MODELLING OPTIMAL HEDGE RATIO IN THE PRESENCE OF FUNDING RISK

Barbara Dömötör
Department of Finance
Corvinus University of Budapest
1093, Budapest, Hungary
E-mail: barbara.domotor@uni-corvinus.hu

KEYWORDS[1]

Corporate risk management, Optimal hedge ratio, Funding liquidity

ABSTRACT[2]

In the broad literature of corporate risk management classic models of optimal hedging assume a one-period hedging decision, and therefore no financing need arises to maintain the hedge position. The multi-period models are usually based on the assumption of no liquidity constraints, and accordingly the eventual financing need can always be met from the market. As a consequence of the recent crisis even interbank deals need to be collateralized, so the funding need of any financial transactions can be disregarded. Another usual assumption of the financial models refers to a zero expected value of the hedging position, which contradicts the also the practice. This study investigates the optimal hedge position as a function of 3 factors that determine the corporate utility function: the risk aversion ratio of the company, the expected value of the hedge position and the financing costs deriving from the hedging itself.

INTRODUCTION

The relevance of corporate risk management is shown from different aspects in the financial theories. The main reason for corporate hedging in all of the theories is some imperfectness of the markets: the presence of taxes (Smith and Stulz, 1985); transaction costs (Dufey és Srinivasulu, 1984); the asymmetric information among market participants (Myers and Majluf, 1984 and Tirole, 2006), or the consequences of unavailable financing can cause financial distresses (Smith and Stulz, 1985; Froot et al. 1993). The latest explanation, the lack of financing, is modeled usually in a two-periods model, in which the hedging itself is concluded in the first, and settled in the second period, and so it has no additional cash-flow consequences (except for the potential upfront fees). The availability of financing is however critical from the aspect of the hedging deal as well, than in contrast to the theory, during the lifetime of the deal it may need to meet financing requirements. These can derive from upfront paying obligations, maturity mismatches, mark-to-market settlements of derivative positions, or cash-collaterals. In case of infinite access to liquidity (in the absence of transaction costs) these issues can be ignored. In reality however meeting these requirements is costly, or even impossible, therefore neglecting liquidity considerations lead to incorrect hedging solutions.

Two different approaches of modelling the liquidity consequences of hedge positions are offered by Deep (2002) and Korn (2003). Deep considers the daily settlement need of the mark-to-market value of futures. On the other hand the model of Korn assumes, that the unrealized loss of forward agreement is to be collateralized. Both models use a concave corporate utility function, the expected value of which is to be maximized.

This paper develops a model based on the concept of Korn, and investigates the effect of the financing cost on the corporate hedging.

The next session describes the applied model, then presents an analytical solution for the lower and upper bound of the optimal hedge ratio. The following part includes the results of the simulation: the optimal hedge ratio is modeled as a function of the affecting factors. The last part concludes and shows some further research possibilities.

THE MODEL

The model assumes a company being exposed to the change of the market price of its product, so its revenue and profit bears market risk. We assume furthermore that hedging of this open position in form of forward agreements is available at the market. The spot price (S) follows geometrical Brownian motion with an expected drift of μ and volatility of σ. According to the stochastic calculus the change of the forward price (F) also follows a geometrical Brownian motion as it can be seen in Equation (1):

$$dF = (\mu - r)Fdt + \sigma F dw \tag{1}$$

[1] The paper is based on the 4. chapter of the thesis proposal The effect of funding liquidity on hedging of market risk. (Dömötör, 2012)

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where r stands for the riskless return and dw - the change of the Wiener-process- is a standard normal distributed random variable.

In contrast to the model of Deep and Korn I do not suppose Equation (1) to be a martingale, so the drift rate can differ from zero in either direction, depending on the relation between μ and r[3].

The model is built up as follows: the company decides at time 0 about its production quantity (Q) and the hedging amount (h), in our case the amount sold on forward. Maturity of the forward agreement and realization of the production are at time 2, and during the lifetime of the derivative position the unrealized loss is to be collateralized at time 1, according to Figure 1.

