NONLINEAR CONTROL OF A SHELL AND TUBE HEAT EXCHANGER

Petr Dostál¹,², Vladimír Bobál¹,², and Jiří Vojtěšek²
¹Centre of Polymer Systems, University Institute, Tomas Bata University in Zlín, Nad Ovčinnou 3685, 760 01 Zlín, Czech Republic.  
²Department of Process Control, Faculty of Applied Informatics, Tomas Bata University in Zlín, Nad Stranemi 4511, 760 05 Zlín, Czech Republic  
{dostalp;bobal;vojtesek}@fai.utb.cz

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Tubular heat exchanger, nonlinear model, steady-state characteristics, external linear model, parameter estimation, polynomial approach, control simulation.

ABSTRACT
The paper deals with design and simulation of nonlinear adaptive control of a shell and tube heat exchanger. The method is based on factorization of the controller on a nonlinear static part and an adaptive linear dynamic part. The nonlinear static part is derived using inversion and subsequent exponential approximation of simulated or measured steady-state characteristics of the exchanger. The linear dynamic part is then obtained from an external linear model of nonlinear elements of the closed-loop. The parameters of the external linear model are recursively estimated via a corresponding delta model. The control law in the 1DOF and 2DOF control system structures is derived using the polynomial approach.

INTRODUCTION
Heat exchangers are an essential part of many technologies in energy and chemical industry, polymer manufacturing, petroleum refineries, and many others. By construction, heat exchangers can be classified into exchangers with direct contact, various types of plate exchangers, and, shell and tube heat exchangers (STHEs), see, e.g. (Smith 2005; Hewitt et al. 1994; Incropera et al. 2011).

As known, STHEs are most common types of heat exchangers. From the system theory, they belong to a class of nonlinear distributed parameter systems with mathematical models in the form of nonlinear partial differential equations. Modelling and simulation of such processes are described in many publications, e.g. in (Luyben 1989; Corriou 2004; Babu 2004; Oggunnaike and Rao 1994). As known, these processes can be hardly controllable by conventional methods that can lead to control of a poor quality. In this case, some advanced control methods should be used such as adaptive, predictive, optimal or nonlinear control, and some others. Obviously, control design always requires a preliminary steady-state and dynamic analysis of the process by simulation tools. Some methods of numerical mathematics used to build simulation models can be found e.g. in (Nevriva et al. 2009; Cook 2002). The aim of the paper is an application of nonlinear control and subsequent control simulation of a simple type of the shell and tube heat exchanger. The control strategy is based on the idea of factorization of the controller on a nonlinear static part (NSP) and an adaptive linear dynamic part (LDP). Similar approaches can be found e.g. in (Chen et al. 2006; Dostál et al. 2011b). The nonlinear static part is obtained from simulated or measured steady-state characteristic of the STHE, its inversion, exponential approximation, and, subsequently, its differentiation. On behalf of development of the linear part, the NSP including the nonlinear model of the STHE is approximated by a continuous-time external linear model (CT ELM). For the CT ELM parameter estimation, an external delta model with the same structure as the CT model is used. The basics of delta models have been described e.g. in (Mukhopadhyay et al. 1992; Garnier and Wang 2008). Although delta models belong into discrete models, they do not have such disadvantageous properties connected with shortening of a sampling period as discrete z-models. In addition, parameters of delta models can directly be estimated from sampled signals. Moreover, it can be easily proved that these parameters converge to parameters of CT models for a sufficiently small sampling period (compared to the dynamics of the controlled process), as shown in (Stericker and Sinha 1993; Dostál et al. 2004).

The 1DOF and the 2DOF control system structures are considered. In the first case, the control system includes only a feedback controller, in the second case, the controller consists of a feedback and a feedforward part. Such structures were described and applied e.g. in (Dostál et al. 2011a; Grimble 1993). Then, resulting CT controllers are derived using the polynomial approach and the pole placement method (Kučera 1993; Míkleš and Fikar 2004).

The simulations are performed on a nonlinear model of the STHE.

