

# SIMULATION OF MULTIVARIABLE CONTINUOUS-TIME DECOUPLING CONTROL

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## KEYWORDS

Multivariable control, Control algorithms, Adaptive control, Polynomial methods, Simulation.

## ABSTRACT

The paper is focused on an implementation of a decoupling multivariable controller in the Matlab/Simulink environment. The control algorithm is based on polynomial theory and pole – placement. A decoupling compensator is used to suppress interactions between control loops. The controller was realized both with fixed parameters and with recursive identification of a model of the controlled system. The internal structure of the controller enables its easy modification and implementation of further similar control algorithms.

## INTRODUCTION

Typical technological processes require the simultaneous control of several variables related to one system. Each input may influence all system outputs. The design of a controller for such a system must be quite sophisticated if the system is to be controlled adequately. There are many different methods of controlling MIMO (multi input – multi output) systems. Several of these use decentralized PID controllers (Cui and Jacobsen, 2002) others apply single input-single-output (SISO) methods extended to cover multiple inputs (Chien et al, 1987). The classical approach to the control of multi-input–multi-output (MIMO) systems is based on the design of a matrix controller to control all system outputs at one time. The basic advantage of this approach is its ability to achieve optimal control performance because the controller can use all the available information about the controlled system. Controllers are based on various approaches and various mathematical models of controlled processes. A standard technique for MIMO control systems uses polynomial methods (Kučera, 1980, Kučera 1991, Vidyasagar 1985) and is also used in this paper. Controller synthesis is reduced to the solution of linear Diophantine equations (Kučera, 1993).

One controller, which enables decoupling control of TITO (two input-two output) systems, is presented. The proposed control algorithm applies a decoupling compensator (Krishnawamy et al, 1991, Peng, 1990, Tade et al, 1986) to suppress undesired interactions

between control loops. The controller was realized both with fixed parameters and with recursive identification of a model of the controlled system.

For purposes of simulation, the controller was realized in the Matlab/Simulink environment as a mask of subsystem. It can be easily inserted into Simulink schemes of the closed loop. No initialization is needed before the simulation start. Blocks for computation of the control law and for recursive identification were realized as S-functions. The internal structure of the controller enables its easy modification and implementation of further similar control algorithms. A simulation experiment is also introduced.

## MODEL OF THE CONTROLLED SYSTEM

A general transfer matrix of a two-input–two-output system with significant cross-coupling between the control loops is expressed as

$$\mathbf{G}(s) = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix} \quad (1)$$

$$\mathbf{Y}(s) = \mathbf{G}(s)\mathbf{U}(s) \quad (2)$$

where  $\mathbf{U}(s)$  and  $\mathbf{Y}(s)$  are vectors of the manipulated variables and the controlled variables.

$$\mathbf{Y}(s) = [y_1(s), y_2(s)]^T \quad \mathbf{U}(s) = [u_1(s), u_2(s)]^T \quad (3)$$

It may be assumed that the transfer matrix can be transcribed to the following form of the matrix fraction:

$$\mathbf{G}(s) = \mathbf{A}^{-1}(s)\mathbf{B}(s) = \mathbf{B}_1(s)\mathbf{A}_1^{-1}(s) \quad (4)$$

where the polynomial matrices  $\mathbf{A} \in R_{22}[s]$ ,  $\mathbf{B} \in R_{22}[s]$  represent the left coprime factorization of matrix  $\mathbf{G}(s)$  and the matrices  $\mathbf{A}_1 \in R_{22}[s]$ ,  $\mathbf{B}_1 \in R_{22}[s]$  represent the right coprime factorization of  $\mathbf{G}(s)$ . The further described algorithm is based on a model with polynomials of second order. This model proved to be effective for control of several TITO laboratory processes (Kubalčík and Bobál, 2006), where controllers based on a model with polynomials of the first order failed. In case of decoupling control using a compensator it is useful to consider matrix  $\mathbf{A}(s)$  as diagonal. The reason is explained in the following section.

