APPLICATIONS OF THE GRAPH THEORY FOR OPTIMIZATION IN MANUFACTURING ENVIRONMENT OF THE ELECTRICAL EQUIPMENTS

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ABSTRACT

Depending on user requirements, manufacturing systems dedicated to electrical equipment must produce a wide range of products. The transition from the manufacturing an assortment of product to another involves additional costs which are necessary to adjust the manufacturing system state to the new technology. The manufacturing optimization requires the launching in fabrication of the assortments of products in a predetermined sequence in order to minimize the cost of changing the technical condition of the system and its adaptation to the technological specificity of the new sort. The graph theory can be successfully used in order to optimize the launching of different type of products and the optimal paths which allow minimal costs. Therefore, one can solve the problem of determining the optimal Hamiltonian path from the point of view of minimal time for scanning a certain path. Several applications of optimum Hamiltonian path will be then presented in this paper. They use either the Chen algorithm or depth-first one, being integrated in the same software application.

INTRODUCTION

The peculiarities of design and manufacture of the electrical equipment are a consequence of many functions that must be fulfilled (switching, protection, instrumentation, amplification etc). A very large range of rated values such as: the rated voltage range from 1 kV to hundreds of kV and the rated current range from hundreds of amperes to tens of kA, have a major influence also.

The wide range of electrical equipment that are manufactured in a dedicated production system includes circuit breakers, contactors, fuses, disconnectors, surge arresters, switches, metal clad switchgear or instrument transformers etc. The constructive solution design of each type of equipment depends on the function that it meets, the rated voltage level and the specific environment requirements.

In this context, the design and manufacturing of the electrical equipment require the use of a wide range of conductors, insulating, magnetic materials, and so on. Accordingly, the manufacturing is characterized by the specific technologies for each type of product, specific machinery or equipment, dedicated tools, measurement and control device which are diversified and tailored to the specific measurements. Providing by the user of the reliable requests of the products and the high efficiency of the manufacturing technologies in order to reduce the costs is one of the major objectives of equipment manufacturers.

The efficiency of the manufacturing technologies is accomplished through the use of group technologies and implementing of systematization criteria of the constructive solutions. However, under these circumstances, the transition from the manufacturing an assortment of a product to another involves additional costs which are necessary to adjust the manufacturing system state to the new technology. This means, for example, new settings of machinery, new tools, new materials acquisition, changing instrumentation and control device, new operators during the production, using of other internal transportation system etc. Therefore, the manufacturing optimization requires the launching in fabrication of the assortments of products in a predetermined sequence in order to minimize the cost of changing the technical condition of the system and its adaptation to the technological specifics of a new sort.

Consequently, an optimal sequence of electrical equipment assortments to be launched in manufacturing process allows the optimization by minimizing of the costs.

Among the amelioration methods which can be applied in the manufacturing environment, the graph theory can be successfully used in order to optimize the launching of different type of products and the optimal paths which allow minimal costs.

Many times, the number of possibilities is very large and then, the optimization problem gets another dimension: the time of selection. For these cases, a formalized form

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is useful in order be implemented in a numerical algorithm run by a computer. Even more, it can define another concern: the methods' optimization that means to find that methods which give, on the shortest way, the optimal solution (Abrudan 1980).
The interest for graph theory has grown in the last decades, when it was applied for different problems in areas as economy, sociology, psychology, engineering etc.
The complex systems and situations can be represented by graphs. Its clearly highlight all the aspects of particular states.
A graph G is completely defined by the nodes array X and by the edges array U.
Mathematically, a graph (G) is:

$$G=\left(X,U\right), \tag{1}$$

where,

$$X=\left\{x_1,x_2,\cdots x_n\right\}, \tag{2}$$

is the nodes array and:

$$U=\left\{\left(x,y\right)\mid x,y\in X\right\}, \tag{3}$$

is the edges array.
If we look a graph as the image of a system, the nodes are the system's components and the edges (x_i,x_j) are interdependencies between components. Even a component x_i does not directly influences the component x_j, it can influences by the way of other components. In this case there is a chain of intermediate components $\{x_1, x_2,\ldots x_k\}$. Each component directly influences the next component and finally influences x_j. Each edge (x_i,x_j) signifies that the system can directly switch from node x_i in node x_j.