MODEL OF THE STHE
Consider an ideal plug-flow shell and tube heat exchanger in the fluid phase and with the counterflow cooling. The fluid flowing in tubes is cooled by a fluid flowing in the shell as shown in Fig. 1. Heat losses and heat conduction along the metal walls of tubes are
assumed to be negligible, but dynamics of the metal walls of tubes are significant. All densities, heat capacities, and heat transfer coefficients are assumed to be constant. Under above assumptions, the STHE model can be described by three partial differential equations (PDEs) in the form

\[
\frac{\partial T_c}{\partial t} + v_c \frac{\partial T_c}{\partial z} = c_1 (T_w - T_c) \tag{1}
\]

\[
\frac{\partial T_w}{\partial t} = c_2 (T_r - T_w) + c_3 (T_c - T_w) \tag{2}
\]

\[
\frac{\partial T_r}{\partial t} - v_c \frac{\partial T_r}{\partial z} = c_4 (T_w - T_r) \tag{3}
\]

with initial conditions

\[T_r(z,0) = T^*_r(z), \quad T_c(z,0) = T^*_c(z), \quad T_w(z,0) = T^*_w(z)\]

and boundary conditions

\[T_r(0,t) = T_{r_0}(t), \quad T_c(L,t) = T_{c_L}(t).\]

The parameters in (1) – (3) are

\[v_r = \frac{4q_r}{n_1 \pi d^2}, \quad v_c = \frac{4q_c}{\pi (d^2 - n_1 d^1)}\]

\[c_1 = \frac{4\alpha_1}{d_1 \rho c_{pr}}, \quad c_2 = \frac{4d_1 \alpha_1}{(d_2^2 - d_1^2) \rho_w c_{pw}}\]

\[c_3 = \frac{4d_2 \alpha_2}{(d_2^2 - d_1^2) \rho_w c_{pw}}, \quad c_4 = \frac{4d_1 d_2 \alpha_2}{(d_3^2 - (d_1^2 + d_2^2)) \rho_c c_{pc}}\]

where \(t\) stands for the time, \(z\) for the axial space variable, \(T\) for temperatures, \(v\) for flow of fluids, \(d\) for fluid flow velocities, \(d_i\) for inner diameter of the tube, \(d_2\) for outer diameter of the tube, \(n_1\) for diameter of the shell, \(\rho\) for densities, \(c_p\) for specific heat capacities, \(\alpha\) for heat transfer coefficients, \(n_1\) is the number of tubes and \(L\) is the length of tubes. Subscripts denote \(r\) describe the refrigerated fluid (RF), \(w\) the metal walls of tubes, \(c\) the cooling fluid (CF), and the superscript \(s\) steady-state values.

From the system engineering point of view, \(T_r(L,t) = T_{rout}\) and \(T_c(0,t) = T_{cout}\) are the output variables, and, \(q_r(t), q_c(t), T_{r_0}(t)\) and \(T_{c_L}(t)\) are the input variables. For the control purposes, the output temperature of the refrigerated fluid \(T_r(L,t) = T_{rout}(t)\) is considered as the controlled output, and, the coolant flow \(q_c(t)\) as the control input, while other inputs can enter into the process as disturbances.

The parameter and steady-state input values with their correspondent units are given in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(L)</td>
<td>8 m</td>
</tr>
<tr>
<td>(n_1)</td>
<td>1100</td>
</tr>
<tr>
<td>(d_1)</td>
<td>0.022 m</td>
</tr>
<tr>
<td>(d_2)</td>
<td>0.024 m</td>
</tr>
<tr>
<td>(d_3)</td>
<td>1 m</td>
</tr>
<tr>
<td>(\rho_r)</td>
<td>985 kg/m³</td>
</tr>
<tr>
<td>(c_{pr})</td>
<td>4.05 kg/K</td>
</tr>
<tr>
<td>(\rho_w)</td>
<td>7800 kg/m³</td>
</tr>
<tr>
<td>(c_{pw})</td>
<td>0.71 kg/K</td>
</tr>
<tr>
<td>(\rho_c)</td>
<td>998 kg/m³</td>
</tr>
<tr>
<td>(c_{pc})</td>
<td>4.18 kg/K</td>
</tr>
<tr>
<td>(\alpha_1)</td>
<td>5.8 kJ/m²/s K</td>
</tr>
<tr>
<td>(\alpha_2)</td>
<td>3.6 kJ/m²/s K</td>
</tr>
<tr>
<td>(T_{r_0})</td>
<td>373 K</td>
</tr>
<tr>
<td>(T_{c_L})</td>
<td>293 K</td>
</tr>
<tr>
<td>(q_c^s)</td>
<td>0.1 m³/s</td>
</tr>
</tbody>
</table>

