DATABASE OF UNSTABLE SYSTEMS: A NEW SITE FOR MODELS OF UNSTABLE PROCESSES

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ABSTRACT

The paper presents a new site for unstable systems which can be used as an information database about unstable processes and stability. It is an open, easily extensible system in the bilingual version (ENG/CZ) containing mathematical models of real unstable processes together with their simulation models in e.g. MATLAB/Simulink environment. Basic information about stability of dynamical systems is also included. This contribution outlines motivation for development of this site, presents its basic structure and suggests areas of prospective use. Several models of unstable processes from the site are also presented and discussed briefly.

INTRODUCTION

Many industrial processes, such as various types of reactors, combustion systems, distillation columns, etc. possess unstable behaviour (Chidambaram 1997; Padma Sree and Chidambaram 2006). Besides industrial, aviation and military areas, there are also many systems and processes in the environmental and social fields that are naturally unstable. Experiments with such systems without proper knowledge can be hazardous (Stein 2003). Designers and control engineers have to understand basic limitations that stem from the process instability (Middleton 1991; Skogestad et al. 2002). Therefore, modelling and simulation tools play an important role in this area. Before a decision is made and implemented on a real unstable process, a proper simulation analysis has to be done in order to ensure safe implementation.

The presented site has been developed to enable students, teachers, scientist, control engineers and many others interested in unstable processes easy access to mathematical models of such systems. All these people can easily use presented models for their own simulation experiments, testing control algorithms, etc. This will broaden awareness about unstable processes and problems they cause. Due to the fact that the database is easily accessible via the Internet (located at http://www.unstable-systems.cz) it can be used by wide range of users for various purposes, e.g. pedagogical, scientific and others.

This paper is structured as follows: after a brief introduction into the stability of dynamical systems the contribution continues by examples of unstable processes including their simplified mathematical models. Further, a basic structure of the developed site is outlined and explained and the paper concludes suggesting possible extensions of the project.

STABILITY OF DYNAMICAL SYSTEMS

Stability is the fundamental required property of control systems. Therefore a great deal of effort has been focused towards proper definition, testing and attainment of system stability.

Defining Stability

Although all people naturally understand the concept of stability and are able to describe what stable behaviour is and what is not, as outlined in e.g. Fig. 1, a proper mathematical definition is not so straightforward. Generally, stability can be formulated as ability to recover from perturbations – short-time disturbances or non-zero initial conditions.

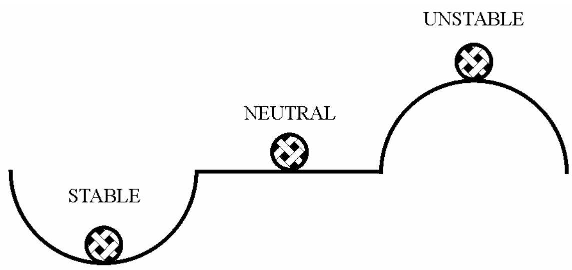

Figure 1: Basic Understanding of Stability – Ball in the Gravitational Field

One of the definitions says that a system is stable if bounded input into the system produces a bounded output from the system. This is so called *BIBO* (Bounded Input – Bounded Output) *stability*, e.g. (Willems 1970; Skogestad and Postlethwaite 2005), illustrated in Fig. 2-3 where examples of some typical step-responses of stable and unstable systems are presented.

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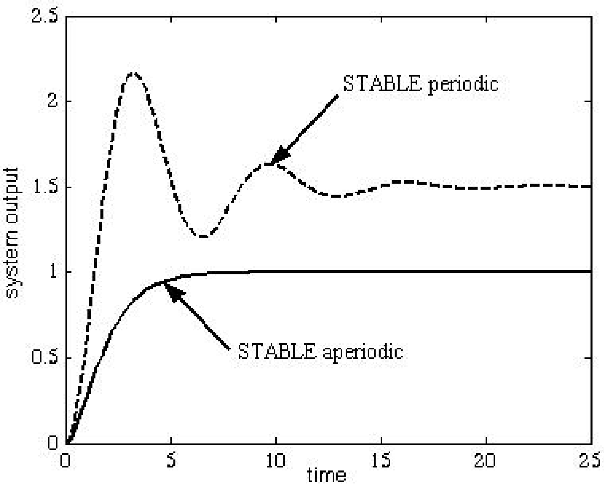