The minimum cost – optimizing criteria of the manufacturing environment

The optimization refers to the process economy. The objective can be assumed to be the minimization of the adaptation effort for variable production task. An important component of the efforts is represented by the transition costs generated by the switching between different technologies, depending on the manufactured products.
Optimizing the system means to optimize the operation of the system and mainly the transition costs among the life of the manufacturing system.
The transition costs can be mathematically described by the transition cost matrix. The manufacturing technology corresponding to the product P_i is represented by the state S_i of the system. When the manufacturing of the product P_i begins, the system being in the state S_j, a transition cost is generated for the transition from S_j to S_i. This transition cost is denoted as c_{ij}.
Generally, the transition cost matrix (C) can be expressed as:

$$\begin{array}{c} \begin{bmatrix} S_1 & \ldots & S_j & \ldots & S_m \end{bmatrix} \\ \Downarrow \quad \ldots \quad \Downarrow \quad \ldots \quad \Downarrow \end{array}$$

$$\begin{bmatrix} P_1 \\ \vdots \\ P_i \\ \vdots \\ P_m \end{bmatrix} \begin{matrix} \Rightarrow \\ \vdots \\ \Rightarrow \\ \vdots \\ \Rightarrow \end{matrix} \quad C=\begin{bmatrix} 0 & \cdots & c_{1j} & \cdots & c_{1m} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ c_{i1} & \cdots & c_{ij} & \cdots & c_{im} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ c_{m1} & \cdots & c_{mj} & \cdots & 0 \end{bmatrix}, \tag{4}$$

where:

$$S=\left\{S_1,S_2,..,S_j,...,S_m\right\}, \tag{5}$$

is the array of the states of the manufacturing system.

$$P=\left\{P_1,P_2,...,P_j,...,P_m\right\}, \tag{6}$$

is the array of the products types which will be manufactured by the analyzed system.
The elements of the transition costs matrix are as follows:

$$c_{ij}=\begin{cases} c_{ij}>\ 0\ ,\ \text{for } i\neq j \\ c_{ij}=\ 0\ ,\ \text{for } i=j \\ c_{ij}\neq c_{ji} \end{cases}. \tag{7}$$

Hamiltonian paths

For a manufacturing system used for a set of products, it must be revealed the optimal sequence of production launches which minimizes the transition costs. The problem can be solved by applying the graph theory. The solution will be the optimal Hamiltonian path.
One of the most popular economic problems is the optimal Hamiltonian path from the point of view of minimal time for scanning a certain path. The minimum time is equivalent to the shortest path which touches once each node. In addition, the final state must be the same as the initial one. The literature signals more algorithms, precise or heuristic, which can give a satisfying solution of the optimal Hamiltonian path without significant delay.

Chen Algorithm
When the Hamiltonian path must be found in a graph without circuits, the Chen algorithm can be applied. It consists in the following steps (Fiedler 1973):
1. The adjacency matrix ($\underline{A}$) is defined. This matrix will be a $n\times n$ matrix for a system with n states:

$$A=\left[a_{ij}\right],1\leq i,j\leq\ n, \tag{8}$$

where,

$$a_{ij}=\begin{cases} 1,\ \text{if the edge } \left(x_i,x_j\right) \text{ exists} \\ 0,\ \text{in rest} \end{cases}. \tag{9}$$

2. The elements of the paths matrix (D) are determined:

$$D = \left[d_{ij} \right], 1 \le i, j \le n , \qquad (10)$$

where,

$$d_{ij} = \begin{cases} 1, \text{ if at least one edge } \left(x_i, x_j \right) \text{ exists} \\ 0, \text{ otherwise} \end{cases} . \qquad (11)$$

3. The applicability of the Chen algorithm is checked (graph without circuits):
If there is an index i with $d_{ii} = 1$, then the graph has circuits and the Chen algorithm can't be applied.
4. If not, the vertex connectivity for each node $p(x_{i..n})$ is computed.
The vertex connectivity of a node x_i, denoted $p(x_i)$, is the maximum number of nodes which can be reached starting from the node x_i.
If, from any node x_i all the superior nodes and only these can be reached, the vertex connectivity of the node x_i is

$$p(x_i) = n - i . \qquad (12)$$

5. The equality is checked:

$$\sum_{i=1}^{n} p(x_i) = \frac{n \cdot (n-1)}{2} . \qquad (13)$$

If (13) is true, the next step is 6. Otherwise the graph does not have Hamiltonian paths and the algorithm stops.
6. The nodes are descendent ordered according to the vertex connectivity and the optimal Hamiltonian path is obtained.