### COMPUTATION MODELS

For computation of both steady-state and dynamic characteristics, the finite differences method is employed. The procedure is based on substitution of the space interval \(z \in (0,L)\) by a set of discrete node points \(\{z_i\}\) for \(i = 1, \ldots, n\), and, subsequently, by approximation of derivatives with respect to the space variable in each node point by finite differences. Two types of finite differences are applied, either the backward or the forward finite difference.

#### Dynamic Model

Applying the finite differences method, PDEs (1) – (3) are approximated by a set of ODEs in the form

\[
\frac{dT_r(i)}{dt} = \left(\frac{v_r}{h} + c_1\right)T_r(i) - \frac{v_r}{h}T_r(i-1) + c_1T_w(i) \tag{5}
\]

\[
\frac{dT_w(i)}{dt} = c_2[\frac{T_r(i) - T_w(i)}{h}] + c_3[\frac{T_c(i) - T_w(i)}{h}] \tag{6}
\]

\[
\frac{dT_c(i)}{dt} = \left(\frac{v_c}{h} + c_4\right)T_c(i) - \frac{v_c}{h}T_c(i+1) + c_4T_w(i) \tag{7}
\]

for \(i = 1, \ldots, n\), \(j = n-i+1\), and, with initial conditions \(T_r(i,0) = T^*_r(i)\), \(T_w(i,0) = T^*_w(i)\) and \(T_c(i,0) = T^*_c(i)\) for \(i = 1, \ldots, n\). In (5) – (7), \(h\) is the diskretization step.

The boundary conditions enter into Eqs. (5) – (7) for \(i = 1\).

Here, the controlled output is computed as

\[T_{rout}(t) = T_r(n,t)\]  

### Steady-State Model

Computation of steady-state characteristics is necessary not only for a steady-state analysis but the steady state values also constitute initial conditions in ODEs (5) – (7). The steady-state model can simply be derived...
equating the time derivatives in (5) – (7) to zero. Then, the steady-state characteristics can be computed by an iterative method.

Steady-State Characteristics
The dependence of the RF output temperature on the coolant flow in the steady-state is in Fig. 2. In subsequent control simulations, the operating interval for \( q_c \) has been determined as

\[
q_c^{\text{min}} \leq q_c(t) \leq q_c^{\text{max}}.
\]

With regard to the purposes of a latter approximation of the steady-state characteristics, the values \( q_c^L < q_c^{\text{min}} \) and \( q_c^U > q_c^{\text{max}} \) are established that denote the lower and upper bound of \( q_c \) used for the approximation. Their values together with values in (9), and, to them corresponding temperatures are in Table 2.

<table>
<thead>
<tr>
<th>Interval for approximation</th>
<th>Operating interval</th>
<th>Operating point</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.04 0.06 0.08 0.10 0.12 0.14 0.16 0.18</td>
<td>300 305 310 315 320 325 330 335</td>
<td>0.04 0.06 0.08 0.10 0.12 0.14 0.16 0.18</td>
</tr>
</tbody>
</table>

Figure 2: Dependence of the RF Output Temperature on the Coolant Flow Rate.

Table 2: Input and Output Values Used in Approximation.

<table>
<thead>
<tr>
<th>( q_c^L ) = 0.05 m³/s</th>
<th>( q_c^U ) = 0.18 m³/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_{\text{out}}^U ) = 332.09 K</td>
<td>( T_{\text{out}}^L ) = 301.21 K</td>
</tr>
<tr>
<td>( q_c^{\text{min}} ) = 0.06 m³/s</td>
<td>( q_c^{\text{max}} ) = 0.16 m³/s</td>
</tr>
<tr>
<td>( T_{\text{out}}^{\text{max}} ) = 325.49</td>
<td>( T_{\text{out}}^{\text{min}} ) = 302.35</td>
</tr>
</tbody>
</table>

CONTROLLER DESIGN
As previously introduced, the controller consist of a nonlinear static part and a linear dynamic part as shown in Fig. 3.

