Employing simulation to analyze the effects of model incongruence – with examples from airline revenue management

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ABSTRACT
Operations research systems are employed to optimize the performance of logistics, transport and supply chain management through solutions based on mathematical models. The scope of these models is constrained not only by the analysts’ knowledge of the system, but also by the computational efficiency of the solution. This restriction can lead to a gap between mathematical models used for solving from conceptual models derived from empirical situations motivating the solutions. This paper claims that simulation models may be employed to analyze and evaluate the effects of the resulting model incongruence. To this end, two types of model incongruence are differentiated, compositional and structural, and two levels of effect are considered separately, short-term and long-term. The concept is illustrated by examples from airline revenue management: A simulation system based on a discrete-event model with multiple agents is used to analyze the effects of parallel flights, competition, and fluctuating demand.

Introduction
According to (Hillier and Lieberman, 2009), operations research “frequently attempts to find a best solution (referred to as an optimal solution) for the problem under consideration”. One step toward this solution is to build “a scientific (typically mathematical) model that attempts to abstract the essence of the real problem”.

As stated above, the scientific models used to generate operations research solutions cannot represent every detail of the empirical situation that motivated the solution. While some degree of abstraction is a defining characteristic of models, the question of how much abstraction is too much keeps coming up in operations research theory and practice.

In (Bertrand and Fransoo, 2002), the authors differentiate “axiomatic” and “empirical” research and claim that the gap between the two views of operations management problems has grown over time. They call for a “well-defined, shared methodological framework for identifying and measuring the relevant characteristics of real-life operational processes”. An approach using simulations to model those characteristics that are known but not included in the axiomatic view of the problem would attempt to link what the authors describe as “axiomatic quantitative research” with what the authors refer to as “empirical model-based quantitative research”. While computer simulation is mentioned in (Bertrand and Fransoo, 2002), it is considered exclusively as a – costly – solution approach, not as an opportunity for evaluating mathematical models in a controlled environment based on empirical knowledge.

(Fisher, 2007) also advocates to reduce the gap between theoretical and empirical research in the areas of operations research and operations management: “the way to avoid the risk of separating ‘into a multitude of insignificant branches’ is to have a healthy injection of empirics”. The author defines a taxonomy of empirical research, differentiating prescriptive and descriptive goals as well as highly and less structured approaches to interacting with the world. As data sources that may be used to inform this research, observations as well as laboratory experiments are listed; simulation models are not considered.

This paper suggests the use of simulation models to evaluate models used in operations research by extending their scope. In this way, the range of data sources providing empirical links for operations research may be systematically extended by simulations. As stated by (Gilbert and Troitzsch, 2005), “simulation allows the researcher to conduct experiments in a way that is normally impossible”. While the simulation models suggested for evaluation will always be based on abstract and thereby limited models of the real world, their scope is not limited by aspects of solution efficiency. They can include more realistic challenges to those abstract models used for solution generation.

By providing a way to measure the effect of differences between the mathematical model and the empirical problem, this approach offers a new view on the application of operations research techniques. According to (Bertrand and Fransoo, 2002), the fact that operations research solutions are usually based only on parts of empirical problems has traditionally been neglected: “the implicit assumption being that these aspects would not af-
fect the effectiveness of the problem solutions”. Whether or not this implicit assumption is correct can be tested in a simulation based on a model closer, if not congruent, to reality.

The use of simulation models as suggested here can provide two types of answers. On the one hand, it can indicate which previously neglected aspects of the empirical model contain the greatest potential for improving the current solution. On the other hand, it can indicate whether possibly costly additions to the mathematical models actually are likely to improve the solution. Both statements provide answers to the same question: What is the cost and gain of extending mathematical models?

In the next section, the concept of model incongruence is defined and an approach for implementing mathematical models within simulations is described in more detail. Two types of model incongruence are differentiated and explained: compositional and structural incongruence. With regard to the effect of incongruence, two levels, short-term and long-term, are considered.

Based on previous research, this paper uses a simulation system implemented for airline revenue management to illustrate the concept. Revenue management as described thoroughly in (Talluri and van Ryzin, 2004) represents an interesting operations research problem in that it entails the forecast of uncertain demand driven by customers’ choices as well as the mathematical optimization of availabilities and prices. Three aspects of airline revenue management, demand correlation between parallel flights, competing offers, and fluctuating demand are considered.