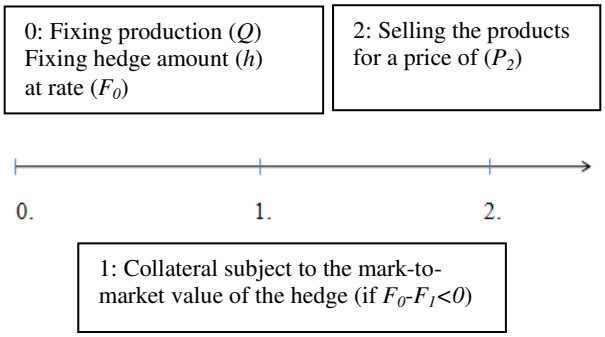

Figure 1: The process of corporate operation

The corporate profit (Π) is realized at time 2:

$$\Pi = S_2 Q - c(Q) + h(F_0 - S_2) + k \min\left[h\frac{(F_0 - F_1)}{1+r};0 \right] \quad (2)$$

The indices refer to the time, the new parameter, k stands for the credit spread to be paid by the hedger company, k is considered to be constant.

The differences between this model and the model of Korn are the non-zero expected value of the forward agreement, the exogenous production amount and lack of adjustment of the hedged amount in time 1.

Although the aim of the company is the maximization of the shareholders' value, and the corporate utility function is meaningless, the corporate decision making is to be modelled in a risk-return framework, which is described by the maximization of the expected utility. The corporate utility function is supposed to be increasing and concave, reflecting a decreasing marginal utility (risk aversion). Based on the literature the model applies a CRRA (constant relative risk aversion) type utility function according to Equation (3):

$$U(\Pi) = \frac{\Pi^{1-\gamma}}{1-\gamma} \quad (3)$$

where γ is a measure of the risk aversion.

The optimal hedge amount (h), which maximizes the expected utility, meets the following requirement:

$$E\left[U'(\Pi) * (F_0 - S_2 + k \min[0; \frac{F_0 - F_1}{1+r}]) \right] = 0 \quad (4)$$

Equation (4) can be written in the next form:

$$E[U'(\Pi)] * E\left[F_0 - S_2 + k \min[0; \frac{F_0 - F_1}{1+r}] \right] = - \operatorname{cov}(U'(\Pi); F_0 - S_2 + k \min[0; \frac{F_0 - F_1}{1+r}]) \quad (5)$$

where the $U'(\Pi)$ is the first derivative according to h, as described in Equation (6)

$$U'(\Pi) = \left[S_2 Q - c(Q) + h(F_0 - S_2) + kh \min[0; \frac{F_0 - F_1}{1+r}] \right]^{-\gamma} \quad (6)$$

The sign of the left hand side of Equation (5) is equal to the sign of the expected value of the short forward position, as the utility function is increasing. If the expected value is positive ($\mu<r$) equality holds only if the covariance term on the right hand side is negative. As the second variable in the covariance is affected negatively by S_2 and F_1, independently from the hedged amount, the negativity of the covariance requires the first part ($U'(\Pi)$) to be a positive function of the stochastic variables. In the absence of financing costs (k=0), this requires h (the hedging amount) to exceed the quantity of the production (Q). From this follows, that it is optimal to overhedge, similarly to the model of Holthausen (1979).

However funding liquidity risk (in the form of financing cost) reduces the optimal hedge ratio, as the effect of F_1 (being positively correlated with S_2) is positive for any positive value of h. The reduction of the optimal hedging depends on the level of the financing costs (k). It can be similarly shown, that the negative expected value of the hedge position causes a lower than 1 optimal hedge ratio, that is further reduced by the eventual financing costs.

In sum this means, that the hedging affects the corporate utility, since the financing cost and the expected value of the hedge position influence the expected value of the profit. The effect of the financing cost to the utility is always negative; the expected value can have both

[3] In case S refers to a foreign-exchange rate, this means that interest rate parity does not hold, which is a stylized fact of FX-markets, investigated by Darvas (2009).

negative and positive impact; while utility increases through variance-reduction.

The result of this threefold effect is a function of the determining parameters: the corporate credit spread, the expected value of the hedge position and the corporate risk aversion factor.

The optimal hedge ratio of the above presented model differs from that of the model of Korn, since risk cannot be eliminated here perfectly, just at a given significance level, as the profit is the function of two not perfectly correlated risk factors (F_1 and S_2).

Despite of the positive correlation of the risk factors, under extreme circumstances the corporate profit can become negative at any hedging level. The worst outcome occurs if the short hedge position is to be financed because of the growing market price of the first period, but this higher market price is not used to complete the hedge position, and the falling market price causes an operating loss on the unhedged part of the firm's production.

