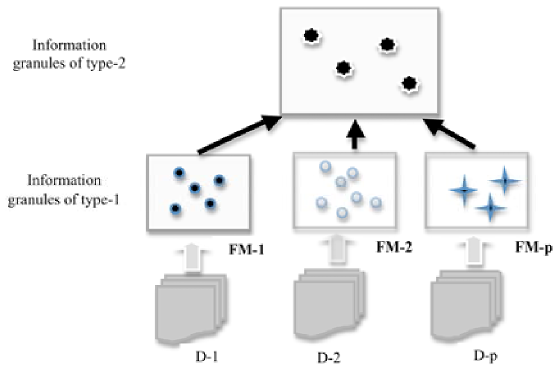
Irrespective of the passive or active approach, in both cases the hierarchical structure gives rise to the hierarchy of information granules. As visualized in Figure 9, there are two stages of information granulation: first information granules are built on basis of data (and those are used in forming fuzzy models at the lower level) and then they are reconciled.

The term information granules of type-2 has to be referred to the original data, so type-1 granulation pertains to the process involving original data whereas the type-2 constructs pertain to information granules built on a basis of prototypes of the information granules at the lower level; in this sense these could be sought of type-2 vis-à-vis original data. Again, here we can distinguish between two ways of forming information granules of higher type, that is (a) selection, and (b) successive granulation. In the selection mode, we choose the most representative subset of information granules from each of the models. As such, this is a combinatorial optimization problem. In the method of successive granulation, the prototypes of information granules produced at the lower level are the objects to be clustered.

It is worth noting that one could have a combination of the hierarchical as well as one-level collaboration mechanisms.



(a)



(b)

Figure 10: From Data to Information Granules of Type-1 and Type-2 - Two Ways of Design: (a) by Selection, and (b) Successive Granulation Using Prototypes Present at the Lower Level of Hierarchy

5. THE DEVELOPMENT OF FUZZY MODELS OF TYPE-2

The hierarchical way of knowledge reconciliation outlined in Section 4, not matter whether being realized in a passive or active way, leads to fuzzy models, which are inherently associated with information granules and directly engage the principle of justifiable granularity. Let us elaborate on two general cases, which illustrate a way in which granulation of information comes to the picture.

Fuzzy models with granular outputs. The prototypes (or *metaprototypes*, being more descriptive) developed at the higher level are associated with the corresponding outputs. Consider a certain prototype, $\mathbf{v}_i$. We determine the outputs of the fuzzy models present at the lower level, that is FM-1($\mathbf{v}_i$), FM-2($\mathbf{v}_i$), and FM-p($\mathbf{v}_i$). We repeat the same calculations for all other prototypes. Apparently, the mapping from the set of prototypes to the output space is one-to-many as for each input $\mathbf{v}_i$ we typically encounter several different numeric outputs. Through the use of any of the technique of granulation (see Section 2), we arrive at a collection of numeric inputs-granular output pairs of the form

$$\{ (\mathbf{v}_1, \mathcal{G}(\text{FM-1}(\mathbf{v}_1), \text{FM-2}(\mathbf{v}_1), .., \text{FM-p}(\mathbf{v}_1))), (\mathbf{v}_2, \mathcal{G}(\text{FM-1}(\mathbf{v}_2), \text{FM-2}(\mathbf{v}_2), \ldots, \text{FM-p}(\mathbf{v}_2))), \ldots (\mathbf{v}_i, \mathcal{G}(\text{FM-1}(\mathbf{v}_i), \text{FM-2}(\mathbf{v}_i), \ldots, \text{FM-p}(\mathbf{vi}_1))), \ldots (\mathbf{v}_c, \mathcal{G}(\text{FM-1}(\mathbf{v}_c), \text{FM-2}(\mathbf{v}_c), \ldots, \text{FM-p}(\mathbf{v}_c))) \} \quad (14)$$

where $\mathcal{G}$(FM-1($\mathbf{v}_i$), FM-2($\mathbf{v}_i$), and FM-p($\mathbf{v}_i$)) is a result of granulation of the corresponding set of numeric outputs of the fuzzy models. Considering the data set (17) visualized schematically in Figure 11, a fuzzy model can be built in different ways, say a collection of rules, neural network with granular outputs, schemes of case-based reasoning (CBR) or fuzzy regression.

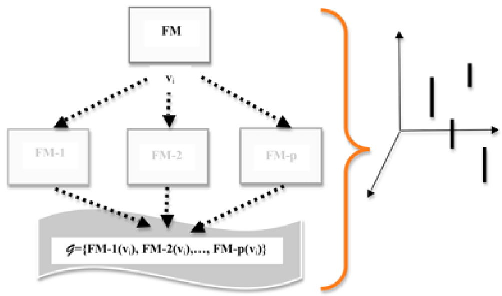


Figure 11: The Formation of Granular Outputs and a Realization of Granular Model of the CBR Architecture

6. CONCLUSIONS

The study has focused on the new category of collaborative fuzzy modeling, which has emerged when dealing with numerous sources of knowledge (local fuzzy models build on a basis of locally accessible data).

Granularity of information plays a pivotal role in fuzzy modeling: while type-1 information granules are the building blocks of fuzzy models, information granules of higher type are essential to formalize and quantify the effect of collaboration and reconciliation of knowledge, which inherently has to quantify a variety of

sources of knowledge coming from individual fuzzy models. The passive and active modes of collaboration help reach a significant level of consensus and, what is equally essential, quantify the level through higher type of information granules. The detailed algorithmic aspects can be realized in different ways and those topics could be a subject of further studies.
The principle of justifiable granularity in application to granular mappings can be exploited in the design of granular models such as granular neural networks, type-2 rule-based systems, or granular regression. In particular, one can consider an admissible level of granularity (treated as a knowledge representation resource), which has to be distributed in an optimal fashion so that the highest level of coverage of numeric data can be achieved. In the formulation of the problem done in this way, one can envision the use of methods of evolutionary optimization as a vehicle to allocate optimal levels of granularity to the individual parameters of the model (say, connections of the neural network).

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