„Depth-First" Algorithm
Another algorithm used for minimum Hamiltonian path determination is the "Depth-First" Algorithm.
In this strategy, the nodes are explored depending on the depth and the upper level is resumed only if the search arrives in a dead-end. The depth search is not completed for infinite trees, when cycles occur. If a constraint is defined for avoiding repeated states, the strategy can be completed. On the other hand, the strategy can achieve the solution in the most depth, without taking into account the cost and consequently, it is not optimal. If additional minimal costs constraints are added, the algorithm can be optimized.
The algorithm consists in following steps (Chin et al. 1982):

1. The adjacency matrix (A) is defined as in (8) and (9).
2. The transition costs matrix C is defined (the costs for transition from state $\underline{i}$ to state j):

$$C = \begin{bmatrix} c_{11} & \cdots & c_{1n} \\ \vdots & \ddots & \vdots \\ c_{n1} & \cdots & c_{nn} \end{bmatrix} . \qquad (14)$$

The searching algorithm expands each node and explores the edges down to the final node. So, a path is stated.
In order to retrieve the rest of the paths, the algorithm returns to the previous node and tries to find another path, different by the one already stated. The algorithm ends when all the paths were checked and consequently all the Hamiltonian paths were found.
The most important advantage of the depth-first algorithm resides in the requested memory resources which a minimal due to the linear complexity. The algorithm must keep only a single path from the initial node to the current final node, together with all the unexpanded nodes which are "brothers" (have a common predecessor) with the nodes in the current path.
The main disadvantage of the algorithm is that is not optimal and it is not completed if additional constraints are not considered.

APPLICATIONS FOR PREDETERMINED SEQUENCE OF THE PRODUCTS MANUFACTURING

For the complex manufacturing environments, the optimum element should be selected from a large array and consequently, the time for selection increases too much. This why, it is useful to have an algorithm capable to be run on a machine.
Following, several applications of optimum Hamiltonian path will be presented. They use either the Chen algorithm or depth-first one, being integrated in the same software application.
When the software application is launched, the user can choose the algorithm to be used: Chen or depth-first (Fig. 1).

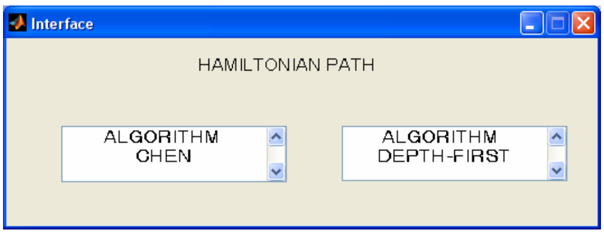

Figure 1: The Selection Algorithms

FINDING THE OPTIMUM HAMILTONIAN PATH BY USING CHEN ALGORITHM

This algorithm can be used only for oriented graphs which does not contain circuits. The returned result will be the single Hamiltonian path of a given graph.

The input data are defined (Fig. 2):
- the number of nodes which represents the number of products and the number of states respectively. For example, $n = 10$;
- the adjacency matrix, A, an $n \times n$ matrix, defined in accordance with (8) and (9):

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \quad (15)$$

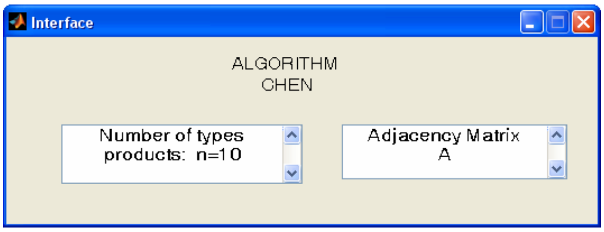

Figure 2: The Input Date of Chen Algorithm

The application determines the paths matrix, in accordance with (10) and (11):

$$D = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \quad (16)$$

The existence of circuits within the given graph is checked. This means to check if for any i we have $d(i,i) = 1$. In this case the application ends (Fig. 3).

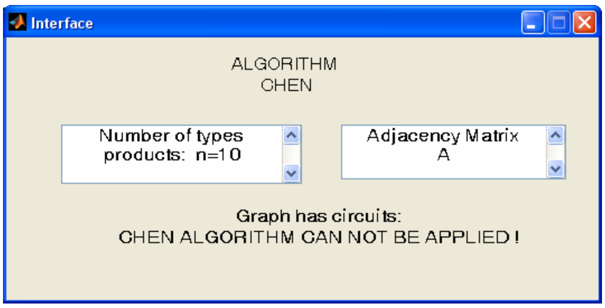

Figure 3: The Message "STOP APPLICATION"

If the graph does not contain circuits, the vertex connectivity is computed for each node based on (12) and then the condition (13) is checked. If this condition is not fulfilled, it means that there is not any Hamiltonian path (Fig.4).