$$\mathbf{A}(s) = \begin{bmatrix} s^2 + a_1s + a_2 & 0 \\ 0 & s^2 + a_7s + a_8 \end{bmatrix} \quad (5)$$

$$\mathbf{B}(s) = \begin{bmatrix} b_1s + b_2 & b_3s + b_4 \\ b_5s + b_6 & b_7s + b_8 \end{bmatrix} \quad (6)$$

Differential equations describing dynamical behavior of the system are as follows

$$y_1'' + a_1y_1' + a_2y_1 = b_1u_1' + b_2u_1 + b_3u_2' + b_4u_2 \quad (7)$$

$$y_2'' + a_3y_2' + a_4y_2 = b_5u_1' + b_6u_1 + b_7u_2' + b_8u_2 \quad (8)$$

## DESIGN OF THE DECOUPLING CONTROLLER

One of possible approaches to control of multivariable systems is the serial insertion of a compensator ahead of the system (Krishnawamy et al, 1991, Peng, 1990, Tade et al, 1986). The compensator then becomes a part of the controller. The objective, in this case, is to suppress undesirable interactions between the input and output variables so that each input affects only one controlled variable. The block diagram for this kind of system is shown in Figure 1 ( $\mathbf{R}$  is a transfer matrix of a controller and  $\mathbf{C}$  is a decoupling compensator).

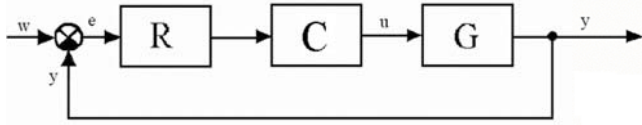


Figure 1: Closed loop system with compensator

The resulting transfer function  $\mathbf{H}$  (the operator  $s$  will be omitted from some operations for the purpose of simplification) is then determined by

$$\mathbf{H} = \mathbf{GC} = \mathbf{A}^{-1}\mathbf{BC} = \mathbf{A}^{-1}\mathbf{H}_1 \quad (9)$$

The decoupling conditions are fulfilled when matrix  $\mathbf{H}$  is diagonal. As it was mentioned above the matrix  $\mathbf{A}$  is supposed to be diagonal. The reason for this simplification is apparent from equation (9). When matrix  $\mathbf{A}$  is assumed to be non-diagonal it has to be included into the compensator in order to obtain a diagonal matrix  $\mathbf{H}$ . The order of the controller and consequently complexity of its design would increase.

The compensator is defined as the adjugated matrix  $\mathbf{B}$

$$\mathbf{C} = \text{adj}(\mathbf{B}) \quad (10)$$

The matrix  $\mathbf{H}_1$  then takes following form

$$\mathbf{H}_1 = \mathbf{B}\text{adj}(\mathbf{B}) = \begin{bmatrix} \det(\mathbf{B}) & 0 \\ 0 & \det(\mathbf{B}) \end{bmatrix} \quad (11)$$

Generally, the vector of input reference signals  $\mathbf{W}$  is specified as

$$\mathbf{W}(s) = \mathbf{F}_w^{-1}(s)\mathbf{h}(s) \quad (12)$$

Further the reference signals are considered as step functions. In this case  $\mathbf{h}$  is a vector of constants and  $\mathbf{F}_w$  is expressed as

$$\mathbf{F}_w(s) = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} \quad (13)$$

The controller can be described both by left and right matrix fractions as well as the controlled system

$$\mathbf{G}_R(s) = \mathbf{P}^{-1}(s)\mathbf{Q}(s) = \mathbf{Q}_1(s)\mathbf{P}_1^{-1}(s) \quad (14)$$

In order to achieve asymptotic tracking of the reference signal, an integrator must be incorporated into the controller. The controller including the integrator can be defined as

$$\mathbf{R} = \mathbf{F}^{-1}\mathbf{Q}_1\mathbf{P}_1^{-1} \quad (15)$$

The component  $\mathbf{F}$  is the integrator. The resulting matrix of the controller can be then defined as follows

$$\mathbf{CR} = \mathbf{CF}^{-1}\mathbf{Q}_1\mathbf{P}_1^{-1} \quad (16)$$

It is possible to derive an equation for the system output, which can be modified by matrix operations to the form

$$\mathbf{Y} = \mathbf{P}_1(\mathbf{AFP}_1 + \mathbf{H}_1\mathbf{Q}_1)^{-1}\mathbf{H}_1\mathbf{Q}_1\mathbf{P}_1\mathbf{W} \quad (17)$$

