MULTIVARIABLE ADAPTIVE CONTROL OF TWO FUNNEL LIQUID TANKS IN SERIES

Adam Krhovják, Petr Dostál and Stanislav Talas

1 Centre of Polymer Systems, University Institute, Tomas Bata University in Zlín, Nad Ovčírou 3685, 760 01 Zlín, Czech Republic.
2 Department of Process Control, Faculty of Applied Informatics, Tomas Bata University in Zlín, Nad Stráněmi 4511, 760 05 Zlín, Czech Republic

{krhovjak;dostalp;talas}@fai.utb.cz

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ABSTRACT
In this paper, we demonstrate continuous-time adaptive control for nonlinear system of two funnel tanks in series. For this purpose continuous-time input-output linear model is considered describing nonlinear system in the neighborhood of an operating point. The two approaches discussed here are direct estimation and alternate delta model. In the case of direct approach the key lies in the differential filters used to compute estimates. In the second strategy parameters of corresponding delta model are recursively estimated. A feedback control configuration with to degrees of freedom is adopted. In order to guarantee stability as well as asymptotic tracking of the step reference and step load disturbance attenuation, polynomial method is taken into account for the resulting controllers. A simulation software tool implementing illustrative model of two funnel liquid tanks in series has been studied in detail.

INTRODUCTION
In practice, most physical processes belong to a class of nonlinear systems. Although considerable research has been devoted to the SISO (Single Input – Single Output) nonlinear systems rather less attention has been paid to the MIMO (Multi Input – Multi Output) systems. Thus, we restrict our attention to the MIMO case.

As we move from linear to nonlinear systems superposition principle known from linear systems does not hold any longer and we are faced with more complicated situation. However, since linear models are so much more traceable, the first step in analyzing a nonlinear system comes with a trick of linearization. This is very intuitive approach but may run into problems trying to control system by using classical controller with a fixed parameters. Thus, the linearization alone is not sufficient. Here it is an idea to develop adaptive techniques eliminating limitations by introducing continuous-time input-output linear model (CT IOLM) with recursively estimated parameters. Thus, the control problem for the nonlinear system has been reduced to a problem of designing adaptive controller for the nonlinear system.

A very considerable amount of effort has been spent in the control community in applying identification techniques to adaptive control. As a result of these experiments two classes of strategies are commonly utilized for learning unknown parameters of CT IOLM.

Since we assume that the reader is familiar with the theory of discrete-time linear systems, it is easy to see that if the sampling period converges to zero, then the parameters of the discrete model do not converge to their continuous counterparts. The way to deal with this trouble has been discussed by (Middleton and Goodwin 1990; Mukhopadhyay et al. 1992; Garnier and Wang 2008), coming with the strategy of an alternative discrete model known as delta. Even though they are considered to be discrete, parameter convergence is guaranteed as sampling period tends to zero. More advanced description is to be found in (Stericker and Sinha 1993). With this in mind, delta model is seen to be powerful enough for the purpose of identification.

On the other hand, there is also a large literature on identification procedure in which one can obtain estimates of the IOLM using the technique of filtering, e.g. (Young 1981; Wahlberg 1990). In this way, input-output variables are needed to go through the differential filters. However, in contrast to delta model strategy, it is not difficult to notice that this approach meet additional calculations, connected to the outputs of differential filters.

Throughout the paper, we have stressed to build an adaptive technique with two different identification strategies for nonlinear system of two funnel liquid tanks. In both cases, estimates of the system are, in turn, used to compute the controller parameters. Such a challenge has been met by organizing the control configuration with two degrees freedom, where both controllers have feedback form. We refer the interest reader to (Dostál et al. 2001; Ortega and Kelly 1984). Finally, we demonstrate a method of control law synthesis based on polynomial method (Kučera 1993; MiklŠ and Fikar 2004) which ensures stability as well as asymptotic tracking of the reference signal. In addition, a great deal of effort has been spent on programming a nonlinear model of two funnel liquid tanks in series, bringing a practical simulation software tool.
MODEL OF TWO FUNNEL LIQUID TANKS

Consider the simple example of two funnel liquid tanks in series shown in Figure 2, where \( q_j \) and \( q_f \) (for \( j = 1, 2 \)) are outlet and inlet streams, respectively. Let us take \( D \) as the maximum diameter and \( H \) as the total height, which are same for both tanks. We define \( h_1 \) and \( h_2 \) as the liquid levels from the bottom.

As an idealization, we can model them by the equations

\[
\begin{align*}
\pi \frac{D^2}{4H} h_1^2 \frac{dh_1}{dt} - q_1 &= q_{i1}, \\
\pi \frac{D^2}{4H} h_2^2 \frac{dh_2}{dt} - q_1 &= q_{i2},
\end{align*}
\]

which are nonlinear. For a more detailed derivation we refer to (Corriou 1994; Bequette 2003). As the liquid moves through the valves, we see dependence of \( q_j \) on the liquid levels as

\[
q_j = k_j \sqrt{h_j - h_0}
\]

where \( k_1, k_2 \) are valve constants.