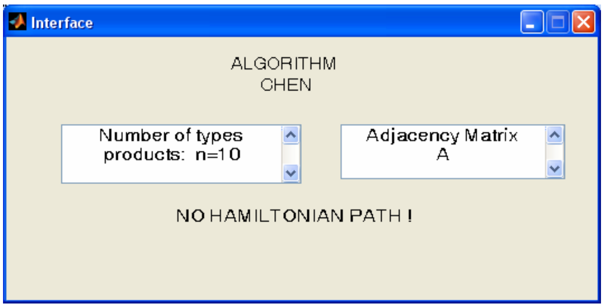

Figure 4: The Message "NO HAMILTONIAN PATH"

If the condition (13) is fulfilled, the optimum Hamiltonian path results as the sequence of scanning the graph's nodes (Fig.5).

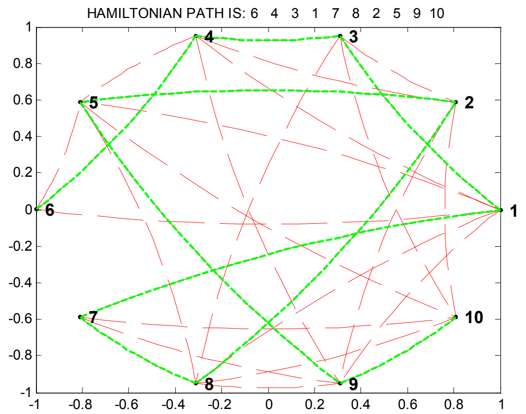

Figure 5: Optimum Hamiltonian Path

FINDING THE OPTIMUM HAMILTONIAN PATH BY USING DEPTH-FIRST ALGORITHM

The developed software application can analyze oriented, non-oriented, with or without circuits graphs. It returns the optimum Hamiltonian path from the point of view of costs (Chin et al. 1982).

The input data are: the transitions costs matrix (C) and the adjacency matrix (A) (Fig.6).

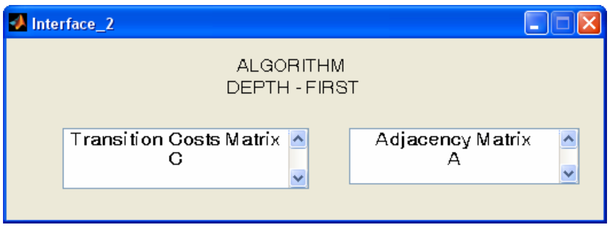

Figure 6: The Input Date of Algorithm Depth-First

As an example, the transitions costs matrix is:

$$C=\begin{bmatrix} 5 & 10 & 10 & 20 & 30 & 14 & 15 & 10 & 17 & 30 \\ 15 & 4 & 17 & 10 & 30 & 40 & 50 & 20 & 30 & 25 \\ 21 & 23 & 35 & 20 & 24 & 60 & 30 & 10 & 25 & 30 \\ 41 & 20 & 35 & 36 & 20 & 42 & 10 & 46 & 9 & 8 \\ 16 & 18 & 11 & 65 & 10 & 14 & 25 & 65 & 40 & 70 \\ 14 & 25 & 32 & 51 & 20 & 30 & 65 & 21 & 41 & 20 \\ 36 & 25 & 14 & 15 & 34 & 26 & 10 & 32 & 19 & 18 \\ 30 & 25 & 26 & 81 & 24 & 65 & 61 & 30 & 27 & 38 \\ 14 & 25 & 34 & 51 & 20 & 42 & 13 & 51 & 62 & 80 \\ 5 & 12 & 43 & 62 & 30 & 20 & 15 & 81 & 49 & 30 \end{bmatrix}.$$

The adjacency matrix (A) is the same as the one considered for Chen algorithm (15).

The software application determines all the possible Hamiltonian paths and computes then the minimum costs one (Fig. 7).