The determinant of the matrix in the denominator ( $\mathbf{AFP}_1 + \mathbf{H}_1\mathbf{Q}_1$ ) is the characteristic polynomial of the MIMO system. The roots of this polynomial matrix determine the behaviour of the closed loop system. They must be placed on the left side of the Gauss complex plane for the system to be stable. Conditions of BIBO stability can be defined by the following Diophantine matrix equation:

$$\mathbf{AFP}_1 + \mathbf{H}_1\mathbf{Q}_1 = \mathbf{M} \quad (18)$$

where  $\mathbf{M} \in R_{22}[s]$  is a stable diagonal polynomial matrix. If the system has the same number of inputs and outputs, matrix  $\mathbf{M}$  can be chosen as diagonal, which allows easier computation of the controller parameters. Correct pole placement of the matrix  $\mathbf{M}$  is very important for good control performance.

$$\mathbf{M}(s) = \begin{bmatrix} s^5 + m_1s^4 + & 0 \\ +m_2s^3 + m_3s^2 + m_4s + m_5 & \\ 0 & s^5 + m_6s^4 + m_7s^3 + \\ & + m_8s^2 + m_9s + m_{10} \end{bmatrix} \quad (19)$$

The degree of the controller polynomial matrices depends on the internal properness of the closed loop. The structures of matrices  $\mathbf{P}_1$  and  $\mathbf{Q}_1$  were chosen so that the number of unknown controller parameters equals the number of algebraic equations resulting from the solution of the Diophantine equation (18) using the method of uncertain coefficients:

$$\mathbf{P}_1(s) = \begin{bmatrix} s^2 + p_1s + p_2 & 0 \\ 0 & s^2 + p_3s + p_4 \end{bmatrix} \quad (20)$$

$$\mathbf{Q}_1(s) = \begin{bmatrix} q_1 s^2 + q_2 s + q_3 & 0 \\ 0 & q_4 s^2 + q_5 s + q_6 \end{bmatrix} \quad (21)$$

For simplification, it was computed the determinant  $\det(\mathbf{B})$ :

$$\det(\mathbf{B}) = (b_1 b_7 - b_3 b_5) s^2 + (b_1 b_8 + b_2 b_7 - b_4 b_5 - b_3 b_6) s + (b_2 b_8 - b_4 b_6) = db_3 s^2 + db_2 s + db_1 \quad (22)$$

The solution of the Diophantine equation results in a set of 10 algebraic equations with unknown controller parameters. Using matrix notation, the algebraic equations are expressed in the following form.

$$\begin{bmatrix} 1 & 0 & db_3 & 0 & 0 \\ a_1 & 1 & db_2 & db_3 & 0 \\ a_2 & a_1 & db_1 & db_2 & db_3 \\ 0 & a_2 & 0 & db_1 & db_2 \\ 0 & 0 & 0 & 0 & db_1 \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix} = \begin{bmatrix} m_1 - a_1 \\ m_2 - a_2 \\ m_3 \\ m_4 \\ m_5 \end{bmatrix} \quad (23)$$

$$\begin{bmatrix} 1 & 0 & db_3 & 0 & 0 \\ a_3 & 1 & db_2 & db_3 & 0 \\ a_4 & a_3 & db_1 & db_2 & db_3 \\ 0 & a_4 & 0 & db_1 & db_2 \\ 0 & 0 & 0 & 0 & db_1 \end{bmatrix} \begin{bmatrix} p_3 \\ p_4 \\ q_4 \\ q_5 \\ q_6 \end{bmatrix} = \begin{bmatrix} m_6 - a_3 \\ m_7 - a_4 \\ m_8 \\ m_9 \\ m_{10} \end{bmatrix} \quad (24)$$