THE THEORETICAL BOUNDS OF THE OPTIMAL HEDGE RATIO

The exact value of the optimal hedge ratio is to be calculated in the function of the parameters of Equation (5) numerically. The bounds of the optimum can be however determined analytically.

The optimal solution has to ensure a positive profit at any price evolution. The theoretically lowest value of S_2 equals to zero. By substituting S_2=0, Equation (2) takes the following form in Equation (7):

$$\Pi = -c(Q) + h(F_0) + k \min\left[h\frac{(F_0 - F_1)}{1+r};0 \right] > 0 \tag{7}$$

In the absence of financing cost the lower bound of the hedge ratio (h/Q) is the same as in the Korn-model, as it is shown in Equation (8), based on (Korn, 2003):

$$\frac{h}{Q} > \frac{\bar{c}}{F_0} \tag{8}$$

This means that the hedge ratio has to exceed the ratio of average cost to the initial forward price.

The minimal hedge amount has to cover not only the operating costs, but the financing cost of the position as well. As the financing cost is an unlimited stochastic variable[4], this coverage can be ensured only at a given significance level. Supposing a maximum of the price change ($\Delta F_{1max\alpha}$) and substituting into Equation (2), the result will be the following:

$$\Pi = -c(Q) + h(F_0) + kh\frac{(\Delta F_{1\max\alpha})}{1+r} > 0 \tag{9}$$

After the rearrangement of Equation (9), the hedge ratio can be seen in Equation (10):

$$\frac{h}{Q} > \frac{\bar{c}}{F_0 - \frac{\Delta F_{1\max\alpha}}{1+r}k} \tag{10}$$

The minimal hedge ratio in this case is given by the ratio of the average cost and the initial forward price reduced by the maximum of the financing costs at a certain (α) level. This ratio ensures a positive end of period profit at any low level of the market price at maturity, even if the hedge position caused financing costs.

The maximum of the hedge ratio is the level, where the financing cost and the negative value of the hedged position are counterbalanced by the realized higher operating income. Denoting the maximum of the price at maturity by $S_{2max} = F_0 + \Delta F_{2max\alpha}$, and substituting it and the maximum of F_1 into Equation (2) we will get Equation (11):

$$\Pi = F_0 + \Delta F_{2\max\alpha} - c(Q) + h(F_0 - F_0 + \Delta F_{2\max\alpha}) + \\ + kh\frac{(F_0 - (F_0 + \Delta F_{1\max\alpha}))}{1+r} > 0 \tag{11}$$

After rearrangement and simplification we receive the upper bound of the hedge ratio in Equation (12):

$$\frac{h}{Q} < \frac{F_0 + \Delta F_{2\max\alpha} - \bar{c}}{\Delta F_{2\max\alpha} + \frac{\Delta F_{1\max\alpha}}{1+r}k} \tag{12}$$

As shown above, the level of the financing costs (k) moderates the measure of over- and underhedge also. If k goes up, the lower bound increases, while the upper bound decreases.

The optimal hedge ratio is determined through Monte Carlo Simulation, using corporate specific parameters (cost function, credit spread, risk aversion) and the chosen parameters of the forward price movement process (drift and volatility).

RESULTS OF THE SIMULATIONS

I run Monte Carlo simulation in MS Excel, based on the generation of 2000 normally distributed random variable for the price change. The initial forward rate was given, F_0=1. In order to catch the fat tail phenomena in Finance - namely the higher probability of the extreme values, than predicted by the normal

[4] As the price movement has no upper limit, the financing cost can be theoretically even infinitive.

distribution, - I set two extremes into the sample manually: F_1=2 and F_2=4, then F_1=2 and F_2=0. These extreme outcomes has no significant effect on the expected value, as their probability is very low (the probability of a 100% increase in the price is $1{,}3*10^{-11}$, based on a normal distribution with 15% standard deviation). The appearance of the extremes however excludes those hedging solutions that would cause negative corporate profit under extreme market circumstances.

The utility of the end of profit was calculated for each outcomes. The aim of the optimization was to find the hedge ratio, where their average, considered to be the expected utility, was maximized.

This paper focuses on the effect of financing costs, so the credit spread is a changing variable on all of the following charts. Table 1 summarizes the investigated set of the parameters.

The cost function is assumed to be linear, the average cost is expressed as a percentage of the initial forward rate.