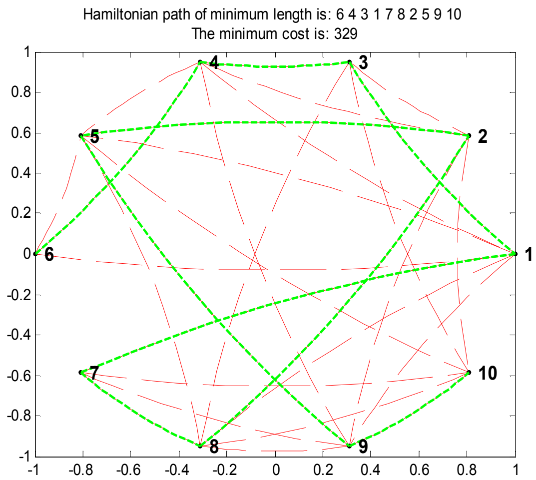

Figure 7: Minimum Costs Hamiltonian Path

If no Hamiltonian path was found, the application displays “No Hamiltonian Path” (Fig.8).

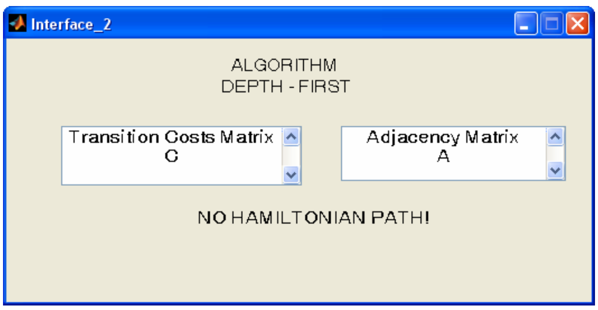

Figure 8: The Message “NO HAMILTONIAN PATH”

If another transitions costs matrix is considered,

$$C_1=\begin{bmatrix} 10 & 5 & 15 & 20 & 30 & 14 & 15 & 10 & 17 & 30 \\ 15 & 4 & 17 & 10 & 30 & 40 & 70 & 25 & 30 & 25 \\ 21 & 23 & 35 & 30 & 24 & 20 & 30 & 10 & 25 & 30 \\ 41 & 20 & 35 & 36 & 20 & 42 & 10 & 46 & 9 & 8 \\ 16 & 50 & 11 & 10 & 10 & 14 & 25 & 65 & 40 & 70 \\ 14 & 10 & 32 & 9 & 20 & 30 & 5 & 21 & 41 & 20 \\ 36 & 5 & 14 & 5 & 10 & 45 & 10 & 32 & 19 & 18 \\ 30 & 5 & 26 & 10 & 30 & 5 & 10 & 30 & 27 & 38 \\ 14 & 25 & 34 & 51 & 20 & 15 & 13 & 51 & 4 & 80 \\ 5 & 12 & 43 & 3 & 30 & 20 & 15 & 81 & 49 & 30 \end{bmatrix},$$

The result will be the one depicted in Fig. 9.

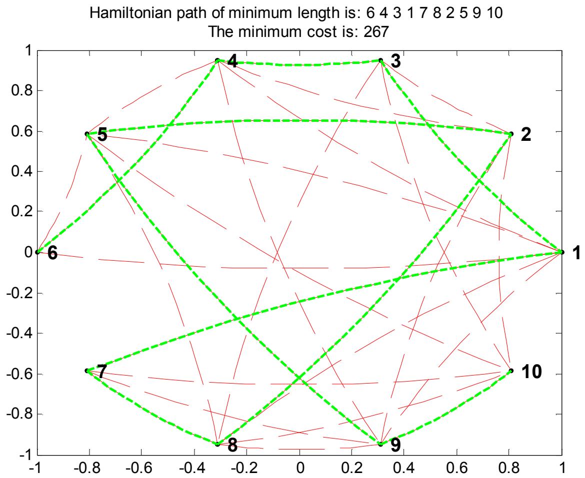

Figure 9: Minimum Costs Hamiltonian Path

As was expected, the Hamiltonian path is the same, but the costs are smaller if the matrix C_1 is considered.

CONCLUSIONS

The optimal Hamiltonian paths were determined for several applications by using algorithms known in the literature. Chen algorithm can be used only for oriented graphs which does not contain circuits. The returned result was being the single Hamiltonian path of a given graph. Depth-First Algorithm can analyze oriented, non-oriented, with or without circuits graphs. It returns the optimum Hamiltonian path from the point of view of costs. Resulted cost can even be reduced by restrictions on the transition costs matrix.

The paper presented more examples for illustration of the flexibility and the performances of the software application.

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