The control law is defined as:

$$\mathbf{F}U = \text{adj}(\mathbf{B})\mathbf{Q}_1\mathbf{P}_1^{-1}\mathbf{E} \quad (25)$$

where  $\mathbf{E}$  is a vector of control errors. This matrix equation can be transcribed to the differential equations of the controller

$$\begin{aligned} & u_1^{(5)} + u_1^{(4)}(p_3 + p_1) + u_1^{(3)}(p_4 + p_1 p_3 + p_2) + u_1^{(2)}(p_1 p_4 + p_2 p_3) + u_1^{(1)} p_2 p_4 = \\ & = e_1^{(5)} b_7 q_1 + e_1^{(4)}(b_7 q_2 + b_8 q_1 + p_3 b_7 q_1) + e_1^{(3)}(b_7 q_3 + b_8 q_2 + p_4 b_7 q_1 + p_3 b_7 q_2 + p_3 b_8 q_1) + \\ & + e_1^{(2)}(b_8 q_3 + p_3 b_7 q_3 + p_3 b_8 q_2 + p_4 b_7 q_2 + p_4 b_8 q_1) + e_1^{(1)}(p_4 b_7 q_3 + p_4 b_8 q_2 + p_3 b_8 q_3) + e_1 p_4 b_8 q_3 - \\ & - e_2^{(5)} b_3 q_4 - e_2^{(4)}(b_3 q_5 + b_4 q_4 + p_1 b_3 q_4) - e_2^{(3)}(b_3 q_6 + b_4 q_5 + p_1 b_3 q_5 + p_1 b_4 q_4 + p_1 b_3 q_4) - \\ & - e_2^{(2)}(b_4 q_6 + p_1 b_3 q_6 + p_1 b_4 q_5 + p_2 b_3 q_5 + p_2 b_4 q_4) - e_2^{(1)}(p_2 b_3 q_6 + p_2 b_4 q_5) - e_2 p_2 b_4 q_6 \end{aligned} \quad (26)$$

$$\begin{aligned} & u_2^{(5)} + u_2^{(4)}(p_3 + p_1) + u_2^{(3)}(p_4 + p_1 p_3 + p_2) + u_2^{(2)}(p_1 p_4 + p_2 p_3) + u_2^{(1)} p_2 p_4 = \\ & = -e_1^{(5)} b_3 q_1 - e_1^{(4)}(b_3 q_2 + b_6 q_1 + p_3 b_3 q_1) - e_1^{(3)}(b_3 q_3 + b_6 q_2 + p_4 b_3 q_1 + p_3 b_3 q_2 + p_3 b_6 q_1) - \\ & - e_1^{(2)}(b_6 q_3 + p_4 b_3 q_3 + p_3 b_3 q_3 + p_4 b_3 q_2 + p_3 b_6 q_2) - e_1^{(1)}(p_3 b_6 q_3 + p_4 b_3 q_3 + p_4 b_6 q_2) - e_1 p_4 b_6 q_3 + \\ & + e_2^{(5)} b_4 q_4 + e_2^{(4)}(b_4 q_5 + b_2 q_4 + p_1 b_4 q_4) + e_2^{(3)}(b_4 q_6 + b_2 q_5 + p_2 b_4 q_4 + p_1 b_4 q_5 + p_1 b_2 q_4) + \\ & + e_2^{(2)}(b_2 q_6 + p_2 b_4 q_5 + p_2 b_2 q_4 + p_1 b_4 q_6 + p_1 b_2 q_5) - e_2^{(1)}(p_2 b_4 q_6 + p_2 b_2 q_5 + p_1 b_2 q_6) + e_2 p_2 b_2 q_6 \end{aligned} \quad (27)$$