Finally, the approach presented here, its motivation and its limitations are summarized. The last section offers an outlook both to further research in the general methodology of analysing model incongruence and the applied area of revenue management.

Evaluating the Effects of Model Incongruence

Building a simulation model based on additional information to evaluate a partial mathematical model rather than directly extending the mathematical model may seem superfluous. However, this impression is based on two possibly faulty assumptions: Firstly, that it is feasible to create an extended mathematical model that can be solved efficiently. Secondly, that such an extended model would necessarily lead to better results. Finally the model that is to be evaluated may have been created before awareness or sufficient empirical information about additional aspects existed.

Using simulation modelling, the effects of model incongruence for optimization can be approximated. After presenting the basis, limits and potential of this approach, this section further analyzes different types of model incongruence. Finally, the effect of different types of incongruence and ways of measuring it are introduced.

Model Incongruence in Operations Research

This paper defines considers the gap between partial mathematical models and reality and defines it as model incongruence. The term is inspired by the concept of “model congruence” taken from econometric research and explained in (Bontemps and Mizon, 2003): In econometric terms, models aim to statistically represent the process generating observed economic data. In this regard, “congruence is a property of a model that has fully exploited all the information implicitly available once an investigate has chosen a set of variables to be used in modelling”. This definition of congruence makes it an unattainable ideal, as the true process that generated the relevant data and all its parameter can usually not be known with certainty.

Model incongruence can be considered while acknowledging that all abstract models are to some extent incongruent to reality in that they are never able to include all variables that actually influence a process or even to parametrize the variables they include with complete accuracy and certainty. However, mathematical models are often more incongruent than strictly necessary: While empirical data may be available on more aspects than those that are included, these are neglected for instance in favor of efficient solutions. If additional data is available, it is possible to create a simulation model that is still incongruent with regard to reality, but less so.

The concept of “enrichment” as described in (Morris, 1967) is an example for a proposed reaction to model incongruence in operations research. Enrichment includes iteratively elaborating simple mathematical models to make them more realistic. The underlying assumption seems to be that a solution based on a model that includes more realistic aspects will provide better results. Given that with the growth of a model comes a growth of complexity and uncertainty – for example if more variables need to be forecasted before an optimization function can be formulated – this is not necessarily true. Incongruence is a feature of abstract models that can be justified if a more realistic model does not provide better solutions.

Analytic Models as Components in Simulation Models

Using a simulation system to evaluate the gap between of a model and the real world extends the process of operations research as described by (Mitroff et al., 1974): The “conceptual model” serves as the basis for not one, but two “scientific models”. In this regard, this paper differentiates between a simulation model including as much detail from the conceptual model as information is available and relevant and an analytic model serving as the basis for the solution. If the simulation model is used to evaluate the performance of the analytic model, the latter can be implemented as a component of the former. For example, a revenue optimization model may be implemented as part of a supply agent in an agent-based simulation.
For the simulation, an extended scientific model including all information about the system considered that is relevant, available and that can be used for calibration and validation has to be implemented. For the solution, a much more parsimonious analytic model is required.

Employing a simulation model for the evaluation of analytic models necessitates the first and possibly the second types of predictions attempted by simulations according to (Troitzsch, 2009). Type 1: If the simulation model can predict the “kinds of behavior [that] can be expected [from a system] under arbitrarily given parameter combinations and initial conditions”, it can evaluate the performance of the solution based on the analytic model under arbitrary conditions. This can be sufficient to evaluate the performance of an analytic model given best-case or worst-case parameters. Type 2: If the simulation model can predict the “behavior a given target system (whose parameters and previous states may or may not have been precisely measured) [will] display in the near future”, it can evaluate the performance of the solution based on the analytic model under realistic conditions. This provides an opportunity to compare the performance of competing models before implementing one of them in the real world.

A major limitation of this approach is its reliance on the validity of the simulation model. The difficulties of validating complex simulation models have been stated for example in (Windrum et al., 2007). If the simulation model is insufficiently validated, it cannot be used to make statements about the effects of the analytic model’s incongruence. With regard to validation and the goal of the evaluation, the differentiation of the two purposes described in the previous paragraph plays a crucial role: Researchers need to be aware of the degree of validation they can provide for the simulation model and the types of predictions they can therefore derive from it.

The approach described here offers a range of opportunities. It provides a way to observe a solution’s performance in a situation that corresponds to the analytic model and in one that includes additional aspects. By measuring the difference in outcomes, the potential gain that can be achieved according to any indicator through extending the analytic model becomes measurable. Competing solutions can be implemented and compared under different conditions ceteris paribus. Finally, a simulation model provides the opportunity for evaluating heuristic improvements of analytic solutions that are not optimal when all empirical aspects are considered.