Figure 2: Stable Systems Responses

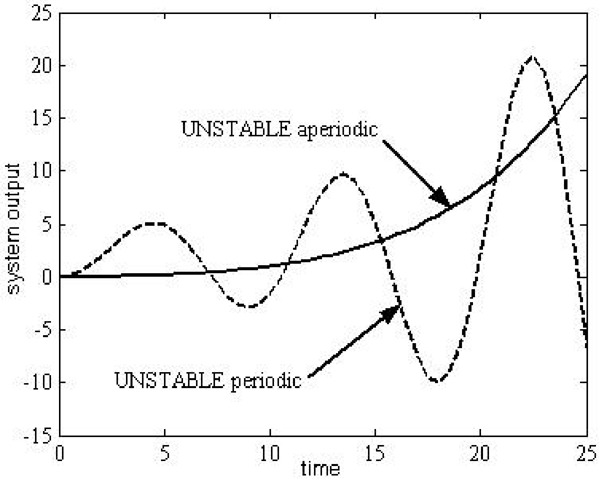

Figure 3: Unstable Systems Responses

Another recognized and more general definition is the *Lyapunov stability*, e.g. (Willems 1970; Parks 1992; Skogestad and Postlethwaite 2005; Åström and Murray 2008). It states, simply speaking, that a system is Lyapunov stable if its output and all states are bounded and converge asymptotically to zero from a sufficiently small initial conditions. Its concept in the state-space (for two states) is graphically illustrated in Fig. 4.

Testing Stability

During the decades many methods of stability testing have been developed. Usage of a particular method depends on the properties of the system to be tested – e.g. if it is linear or nonlinear, continuous-time or discrete-time, time-variant or time-invariant, etc. The methods can be both numerical and graphical. An interested reader can found details in books focused on systems theory or control engineering, e.g. (Willems 1970; Skogestad and Postlethwaite 2005; Åström and Murray 2008; Doyle et al. 2009).

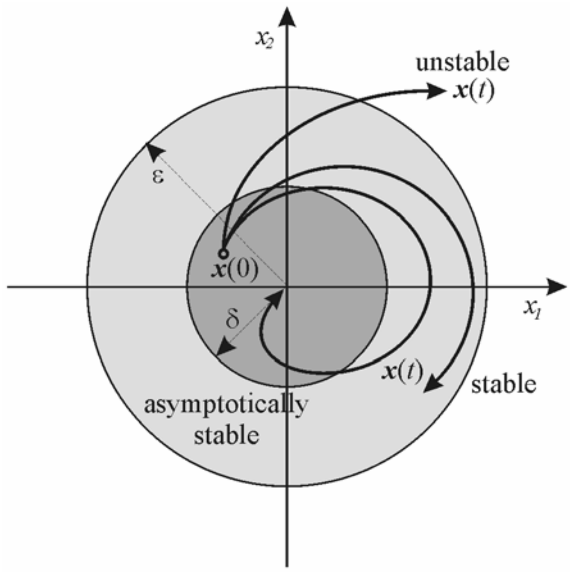

Figure 4: Lyapunov Stability

Attainment of Stability

An unstable system can be stabilized by feedback. There many sources focused on the control system design for unstable processes, e.g. (Chidambaram 1997; Park et al. 1998; Marchetti et al. 2001; Lozano et al. 2004; Padma Sree and Chidambaram 2006; García et al. 2006; Dostál et al. 2008). Many of these works solve the control system design problem connected also with delayed and non-minimum-phase systems which are also problematic to control.

Besides testing and attainment of stability it is often important to test and ensure certain measure of stability, i.e. *relative stability* which gives answer to the question how far the system is from instability. For control systems design, so called *gain* and *phase margins* are frequently used, for details see e.g. (Skogestad and Postlethwaite 2005; Åström and Murray 2008; Doyle et al. 2009).