For purposes of simulation, the controller was realized in the Matlab/Simulink environment as an S-function. It was then necessary to obtain its state equations. Further there it is introduced a conversion of the first differential equation (26) to the state equations. The second differential equation (27) was conversed similarly. Equation (26) can be itemized as follows

$$\begin{aligned} & u_{1A}^{(5)} + u_{1A}^{(4)}(p_3 + p_1) + u_{1A}^{(3)}(p_4 + p_1 p_3 + p_2) + u_{1A}^{(2)}(p_1 p_4 + p_2 p_3) + u_{1A}^{(1)} p_2 p_4 = \\ & = e_1^{(5)} b_7 q_1 + e_1^{(4)}(b_7 q_2 + b_8 q_1 + p_3 b_7 q_1) + e_1^{(3)}(b_7 q_3 + b_8 q_2 + p_4 b_7 q_1 + p_3 b_7 q_2 + p_3 b_8 q_1) + \\ & + e_1^{(2)}(b_8 q_3 + p_3 b_7 q_3 + p_3 b_8 q_2 + p_4 b_7 q_2 + p_4 b_8 q_1) + e_1^{(1)}(p_4 b_7 q_3 + p_4 b_8 q_2 + p_3 b_8 q_3) + e_1 p_4 b_8 q_3 \end{aligned} \quad (28)$$

$$\begin{aligned} & u_{1B}^{(5)} + u_{1B}^{(4)}(p_3 + p_1) + u_{1B}^{(3)}(p_4 + p_1 p_3 + p_2) + u_{1B}^{(2)}(p_1 p_4 + p_2 p_3) + u_{1B}^{(1)} p_2 p_4 = \\ & = -e_2^{(5)} b_3 q_4 - e_2^{(4)}(b_3 q_5 + b_4 q_4 + p_1 b_3 q_4) - e_2^{(3)}(b_3 q_6 + b_4 q_5 + p_1 b_3 q_5 + p_1 b_4 q_4 + p_1 b_3 q_4) - \\ & - e_2^{(2)}(b_4 q_6 + p_1 b_3 q_6 + p_1 b_4 q_5 + p_2 b_3 q_5 + p_2 b_4 q_4) - e_2^{(1)}(p_2 b_3 q_6 + p_2 b_4 q_5) - e_2 p_2 b_4 q_6 \end{aligned} \quad (29)$$

Equation (28) can be transcribed to the transfer function. It is also possible to establish an auxiliary variable  $Z$

$$\begin{aligned} G(s) &= \frac{b_7 q_1 s^5 + (b_7 q_2 + b_8 q_1 + p_3 b_7 q_1) s^4 + (p_4 b_7 q_3 + p_4 b_8 q_2 + p_3 b_8 q_3) s^3 +}{s^5 + (p_3 + p_1) s^4 + (p_4 + p_1 p_3 + p_2) s^3 + (p_1 p_4 + p_2 p_3) s^2 + p_2 p_4 s} \\ &+ \frac{(b_7 q_3 + b_8 q_2 + p_4 b_7 q_1 + p_3 b_7 q_2 + p_3 b_8 q_1) s^2}{s^2} \\ &+ \frac{(b_8 q_3 + p_3 b_7 q_3 + p_3 b_8 q_2 + p_4 b_7 q_2 + p_4 b_8 q_1) s^2}{s^2} \\ &= \frac{U_{1A}}{E_1} = \frac{U_{1A}}{Z} \frac{Z}{E_1} \end{aligned} \quad (30)$$

By means of the variable  $Z$  it is possible to define following equations

$$\begin{aligned} & b_7 q_1 z^{(5)} + (b_7 q_2 + b_8 q_1 + p_3 b_7 q_1) z^{(4)} + (b_7 q_3 + b_8 q_2 + p_4 b_7 q_1 + p_3 b_7 q_2 + p_3 b_8 q_1) z^{(3)} + \\ & + (b_8 q_3 + p_3 b_7 q_3 + p_3 b_8 q_2 + p_4 b_7 q_2 + p_4 b_8 q_1) z^{(2)} + (p_4 b_7 q_3 + p_4 b_8 q_2 + p_3 b_8 q_3) z^{(1)} + \\ & + p_4 b_8 q_3 z = u_{1A} \end{aligned} \quad (31)$$

$$z^{(5)} + (p_3 + p_1) z^{(4)} + (p_4 + p_1 p_3 + p_2) z^{(3)} + (p_1 p_4 + p_2 p_3) z^{(2)} + p_2 p_4 z^{(1)} = e_1 \quad (32)$$

Equation (32) can be conversed to a set of differential equations of the first order (state equations). Choice of the state variables is as follows

$$x_1 = z \quad x_2 = z' \quad x_3 = z'' \quad x_4 = z''' \quad x_5 = z^{(4)} \quad (33)$$