Table 1: The investigated set of the parameters

Parameter		Notation	Figure 2	Figure 3	Figure 4	Figure 5
Corporate specific	Average cost	c^*	10%	50%	10%	10%
	Credit spread	k	changing variable	changing variable	changing variable	changing variable
	Risk aversion	γ	changing variable	changing variable	2	0,5
Forward price process	Drift	μ	0	0	changing variable	changing variable
	Volatility	σ	15%	15%	15%	15%
	Initial forward rate	F_0	1	1	1	1
	Riskless return	r	5%	5%	5%	5%

Figure 2 depicts the optimal hedge ratio, by choosing similar fix parameters, than Deep and Korn: the drift of the forward price is supposed to be zero, volatility of 15% and average operating cost of 10%. Because of the zero expected value of the forward position this factor has no impact to the utility function.

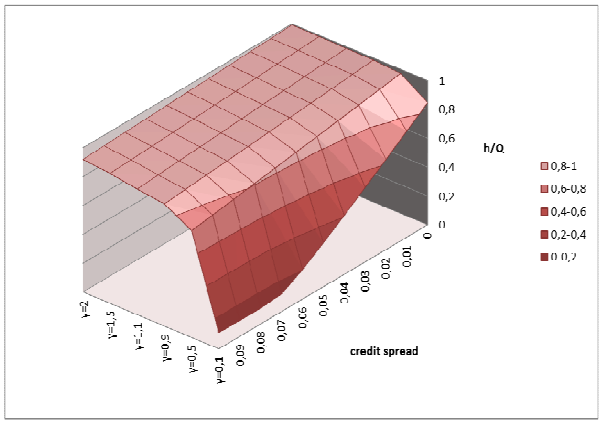

Figure 2: Optimal hedge ratio as a function of the credit spread and risk aversion (forward drift: 0%, volatility: 15%, average cost 10%)

The results are very close to the conclusion of the Korn-model: the operating margin is high enough (90%), so that for a risk averse firm (gamma above 0,5), the utility enhancement deriving from the reduced volatility, exceeds the utility reduction of the potential financing costs of the hedge. As a consequence, 1 percentage point rise of the credit spread reduces the optimal hedging ratio by only 2,5%-point for a firm with 0,5 risk aversion coefficient.

With the fall of the sensitivity towards risks (decreasing gamma) the marginal utility of the hedge offsets less and less the effect of the financing costs. For a firm with a risk aversion factor of 0,1, the optimal hedge ratio drops to the minimum hedging level shown in Equation (10), which ensures the positivity of the profit, if the credit spread hits 7%.

Figure 3 illustrates the optimal hedge ratio taking the same parameters than the former simulation except for the average cost, which is constant 50% here. The increase of the average cost causes a slight enhancement of the hedging ratio in each case, but through its effect on the minimal hedge ratio, the optimum is affected significantly for the less risk averse hedgers.

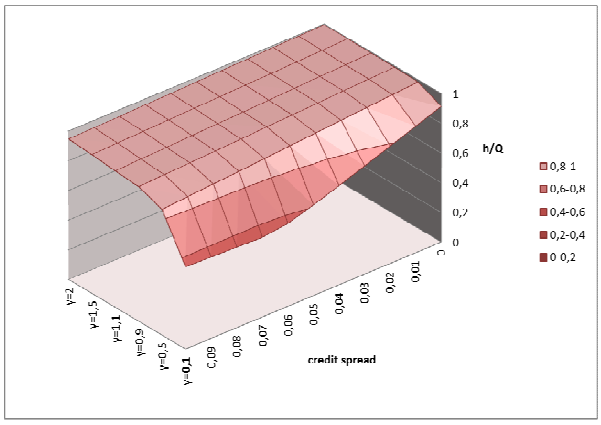

Figure 3: Optimal hedge ratio as a function of the credit spread and risk aversion (forward drift: 0%, volatility: 15%, average cost 50%)

The following simulations show the effect of the non-zero drift of the forward position, namely the drift of Equation (1) differs from zero. Although in case of currencies, according to the uncovered interest rate parity, the expected value of the forward position is zero, it can be shown that carry trade has a significant role in financial markets.

The expected value of the hedge position takes a more significant effect on the optimal hedge ratio, than financing costs. The positive drift (μ) of the forwad price causes an expected loss for a hedger in short position, that leads to a substantial reduction of the hedge ratio even for a more risk averse (γ=2) firm.