The so-called *robust stability* is next important term in control engineering. It is used for the case we want to test/achieve stability not only for one system but for a certain class of systems, typically a nominal system and some neighbourhood, which is useful in the case of uncertain models. An interested reader is referred to books devoted to the robust systems design, e.g. (Barmish 1994; Bhattacharyya et al. 1995).

EXAMPLES OF UNSTABLE SYSTEMS

As explained in the introduction section unstable processes are common in many areas of our daily lives. Several such systems are briefly described in this section. Mathematical models together with simulation files and original sources are also available online from the developed site (Gazdoš and Kolařík 2012).

Nonideal CSTR

This process is represented by a continuous stirred-tank reactor (CSTR) with nonideal mixing decribed by the Cholette's model. The process can be sketched as illustrated in Fig. 5.

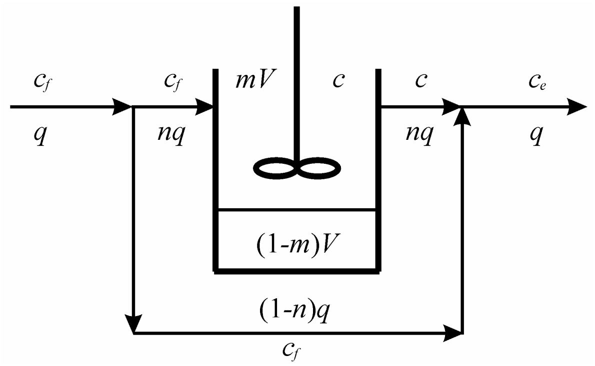

Figure 5: Schematic Representation of a Nonideal CSTR

A simplified mathematical model of the process dynamics can be expressed by these formulas (Liou and Chien 1991; Padma Sree and Chidambaram 2006):

$$\begin{gathered}\frac{dc(t)}{dt}=\frac{nq}{mV}\left[c_f(t)-c(t)\right]-\frac{k_1c(t)}{\left[1+k_2c(t)\right]^2}\\ nc(t)+(1-n)c_f(t)=c_e(t)\end{gathered} \quad (1)$$

In the picture and equations, $c(t)$ is the concentration of the reactant in the well mixed zone, $c_e(t)$ is the concentration in the exit stream (controlled variable) and $c_f(t)$ is the feed concentration (manipulated variable). Further, n is the fraction of the reactant feed that enters the zone of perfect mixing and m is the fraction of the reactor total volume V where the reaction occurs. Constants k_1, k_2 describe the reaction rate and q is the inlet flow rate. Parameters of the model are presented in the first part of Table 1 below; its second part describes values of the input, output and state variables in a nominal operating point.

Table 1: Parameters of the Nonideal CSTR

Symbol	Value [unit]
n	0.75 [-]
m	0.75 [-]
V	1 [l]
q	0.033 [l/s]
k_1	10 [s^{-1}]
k_2	10 [l/mol]
c_f	6.484 [mol/l]
c_e	1.8 [mol/l]
c	0.2387 [mol/l]

Linearization of the model (1) around the given nominal operating point gives the transfer function model as:

$$\frac{\Delta c_e(s)}{\Delta c_f(s)}=\frac{0.7725s-0.1727}{3.1s-1}. \quad (2)$$

A state-space representation of the linearized model in the general form:

$$\begin{aligned}\boldsymbol{x}'(t)&=\boldsymbol{A}\boldsymbol{x}(t)+\boldsymbol{B}\boldsymbol{u}(t)\\ \boldsymbol{y}(t)&=\boldsymbol{C}\boldsymbol{x}(t)+\boldsymbol{D}\boldsymbol{u}(t)\end{aligned}, \quad (3)$$

where $\boldsymbol{x}(t)$ defines a vector of state variables, $\boldsymbol{y}(t)$ a vector of output variables and $\boldsymbol{u}(t)$ a vector of input variables can be obtained e.g. in this form (using the MATLAB function *ssdata*):

$$\begin{gathered}\boldsymbol{A}=[0.3226],\ \boldsymbol{B}=[0.1250],\\ \boldsymbol{C}=[0.1974],\ \boldsymbol{D}=[0.2492].\end{gathered} \quad (4)$$