And the state equations are

$$\begin{aligned} & x_1' = x_2 \\ & x_2' = x_3 \\ & x_3' = x_4 \\ & x_4' = x_5 \\ & x_5' = e_1 - (p_3 + p_1) x_5 - (p_4 + p_1 p_3 + p_2) x_4 - (p_1 p_4 + p_2 p_3) x_3 - p_2 p_4 x_2 \end{aligned} \quad (34)$$

On the basis of the state variables, which are substituted to equation (31), it is possible to derive the first part of the manipulated variable  $u_{1A}$

$$\begin{aligned} u_{1A} &= b_7 q_1 (e_1 - (p_3 + p_1) x_5 + (p_4 + p_1 p_3 + p_2) x_4 + (p_1 p_4 + p_2 p_3) x_3 + p_2 p_4 x_2) + \\ &+ (b_7 q_2 + b_8 q_1 + p_3 b_7 q_1) x_5 + (b_7 q_3 + b_8 q_2 + p_4 b_7 q_1 + p_3 b_7 q_2 + p_3 b_8 q_1) x_4 + \\ &+ (b_8 q_3 + p_3 b_7 q_3 + p_3 b_8 q_2 + p_4 b_7 q_2 + p_4 b_8 q_1) x_3 + (p_4 b_7 q_3 + p_4 b_8 q_2 + p_3 b_8 q_3) x_2 + \\ &+ p_4 b_8 q_3 x_1 \end{aligned} \quad (35)$$

Similarly it is possible to transcribe equation (29)

$$\begin{aligned} & -b_3 q_4 z^{(5)} - (b_3 q_5 + b_4 q_4 + p_1 b_3 q_4) z^{(4)} - (b_3 q_6 + b_4 q_5 + p_1 b_3 q_5 + p_1 b_4 q_4 + p_1 b_3 q_4) z^{(3)} - \\ & - (b_4 q_6 + p_1 b_3 q_6 + p_1 b_4 q_5 + p_2 b_3 q_5 + p_2 b_4 q_4) z^{(2)} - (p_2 b_3 q_6 + p_2 b_4 q_5) z^{(1)} - \\ & - p_2 b_4 q_6 z = u_{1B} \end{aligned} \quad (36)$$

$$z^{(5)} + (p_3 + p_1) z^{(4)} + (p_4 + p_1 p_3 + p_2) z^{(3)} + (p_1 p_4 + p_2 p_3) z^{(2)} + p_2 p_4 z^{(1)} = e_2 \quad (37)$$

State variables were chosen similarly as in the previous case

$$x_6 = z \quad x_7 = z' \quad x_8 = z'' \quad x_9 = z''' \quad x_{10} = z^{(4)} \quad (38)$$

The state equations are then as follows

$$\begin{aligned} x_6' &= x_7 \\ x_7' &= x_8 \\ x_8' &= x_9 \\ x_9' &= x_{10} \\ x_{10}' &= e_2 - (p_3 + p_1)x_{10} - (p_4 + p_1p_3 + p_2)x_9 - (p_1p_4 + p_2p_3)x_8 - p_2p_4x_7 \end{aligned} \quad (39)$$

The second part of the manipulated variable  $u_{1B}$  can be computed similarly like the part  $u_{1A}$  by substitution of the state variables to equation (36)

$$\begin{aligned} u_{1B} &= -b_3q_4(e_2 - (p_3 + p_1)x_{10} + (p_4 + p_1p_3 + p_2)x_9 + (p_1p_4 + p_2p_3)x_8 + p_2p_4x_7) - \\ &\quad - (b_3q_5 + b_4q_4 + p_1b_3q_4)x_{10} - (b_3q_6 + b_4q_5 + p_1b_3q_5 + p_1b_4q_4 + p_3b_3q_4)x_9 - \\ &\quad - (b_4q_6 + p_1b_3q_6 + p_1b_4q_5 + p_2b_3q_5 + p_2b_4q_4)x_8 - (p_1b_4q_6 + p_2b_3q_6 + p_2b_4q_5)x_7 - \\ &\quad - p_2b_4q_6x_6 \end{aligned} \quad (40)$$

The manipulated variable  $u_1$  is then defined by the following sum

$$u_1 = u_{1A} + u_{1B} \quad (41)$$

An expression for computation of the manipulated variable  $u_2$  is obtained similarly on the basis of differential equation (27).