![Controller Scheme](image)

Figure 3: Controller Scheme.

Here, the control input and the controlled output variables are considered in the form

\[
u(t) = \Delta q_c(t) = q_c(t) - q_c^s, \quad y(t) = T_{\text{out}}(t) - T_{\text{out}}^s
\]

where \( q_c^s = 0.9, T_{\text{out}}^s = 312.53 \) represents an operating point around which the changes take place during the control.

The LDP creates a linear dynamic relation which represents a difference of the RF output temperature adequate to its desired value. Then, the NSP generates a static nonlinear relation between \( u_0 \) and a corresponding increment (decrement) of the coolant flow rate.

Nonlinear Static Part of the Controller
The NSP derivation appears from a simulated or measured steady-state characteristics. The coordinates on the graph axis are defined as

\[
\xi = \frac{\xi - q_c^L}{q_c^U - q_c^L}, \quad \psi = \frac{T_{\text{out}}^U - T_{\text{out}}^L}{T_{\text{out}}^U - T_{\text{out}}^L}
\]

where

\[
q_c^L \leq q_c^s \leq q_c^U.
\]

In term of the practice, it can be supposed that the measured data will be affected by measurement errors. The simulated steady-state characteristics that corresponds to reality is shown in Fig. 4.

Changing the axis, the inverse of this characteristic can be approximated by a function from the ring of polynomial, exponential, rational, eventually, by other type functions. Here, the second order exponential approximate function has been found in the form

\[
\xi = -1.3 + 0.619 e^{-0.505 \psi} + 1.606 e^{-0.213 \psi}.
\]

The inverse characteristic together with its approximation is in Fig. 5. Now, a difference of the coolant flow rate \( u(t) = \Delta q_c(t) \) in the output of the NSP can be computed for each \( T_{\text{out}} \) as

\[
u(t) = \Delta q_c(t) = \frac{q_c^U - q_c^L}{T_{\text{out}}^U - T_{\text{out}}^L} \left( \frac{d \xi}{d \psi} \right)_{\psi(T_{\text{out}})} u_0(t)
\]
Figure 5: Approximation of the Inverse Characteristics.

where the derivative of \( \xi \) with respect to \( \psi \) takes the form

\[
\frac{d\xi}{d\psi} = -2.789 e^{-4.505\psi} - 0.342 e^{-0.213\psi}.
\] (15)

Its plot is shown in Fig. 6.

Figure 6: Derivative of Inverse Characteristics.

**CT and Delta External Linear Model**

The nonlinear component (NC) of the closed-loop consisting of the NSP of the controller and the STHE nonlinear model is shown in Fig. 7.

The CT external linear model of this nonlinear component is chosen on the basis of step responses simulated around the above defined operating point. Step responses are shown in Fig. 8. Taking into account profiles of curves in Fig. 8 with zero derivatives in \( t = 0 \), the second order CT ELM has been chosen in the form of the second order linear differential equation

\[
\ddot{y}(t) + a_1 \dot{y}(t) + a_0 y(t) = b_0 u(t)
\] (16)

and, in the complex domain, as the transfer function

\[
G(s) = \frac{b_0}{s^2 + a_1 s + a_0}.
\] (17)

Establishing the \( \delta \)-operator

\[
\delta = \frac{q - 1}{T_0}
\] (18)

where \( q \) is the forward shift operator and \( T_0 \) is the sampling period, the delta ELM corresponding to (17) takes the form

\[
\delta^2 y(t') + a_1' \delta y(t') + a_0' y(t') = b_0' u(t')
\] (19)

where \( t' \) is the discrete time. When the sampling period is shortened, the delta operator approaches the derivative operator, and, the estimated parameters \( a', b' \) reach the parameters \( a, b \) of the CT model.