In the case of the presented reactor the variables $\boldsymbol{x}(t)$, $\boldsymbol{y}(t)$ and $\boldsymbol{u}(t)$ are only scalar and correspond to the reactor variables $c(t)$, $c_e(t)$ and $c_f(t)$ respectively (consequently $\boldsymbol{A},\boldsymbol{B},\boldsymbol{C},\boldsymbol{D}$ are also only scalars).
From the control theory point of view, the models (2)-(4) represent a first-order proper system which is unstable (one positive *pole*, i.e. denominator root, located at $p_1=0.3226$) with non-minimum-phase behaviour (one positive *zero*, i.e. numerator root, located at $z_1=0.2236$), and with gain $k=0.1727$ [-]. Such systems which are both unstable and non-minimum-phase are not so easy to control. The step-response of the model recorded in Fig. 6 clearly demonstrates instability of the system.

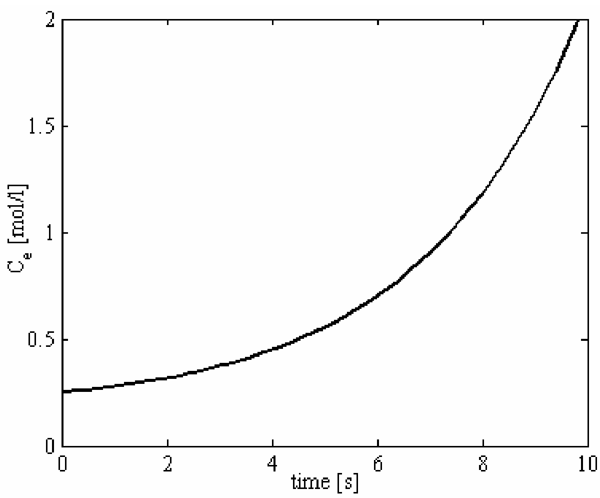

Figure 6: Nonideal CSTR Step-Response

Ballistic Missile

A ballistic missile sketched simply in Fig. 7 can represent another unstable system from the military industry. Although it is completely different from the previous one, it shares the property of instability and consequently problematic control.

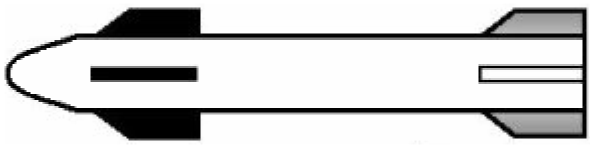

Figure 7: Ballistic Missile

When controlling altitude of the ballistic missile, the transfer function relating the altitude $y(t)$ to the thrust chamber deflection $\delta(t)$ has the following form (Blakelock 1991; Padma Sree and Chidambaram 2006):

$$\frac{\Delta y(s)}{\Delta\delta(s)} = \frac{7.21(s+0.0526)}{(s+1.6)(s-1.48)(s-0.023)}. \tag{5}$$

Its state-space representation in the general form (3) can be obtained e.g. as (MATLAB function *ssdata*):

$$\begin{aligned} \boldsymbol{A} &= \begin{bmatrix} -0.0970 & 1.1854 & -0.0545 \\ 2 & 0 & 0 \\ 0 & 0.5 & 0 \end{bmatrix}, \boldsymbol{B} = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}, \\ \boldsymbol{C} &= [0 \quad 1.8025 \quad 0.1896], \boldsymbol{D} = [0]. \end{aligned} \tag{6}$$

From the control theory point of view the missile represents a strictly proper unstable system of 3rd order. The instability is given by the two poles (denominator roots) located in the right half of the complex plane ($p_1 = 0.023$, $p_2 = 1.48$), as illustrated in Fig. 8 where the *Pole-Zero Map* is presented. The system also has relatively fast dynamics with time-constants in seconds.