## RECURSIVE IDENTIFICATION

The controller was also realized as a self-tuning controller with recursive identification of a model of the controlled system. The recursive least squares method (Bobál, 2005) proved to be effective for self-tuning controllers and was used as the basis for our algorithm. For our two-variable example we considered the disintegration of the identification into two independent parts. It is not possible to measure directly input and output derivatives of a system in case of continuous – time control loop. One of the possible approaches to this problem is establishing of filters and filtered variables to substitute the primary variables. This approach is described in detail in (Wahlberg, 1990). The filtered variables are then used in the recursive identification procedure.

## IMPLEMENTATION OF THE CONTROLLER

A Simulink scheme of the closed loop system is in Figure 3. The controller uses two input signals and provides two outputs. The inputs are the reference signals ( $w_1, w_2$ ) and the vector of actual outputs of controlled process ( $y_1, y_2$ ). The main controller output is the vector of control signals – the input signals of the controlled process. The second controller output consists of the current parameter estimates of the controlled process model.

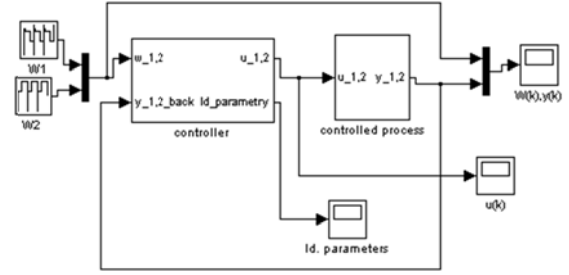


Figure 2: SIMULINK scheme of closed loop system

The controller is constructed as a mask of a subsystem, which consists of Simulink blocks and has inputs, outputs and parameters. Internal controller structure consists of Simulink blocks which provide, among others, the possibility of easy creation of a new controller by a modification of the proposed controller. The structure of the controller is presented in Figure 3. It consists of two basic parts: an on – line identification block and a block for computation of the control law and the manipulated variables. Controller's parameters are set using dialog windows.

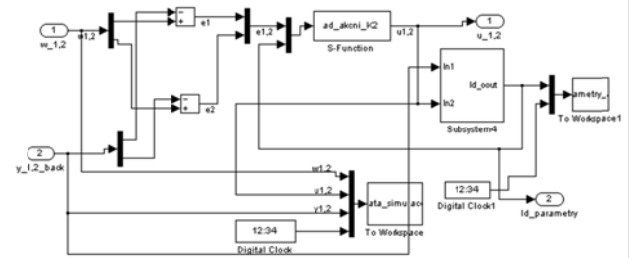


Figure 3: Internal structure of the controller

In Figure 4 is an internal structure of the subsystem for the system identification. It consists of the filters for filtering of the controlled and manipulated variables and a block for recursive identification.

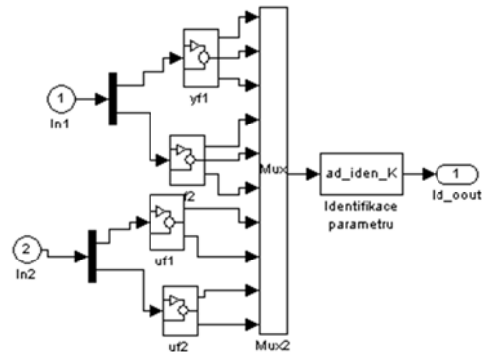


Figure 4: Filtering of variables and recursive identification

## SIMULATION VERIFICATION

Verification by simulation was carried out on a range of plants with various dynamics. The control of the model below is given here as an example. The controller's

synthesis is based on the model with diagonal matrix  $A$ , which is obtained by recursive identification and which describes the dynamics of the system with full matrix  $A$ .