Compositional and Structural Model Incongruence

Within the available knowledge of the empirical situation, a simulation model’s congruence can be adjusted to be as realistic as possible or closer to that of the analytic model. By implementing the analytic model within the simulation model, researchers gain the opportunity to define not only the extent to which an analytic model matches the situation (its congruence to the simulation model) but also the type of incongruence it is confronted with.

When considering different types of incongruence, the definition of different types of uncertainty as provided by (Walker et al., 2003) can serve as a template. Uncertainty, defined as “any deviation from the unachievable ideal of completely deterministic knowledge of the relevant system”, is a reason for the simulation model’s incongruence. If everything about the system (the data generation process) considered could be known and measured, perfect congruence would be achievable. When an analytic model is incongruent to a simulation model incorporating all data available about the empirical situation, this may be described as a type of voluntary uncertainty, self-imposed in order to increase the solution’s efficiency.

In (Walker et al., 2003), two types of uncertainty are defined based on location: “context uncertainty” and “model structure uncertainty”. Context uncertainty occurs when knowledge about the boundaries of the modeled system is ambiguous; it may result in “the wrong question being answered”. Model structure uncertainty occurs when knowledge about the causal relationships within the modeled system is ambiguous, it may result in answers that are plain wrong.

Analogously, two types of incongruence may be defined: Compositional and structural. Compositional incongruence occurs when parts of empirical an situation that may be relevant to the problem are not included in the model. Structural incongruence occurs when relationships and behaviors between components of the model are not accurately modeled.

An example for compositional incongruence from the area of airline revenue management is the exclusion of competing offers from the model. An example for behavioural incongruence is the exclusion of customer preferences – while customer preferences may be considered, the cause-and-effect relationship between experience and expectations with regard to request times is neglected.

Measuring Effects of Model Incongruence

Before measuring the effects of model incongruence, its extent needs to be quantified. This can be achieved by considering the two types of model incongruence introduced in the previous section and checking the components and structural relationships that are known to exist in the empirical situation and are not included in the analytic model. The range of components and the impact of the relationships has to be measured based on empirical data if available. If this can be achieved, a realistic degree of incongruence (or several scenarios thereof) can be implemented in the simulation model.

An analytic model’s incongruence can be expected to have both positive and negative effects on the performance of solutions derived from it. As stated before, incongruence between the analytic model and the simulation model may be due not to necessity, but to choice – a partial analytic model may provide a more computationally efficient solution. The effects of model size on computational efficiency, however, are usually explicitly
considered and often stated as a reason for the limitation of analytic models. Examples for this type of discussion can be found in (\textsuperscript{?}).

This section focuses on the effects of an analytic model’s incongruence on the effectiveness of a solution. A solution’s effectiveness may be measured by any indicators taken from the empirical problem that motivated the solution. For example, an approach to optimizing revenue through allocating capacity may aim to increase revenue, but the underlying problem may also consider productivity indicators such as bookings or market share. Both a short-term and a long-term evaluation of the solution’s performance according to all indicators that are relevant to the empirical situation are possible.

**Short-term evaluations** of the indicators that are included in the analytic model are often provided to justify a solution’s superiority – short-term evaluations of indicators not included in the analytic model become possible only if the analytic model is evaluated in reality or in a more congruent simulation model.

A simulation model allows one to measure not just the short-term effect of implementing one solution as opposed to another, but also the long-term consequence of applying the solution repeatedly. As stated in (Gilbert and Troitzsch, 2005), the ability to speed up time and thereby model long periods of simulation-time in comparatively little real-time is one of the advantages of simulations.

**Long-term evaluations** of relevant indicators in a simulation model can expose effects of an analytic model’s incongruence that may not be immediately visible when the solution implemented in the real world. In the real world, external factors such as technological progress, market composition or economic activity may change over time and influence or obscure the solution’s success. In a simulation model, such external factors can be either kept constant or modulated. Keeping all external factors constant provides a way of evaluating whether the analytic model’s incongruence leads to a feedback cycle increasing the gap between the solution and the empirical situation or whether the solution adapts and compensates for the incongruence. Modulating external factors in a controlled environment also provides an opportunity for evaluating the solution’s robustness over time.