**Delta Model Parameter Estimation**

Substituting \( t' = k - 2 \), equation (19) can be rewritten to the form

\[
\delta^2 y(k-2) + a_1' \delta y(k-2) + a_0' y(k-2) = b_0' u(k-2).
\] (20)

Establishing the regression vector

\[
\Phi_\delta(k) = \begin{pmatrix} y(k-2) & y(k-3) & y(k-4) \\ \delta y(k-2) & \delta y(k-3) & \delta y(k-4) \end{pmatrix}
\] (21)

where

\[
\delta y(k-2) = \frac{y(k-1) - y(k-2)}{T_0}
\] (22)

the vector of delta model parameters

\[
\Theta_\delta(k) = \begin{pmatrix} a_1' a_0' b_0' \end{pmatrix}
\] (23)

is recursively estimated by the least squares method with exponential and directional forgetting from the ARX model, e.g. (Bobál et al. 2005).

\[
\delta^2 y(k-2) = \Theta_\delta^T(k) \Phi_\delta(k-1) + e(k)
\] (24)

where

\[
\delta^2 y(k-2) = \frac{y(k-2) - 2 y(k-1) + y(k-2)}{T_0}
\] (25)
Linear Dynamic Part of the Controller

In the control simulations, the 1DOF and 2DOF control system structures are considered. While the 1DOF structure includes only the feedback controller $Q$, a controller in the 2DOF structure consist of the feedback part $Q$ and the feedforward part $R$ as shown in Figs. 9 and 10. In both figures, $w$ denotes the reference signal, $y$ the controlled output and $u_0$ the input to the ELM. The reference $w$ and the disturbance $v$ are considered as step functions with transforms

$$W(s) = \frac{w(s)}{s}, \quad V(s) = \frac{v(s)}{s}. \quad (26)$$

The controller transfer functions in both structures are considered as

$$Q(s) = \frac{q(s)}{p(s)}, \quad R(s) = \frac{r(s)}{p(s)} \quad (27)$$

where $q$, $r$ and $p$ are coprime polynomials in $s$ fulfilling conditions of properness $\deg r \leq \deg p$ and $\deg q \leq \deg p$. For both step input signals, the polynomial $p(s)$ takes the form $p(s) = s \tilde{p}(s)$.

It is well known from the algebraic control theory that controllers ensuring stability of the control system result in the polynomial ring from a solution of the polynomial equation

$$a(s) \tilde{p}(s) + b(s) q(s) = d(s) \quad (28)$$

in the 1DOF structure, and, from a couple of polynomial equations

$$a(s) \tilde{p}(s) + b(s) q(s) = d(s) \quad (29)$$
$$s \tilde{t}(s) + b(s) r(s) = d(s) \quad (30)$$

in the 2DOF structure. In both cases, the polynomial $d$ on their right sides is a stable polynomial.

Then, the controller's transfer functions take forms

$$Q(s) = \frac{q(s)}{s \tilde{p}(s)} = \frac{q_2 s^2 + q_1 s + q_0}{s(s + p_0)} \quad (31)$$

$$R(s) = \frac{r(s)}{s \tilde{p}(s)} = \frac{r_0}{s(s + p_0)} \quad (32)$$

where $Q(s)$ is the same for both structures. Moreover, the equality $r_0 = q_0$ can easily be obtained.

The controller parameters then follow from solutions of polynomial equations (29) and (30) and depend upon coefficients of the polynomial $d$. In this paper, the polynomial $d$ with roots determining the closed-loop poles is chosen as

$$d(s) = n(s)(s + \alpha)^2 \quad (33)$$

where $n$ is a stable polynomial obtained by spectral factorization

$$a^2(s) a(s) = n^2(s) n(s) \quad (34)$$

and $\alpha$ is a selectable parameter that can usually be chosen by way of simulation experiments. Note that a choice of $d$ in the form (33) provides the control of a good quality for aperiodic controlled processes. The polynomial $n$ has the form

$$n(s) = s^2 + n_1 s + n_0 \quad (35)$$

with coefficients

$$n_0 = \sqrt{d_0^2}, \quad n_1 = \sqrt{d_1^2 + 2n_0 - 2d_0} \quad (36)$$

Then, the controller parameters can be obtained from solution of the matrix equation