The optimal hedge ratio is modelled as a function of the financing costs and the expected value of the hedge in the following figures. The volatility and the average cost are the initial constant rates (15% and 10% respectively), the risk aversion coefficient (γ) is set to 2 in Figure 4. As the chart shows, 1 %-point increase of the forward drift causes some 20%-point lower optimal hedge ratio. In case of negative drift – which causes the positivity of the expected value of the position – the optimal hedge ratio exceeds 100%.

A minor difference from zero drift leads to significant under- or overhedging in the optimum. Moreover the bounds of the optimal hedge ratio are reached at a 5% drift of the forward price, in our case the upper bound of 130% and the lower bound of 11% (credit spread=11%).

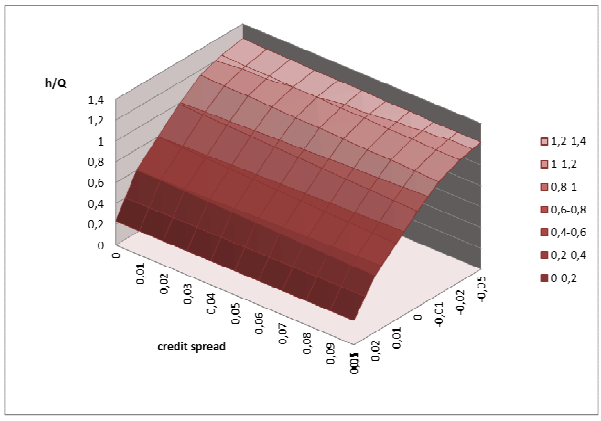

Figure 4: Optimal hedge ratio as a function of the financing cost and forward drift (γ=2, volatility: 15%, average cost 10%)

With the fall of the risk aversion and so the marginal utility of variance reduction, the optimal hedge ratio converges faster to the upper or lower bound. As Figure 5 shows, 1% positive (negative) drift of the forward price is enough to shift the optimal hedging level to the minimum (maximum) quantity, if the risk aversion factor is 0,5.

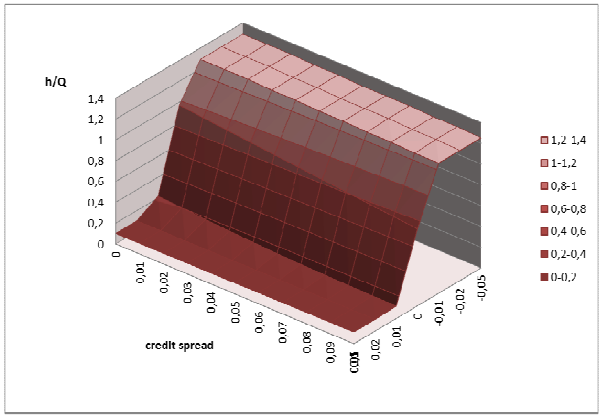

Figure 5: Optimal hedge ratio as a function of the credit spread and forward drift (γ=0,5, volatility: 15%, average cost 10%)

If the expected value of the forward hedge exceeds 1%, the financing cost affects the optimal hedging only by its effect on the minimum/maximum hedging ratio.

CONCLUSION

The paper investigates the corporate hedging decision, by considering funding liquidity risk and expected value of the hedge position itself. Funding cost is quantified through the financing cost of the collateral to be placed for the mark-to market loss of the hedging position. The bounds of the optimal hedge ratio are

presented and the optimum is modelled as a function of the corporate credit spread (financing cost) and expected drift of the forward price. The analysis shows, that 1 percentage point increase of the credit spread causes some (2-3) percentage point decrease in the optimal hedging ratio. The effect of the expected value of the forward position proved to be more significant in the simulations, 1 percentage-point (+/-) change of the expected value leads to a dramatic (20-30%-point) change of the optimal hedging ratio. It seems that this later effect can better explain the empirical fact of corporate under- or overhedge. The above analysis assumes static credit spread, at which financing is always available. Releasing this assumption is the topic of further research.

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AUTHOR BIOGRAPHIES

BARBARA DÖMÖTÖR is an Assistant Professor of the Department of Finance at Corvinus University of Budapest. Before starting her PhD studies in 2008, she worked for several multinational banks. She is now working on her doctoral thesis about corporate hedging. She is lecturing Corporate Finance, Financial Risk Management and Investment Analysis, her main research areas are financial markets, financial risk management and corporate hedging. Her e-mail address is: barbara.domotor@uni-corvinus.hu