Finally, if the aim of considering an analytic model through its implementation in a simulation is to improve the current solution, the cost of model incongruence has to be compared to the expected cost of extending the analytic model. This challenge may be supported through simulation modelling, too, as prototypes of a solution based on an extended analytic model may be implemented in a simulation model to test their efficiency and the extent to which they improve the results, that is, their effectiveness.

**Examples from Airline Revenue Management**

As stated in the introduction, revenue management can be considered to provide interesting examples for the evaluation of model incongruence in operations research models, as it considers a system with uncertain, dynamic aspects due to the inclusion of customer demand in the objective function. The main objective of airline revenue management is to maximize revenue by differentiating the availability of booking classes with different sets of restrictions and different prices based on a segmentation of expected demand. This is achieved by computing optimal inventory controls based on a forecast of customer requests. For an introduction to revenue management as well as mathematical methods of demand modeling and optimization, please refer to (Talluri and van Ryzin, 2004).

Analytic revenue management models, used both for the demand forecast and the availability optimization, are mathematical models in the sense of operations research and accordingly subject to model incongruence. In revenue management literature, model aspects that have been identified to be incongruent are often referred to as “challenges”. An implicit assumption appears to be that with the extension of the analytic model to meet such challenges and thereby to remove incongruence, revenue management methodology can be continuously improved. Where the extension of the model seems infeasible, an improvement of the solution through human analysts is recommended as in (Mukhopadhyay et al., 2007) or (Isler and Imhof, 2008).

Focusing on indicators of forecast accuracy, the approach introduced here has previously been applied to airline revenue management in (Cleophas et al., 2009). An improved and extended version of the simulation system presented in that paper was employed to generate the results presented here: The system was re-created as a web-based tool for decision support and training as described in (Cleophas, 2012), improving the verification of all components through professional software engineering. Additional revenue management algorithms with regard to forecasting and optimization as well as tools for manipulating availabilities and an improved analysis interface were added.

The simulation system the principles offered in (Frank et al., 2008) with regard to a discrete-event based simulation framework and extends them through the implementation of an agent-based demand model. The demand data as well as the price structure used to parametrize the simulation model were calibrated using data from Lufthansa AG.

This section provides three examples from airline revenue management to illustrate the approach introduced in this paper. In the first example, the independent optimization of parallel flights is considered when customers flexibly decide which flight to book and will choose the one for which cheaper tickets are available. In the second example, the parallel flights are offered by two competing airlines, where one airline applies a pricing structure
that is 5% cheaper than that of the other airline. In the third example, economic cycles are implemented through demand shifts over time, to which the demand forecast has to adapt in order to provide reliant inputs to the availability optimization.

The revenue management system employed for the simulations described here is a hybrid system considering price-sensitive demand as documented in (Figg et al., 2009). The results shown in this section are based on the bookings generated in 150 simulation runs after 50 initialization runs. All confidence intervals provided are based on a single-sample t-test with error probabilities of at most 5%.

Parallel Flights as Internal Competition

Considered for example by (Zhang and Cooper, 2005), parallel flights are often mentioned as one of the challenges of traditional airline revenue management systems. Customers are indifferent with regard to which of two flights to book if the flights are offered at the same time and by the same carrier, take the same time and follow the same route. Yet, traditional revenue management systems forecast and optimize all flights that are not part of an itinerary independently, not taking that indifference into consideration.

In the case of parallel flights, there is no compositional incongruence: All relevant parts of the system (customer demand, flights offered, booking classes and prices offered) are considered by the analytic model. However, the analytic model is structurally incongruent – the flexible choice behavior of customers that are indifferent to which of two parallel flights to book is not included. Accordingly, one could expect to see revenue diminished when parallel flights are considered independently rather than optimizing them as a unified capacity.

In the example of parallel flights, both short-term and long-term effects of model incongruence are to be expected. In the short-term, revenue may decrease as customers can choose the cheapest ticket available not just on the flight which they are predicted to request, but also on its parallel equivalent. In the long-term, forecast adaptations may lead to a systematic increase of the incongruence or may balance out the effects.

Underbidding by External Competition

The idea that competitors’ offers may undermine revenue management success when customers buy the cheapest ticket available and are indifferent with regard to carrier brands is described in (Isler and Imhof, 2008). Revenue management models that do not explicitly consider competing offers may be regarded to suffer from compositional incongruence. Even if the existence of cheaper alternatives is regarded as given, structural incongruence may still exist if the model does not consider the fact that the competitor may also implement revenue management. In such a case, causal relationships between the optimized availabilities and observed demand of one carrier and those of a competing carrier is neglected.