$$A(s) = \begin{bmatrix} s^2 + 2s + 0,7 & 0,2s + 0,4 \\ -0,5s - 0,1 & s^2 + 2s + 0,7 \end{bmatrix} \quad (42)$$

$$B(s) = \begin{bmatrix} 0,5s + 0,2 & 0,1s + 0,3 \\ 0,5s + 0,1 & 0,3s + 0,4 \end{bmatrix} \quad (43)$$

Figure 5 shows the plant's step response

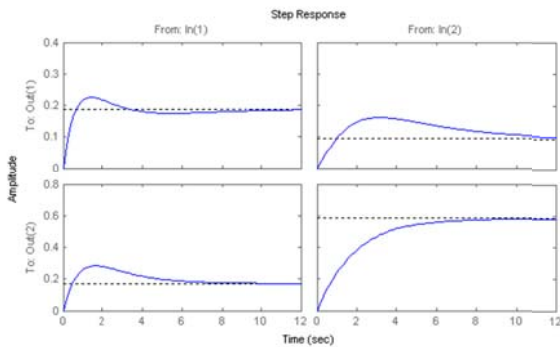


Figure 5: Step response of the controlled system

The matrix  $M(s)$  on the right side of the diophantine equation (18) obtained from experiments is

$$M(s) = \begin{bmatrix} s^5 + 5s^4 + 10s^3 + & 0 \\ +10s^2 + 5s + 1 & \\ 0 & s^5 + 5s^4 + 10s^3 + \\ & +10s^2 + 5s + 1 \end{bmatrix} \quad (44)$$

The time responses of the control are shown in Figure 6.

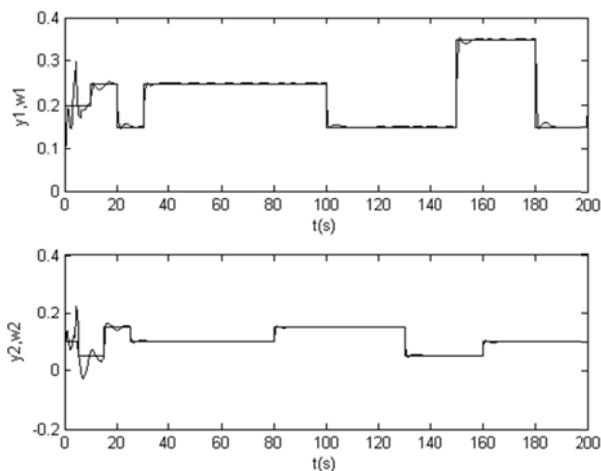


Figure 6: Adaptive control with decoupling controller

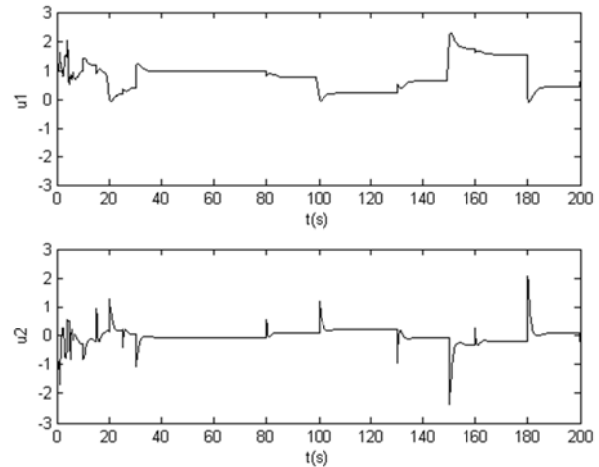


Figure 7: Adaptive control with decoupling controller-manipulated variables

From the courses of the variables in Figure 6 it is obvious that the basic requirements on control were satisfied. The system was stabilized and the asymptotic tracking of the reference signals was achieved. With regards to decoupling, interactions between the control loops are negligible.

## CONCLUSIONS

A decoupling TITO controller was designed and implemented in the Matlab/Simulink environment. The simulation results proved that the method is suitable for control of linear systems. With regards to decoupling, it is clear that the compensator reduces interactions between the control loops. The internal structure of the implemented controller enables its easy modification and implementation of further similar control algorithms.

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