$$\begin{pmatrix}
1 & 0 & 0 & 0 & p_0 \\
0 & b_0 & 0 & 0 & q_2 \\
0 & 0 & b_0 & 0 & q_1 \\
0 & 0 & 0 & b_0 & q_0 \\
0 & 0 & 0 & 0 & d_0
\end{pmatrix} = \begin{pmatrix}
d_3 - a_1 \\
d_2 - a_0 \\
d_2 - a_0 \\
d_1 \\
d_0
\end{pmatrix} \quad (37)$$

where

$$d_3 = n_1 + 2\alpha, \quad d_2 = 2\alpha n_1 + n_0 + \alpha^2 \quad (38)$$
$$d_1 = 2\alpha n_0 + \alpha^2 n_1, \quad d_0 = \alpha^2 n_0$$

Evidently, the controller parameters can be adjusted by the selectable parameter $\alpha$.

**SIMULATION RESULTS**

All control simulations were performed around the above defined operating point.

For the start (adaptation phase 15 min), a P controller with experimentally tuned small gain was used in all simulations.

An effect of the parameter $\alpha$ on the controlled output and the control input responses in the 1DOF structure is presented in Figs. 11 and 12. While a difference between controlled outputs for selected values $\alpha$ is not very significant, an increasing $\alpha$ results in higher changes of the control input. Both signals significantly respond to further increase of the value $\alpha$ as it can be seen in Figs. 13 and 14 for the 1DOF structure. There, the controlled output exhibits higher overshoots, and,
the control input oscillates between constraints. Therefore, the selection of an appropriate value $\alpha$ is very important especially in control of a real process. The controlled output and the control input in both control system structures for a selected value $\alpha$ are compared in Figs. 15 and 16. There, a difference in controlled outputs is minimal, but the control input in the 2DOF structure shows smaller changes. The big difference in the use of both structures for the higher $\alpha$ is but to see in Figs. 13 and 14 where the results from the 1DOF structure are unacceptable while the 2DOF structure provides good control responses.

CONCLUSIONS

The paper dealt with design and simulation of nonlinear adaptive control of a shell and tube heat exchanger. The control strategy is based on factorization of a controller into the linear and the nonlinear part. The design of the controller nonlinear part employs steady-state characteristics of the process and their additional modifications. Then, the nonlinear part of the closed-loop is approximated by a continuous time external linear model with parameters obtained via recursive parameter estimation of a corresponding delta model. The resulting linear part of the controller is considered for both 1DOF and 2DOF control system structures and derived using the polynomial approach. The simulation results demonstrate improved usability of the 2DOF structure especially in terms of the control input characteristics.

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AUTHOR BIOGRAPHIES

PETR DOSTÁL studied at the Technical University of Pardubice, where he obtained his master degree in 1968 and PhD. degree in Technical Cybernetics in 1979. In the year 2000 he became professor in Process Control. He is now Professor in the Department of Process Control, Faculty of Applied Informatics of the Tomas Bata University in Zlin. His research interest are modeling and simulation of continuous-time chemical processes, polynomial methods, optimal and adaptive control.

VLADIMÍR BOBÁL was born in Slavičín, Czech Republic. He graduated in 1966 from the Brno University of Technology. He received his Ph.D. degree in Technical Cybernetics at Institute of Technical Cybernetics, Slovak Academy of Sciences, Bratislava, Slovak Republic. He is now Professor in the Department of Process Control, Faculty of Applied Informatics of the Tomas Bata University in Zlin. His research interests are adaptive control systems, system identification and CAD for self-tuning controllers.

JÍRI VOJTĚŠEK was born in Zlin, Czech Republic in 1979. He studied at Tomas Bata University in Zlin, Czech Republic, where he received his M.Sc. degree in Automation and control in 2002. In 2007 he obtained Ph.D. degree in Technical cybernetics at Tomas Bata University in Zlin. He now works as an assistant professor at the Department of Process Control, Faculty of Applied Informatics of the Tomas Bata University in Zlin. His research interests are modeling and simulation of continuous-time chemical processes, polynomial methods, optimal, adaptive and nonlinear control.

Dr. Vojtesek is the Chairman of the European Conference of the Modelling and Simulation (ECMS) and IPC member of the IASTED.