Figure 2: Revenue per Seat Kilometres Offered Under Competition

In the example given by figure 2, only the short-term effects of a situation with competition in a revenue management system that does not consider competition are illustrated. In this simulation, the competing carrier implements a price structure that consistently is 5% lower than that of the original carrier. As a result, the original carrier’s revenue per seat kilometer offered is reduced by up to 6%, while the competitor gains 4% more in revenue per seat kilometer offered than what was earned in the monopoly situation. On the long-term, the situation may be even more grave as forecast adaptations and mutual underbidding can lead to a price war.
Forecast Adaptation: Economic Cycles

The final example considers the fact that passenger demand is usually not static but changes over time due to external factors such as economic cycles. An example of research considering the value of human analysts’ adjusting passenger demand forecasts is provided by (Mukhopadhyay et al., 2007); most forecast models employed for revenue management adapt to changes in demand as observed through changes in booking numbers. However, lacking a scientific model of demand, these adaptations are usually realized by describing patterns rather than by analyzing causal relationships.

When demand is expected to change over time and the opportunity for adaptation is part of the model, compositional incongruence may not be regarded as given. However, due to the causal relationships between external factors and passenger demand, the external factors (such as for example economic indicators) may be relevant to the model. Ignoring them therefore can lead to compositional incongruence, as ignoring the causes for shifts in demand can constitute structural incongruence.

As shifts in demand can only occur over a longer period of time, they may be considered as a long-term phenomenon that is best observed in this time frame. For that reason, short-term effects are not considered in further detail in this example.

Figure 3 shows the change of revenue as demand volume changes over multiple simulation runs. Demand volume is first reduced by 30%, then increased by 30%, reduced by 30% again, and finally set back to the initial status. As illustrated by the diagram, a 30% decrease in overall demand does not necessarily lead to a corresponding decrease in revenue: the system is robust enough to restrict losses to at most 25%. However, a 30% increase in overall demand cannot be translated into a corresponding increase in revenue, either: possibly due to restricted capacity as well as the cost of adapting the forecast, revenue can be increased only by at most 10%. Finally, the demand shifts seem not to diminish the system’s performance on the long run: as the demand volume is returned to its initial status, revenue does no longer deviate significantly from what was earned initially.

Conclusion

In this paper, an approach of using simulations to measure the effects of model incongruence has been introduced. Extending the model of operations research as described by (Mitroff et al., 1974), the scientific model differentiated into a simulation model and an analytic model: While the analytic model includes those components and relationships of the conceptual model that allow for a computationally efficient solution, the simulation model may include all relevant components and relationships for which sufficient empirical data is available.

The approach presented here allows one to evaluate and prioritize extensions of the analytic model with regard to their effects on relevant indicators. For the example shown in the previous section, the conclusion may be that the extension of revenue management models with regard to external competition holds greater potential than extensions with regard to internal competition. Of course, such a conclusion is subject to further analysis as well as to considerations of feasibility.

To allow for the analysis and control of model incongruence, two types of incongruence are differentiated. Analogously to the uncertainty that can cause it, incongruence may be compositional or structural. The exclusion of relevant parts of the conceptual model in the analytic model leads to compositional incongruence; the exclusion of relevant causal relationships leads to structural incongruence. The methodological part of the paper concludes that the effects of model incongruence have to be considered on two levels, the short- and the long-term.

The theoretical concept was illustrated using three examples from airline revenue management. Based on simulation results, the short- and long-term effects of different types of model incongruence were illustrated.

The approach suggested here regards the simulation model as being independent of the type of simulation (agent-based, system-dynamic, discrete-event-based) used. This independence of modeling and methodology is intentional: The type of simulation suited for the implementation of the simulation model depends on the conceptual model and the aspects of the system considered. For example, in airline revenue management, customers may be modeled as agents or their requests may be modeled as events depending on the choice-behavior considered. The improvement of analytic models through human analysts may even benefit from participatory agent-based modeling as described in (Guyot and Honiden, 2006).

At this point, the causes for compositional and structural incongruence have not been considered in further detail. The concept may be improved by considering the role of uncertainty with regard to the validity both of analytic and of simulation models. An in-depth treatment of the enrichment approach using the view of evaluation in a simulation model may provide further insight.
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AUTHOR BIOGRAPHIES

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