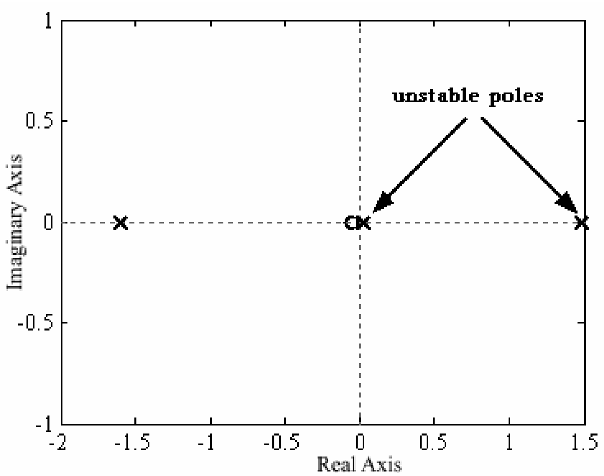

Figure 8: Pole-Zero Map of the Ballistic Missile

X-29 Aircraft

The X-29 sketched in Fig. 9 was an experimental aircraft that tested forward-swept wing, canard control surfaces, and other novel aircraft technologies. It was deliberately designed with static instability to increase its maneuverability and speeds of command response. However, as a consequence, it was impossible to pilot this airplane conventionally by manual flight controls and it required the use of so-called fly-by-wire (computerized) control system. In addition special hardware (sensors, control processors and actuators) had to be used in order to stabilize the system over all flight regimes and all loading conditions. Considerable effort has been devoted to the design of flight control system for this airplane, e.g. (Rogers and Collins 1992; Clarke et al. 1994; Stein 2003).

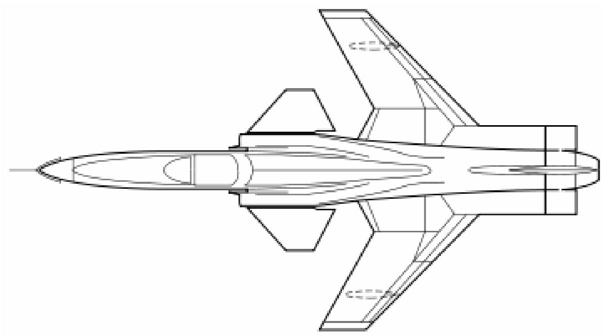

Figure 9: X-29 Aircraft

The benefits of instability (better maneuverability and faster reaction) were desired in the transonic and supersonic flight regimes, so the airplane was designed to be modestly unstable in those regimes. However, due to a basic aerodynamic phenomenon the X-29's slight instability at supersonic speeds turned into a much more dramatic instability at subsonic speeds. A simplified linearized model at one such flight condition given by a transfer function has the following form:

$$G(s) = \frac{s-26}{s-6}. \tag{7}$$

As can be clearly seen, the airplane's real pole (denominator root) is as large as +6 rad/s which makes this system nearly impossible to control manually – it can be compared, simply speaking, to balancing a 1-ft-long stick (Stein 2003). Besides the unstable pole the systems has also strong non-minimum-phase behaviour, i.e. inverse response (a *zero* – numerator root, located at $z_1 = 26$). These facts make this system very difficult to control. State-space realization in the general form (3) can be obtained e.g. as (MATLAB function *ssdata*):

$$\begin{aligned} \boldsymbol{A} &= [6], \boldsymbol{B} = [4], \\ \boldsymbol{C} &= [-5], \boldsymbol{D} = [1]. \end{aligned} \tag{8}$$

Next figure (Fig. 10) shows sensitivity function $S(\omega)$ of a X29 prototype.

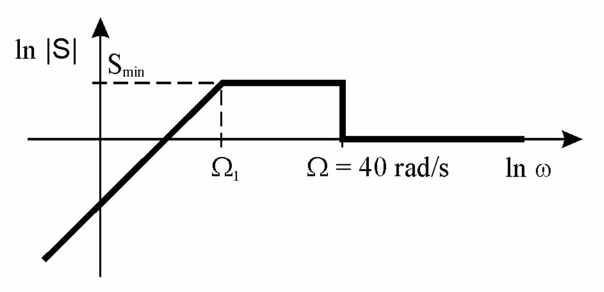

Figure 10: X-29 Sensitivity Function

From the plot it can be seen that the system has considerably limited bandwidth (up to 40 rad/s) which is given by the used HW components (sensors, control processors, actuators), airplane mechanical structure and aerodynamics conditions. Consequently it narrows possibilities of convenient control system design.

All here discussed models are available online at the presented site (Gazdoš and Kolařík 2012) with short description, downloadable model in the MATLAB/ Simulink environment and selected references related to modelling, simulation, control system analysis and design of the processes/systems. Next section briefly presents main structure of the site devoted to unstable systems and outlines main possibilities it offers.

SITE STRUCTURE AND POSSIBILITIES

The database of unstable system has been developed as an open, easily extensible system in the bilingual version (English/Czech). It is easily accessible on-line at the web-address http://www.unstable-systems.cz. A starting version of the site was implemented within the final work (Kolařík 2012) using a free and open source content management system Joomla! of the 2.5 version, available online at http://www.joomla.org and described in e.g. (Marriott and Waring 2013). This makes administration of the site very easy. Basic structure of the web is presented in Fig. 11. Apart from this there are also user-related services such as registration, profile editing and web administration.

HOME Bookmark

The *HOME* bookmark is the starting point of the site. It introduces a purpose of the site, enables registration, shows latest news and recent posts together with simple statistics and some useful links. Registered users have the following possibilities:

- access to files with simulation models of the systems
- access to latest news via a newsletter
- possibility of articles rating
- possibility to add comments to the systems models

It is also possible to search within the site and change language (ENG/CZ) here.

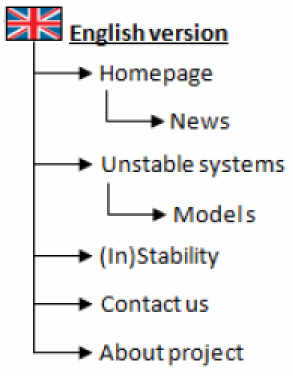

Figure 11: Basic Structure of the Website

UNSTABLE SYSTEMS Bookmark

This bookmark is the main part of the site which contains mathematical models of unstable processes. So far (March 2013) it contains following models:

- ballistic missile
- fluidized bed reactor
- inverted pendulum
- magnetic levitation system
- nonideal continuous stirred-tank reactor
- X-29 aircraft

Every model has the following information:

- brief description of the system
- scheme of the process or picture of the system
- simplified mathematical model
- definition of used variables and parameters
- downloadable model in e.g. MATLAB/Simulink environment
- sources of further information

It is also possible to generate a printable version of the models or send a link of a model by e-mail. Registered users can download the simulation models, add comments and rate them. Besides this it is also possible to search within the models and sort them.

(IN)STABILITY Bookmark

This part of the site explains basics about stability of systems – general understanding of the term, several definitions, such as *BIBO* (Bounded Input – Bounded Output) and *Lyapunov* stability. It also offers further reading on this subject.

Next bookmark *CONTACT US* enables to send questions, suggestions and remarks, etc. on the site and models to the authors. The final bookmark *ABOUT PROJECT* briefly introduces basic information about this project including main authors, brief description and terms of use.

CONCLUSIONS

Modelling and simulation tools play important roles in our lives nowadays. In the case of unstable systems analysis and control design, they role is crucial, however. Experiments with such systems without the proper knowledge about possible consequences can be very hazardous. The goal of this contribution is to present a starting project of the web-based database of unstable systems. This site can help students, teachers, designers, scientists and many others to understand basic properties of unstable systems. This is done via available models of the systems and suggested further readings. The developed web-site is an open, constantly developing system which is still "under construction". Therefore experiences of users, their remarks, suggestions and comments are welcome.

Further development of the site will be focused on the completing an extension of the given information concerning the systems stability, description of the models and suggested further readings. The number of available unstable systems models will of course grow as well as the number of simulation files (not necessarily limited to the MATLAB/Simulink environment). Another interesting extension can be seen in the possibility to implement simulation directly into the site, e.g. using the popular *Easy Java Simulations* open-source software tool. At the moment only the site administrator can add articles into the system. The possibility to do this by registered users is also being considered, which would help to develop the database.

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