The equilibrium points of the system are determined by setting \( h_1 = h_2 = 0 \) and solving for \( h_1 \) and \( h_2 \). Therefore the equilibrium points correspond to the solution of

\[
0 = \frac{4H^2}{\pi D^2 h_1^2} (q_{i1} - q_1)
\]

which is not available, we estimate \( h_1, h_2 \) by

Suppose \( h_1 = 0 \) and \( q_{i1} \neq 0 \), and consider the change of variables

\[
\begin{align*}
\delta_1(t) &= q_{i1}(t) - q_{i1}, \\
\delta_1(t) &= q_{i2}(t) - q_{i2}\n\end{align*}
\]

In the new variables \( \bar{u} \) and \( \bar{y} \), system has equilibrium at the origin. Therefore, in a small neighborhood of the origin, we may approximate the nonlinear system (1), (2) by its linearization about the origin. In particular, this implies that the continuous-time IOLM takes the form

\[
\begin{bmatrix}
s + a_{01} & a_{02} \\
a_{03} & s + a_{04}
\end{bmatrix}
\begin{bmatrix}
y_1 \\
y_2
\end{bmatrix}
= \begin{bmatrix}
b_{01} & 0 \\
b_{04}
\end{bmatrix}
\begin{bmatrix}
u_1 \\
u_2
\end{bmatrix}
\]

\[ (9) \]

ADAPTIVE CONTROLLER DESIGN

In this section we will briefly discuss identification procedure and controller synthesis problem using polynomial approach.

Let us start by considering an adaptive control scheme depicted in Figure 1.

To give the reader a bird's eye view of the identification block, let us run two scenarios, answering the question of update law.

CT IOLM estimates

Since measurement of \( \dot{y}(t) \) and \( \dot{u}(t) \) is not available, we define vectors, representing filtered variables \( y_j(t) \) and \( u_j(t) \) by

\[
\begin{align*}
\dot{y}_j + c_{0j} y_j &= y_{i1} + \dot{y}_{i2} + c_{02} y_{i2} = y_2 \\
\dot{u}_j + c_{0u} u_j &= u_{i1} + \dot{u}_{i2} + c_{0u} u_{i2} = u_2
\end{align*}
\]

From the form of ARX according to (Dostál et al. 2004) it is clear that the parameters may be estimated by

\[
\begin{align*}
\dot{y}_j(t) &= b_{0j} \mu_j(t) - a_{0j} y_j(t) - a_{02} y_j(t) + \epsilon_j(t) \\
\dot{y}_j(t) &= b_{0j} \mu_j(t) - a_{0j} y_j(t) - a_{02} y_j(t) + \epsilon_j(t)
\end{align*}
\]

Delta model estimates

One of the difficulties in identifying ARX, proposed in previous subsection, is that we are required to have additional machinery associated with differential filters. As we will soon see, an alternative mechanism by which we identify parameters of the system, does not require any extra calculation.

For the sake of comparison let us consider competing delta model, which can be written as

\[
\begin{bmatrix}
\delta + \hat{a}_{01} & \hat{a}_{02} \\
\hat{a}_{03} & \delta + \hat{a}_{04}
\end{bmatrix}
\begin{bmatrix}
y_1 \\
y_2
\end{bmatrix}
= \begin{bmatrix}
b_{01} & 0 \\
b_{04}
\end{bmatrix}
\begin{bmatrix}
u_1 \\
u_2
\end{bmatrix}
\]

\[ (14) \]
Following the convention, the \( \delta \)-operator is defined as
\[
\delta = \frac{q - 1}{T_0}
\]  
(15)
where \( q \) is the forward shift operator and \( T_0 \) corresponds to the sampling period.

This should come as no surprise in view of our earlier discussion on CT estimates that the equation (14) provides an essential factor used to formulate ARX model as follows
\[
\begin{align*}
\delta y_i(\hat{k} - 1) &= b_{i0}u_i(\hat{k} - 1) - a_{i1}y_i(\hat{k} - 1) - a_{i2}y_i(\hat{k} - 2) + e_i(\hat{k}) \\
\delta y_i(\hat{k} - 1) &= b_{i0}u_i(\hat{k} - 1) - a_{i1}y_i(\hat{k} - 1) + e_i(\hat{k})
\end{align*}
\]  
(16)
(17)
where the left-hand side of the above equations (16), (17) is given by
\[
\delta y_i(\hat{k} - 1) = \frac{y_i(\hat{k}) - y_i(\hat{k} - 1)}{T_0}
\]  
(18)

Multivariable controller synthesis

Since the basis for the design of identification algorithms has been previously explored, we would like to find an adaptive controller that is a prescription for designing \( u \) such that \( y \) asymptotically tracks \( w \) with all generated signals remaining bounded.

The model \( G \) is a multi-input multi-output continuous-time IOLM, with transfer function matrix
\[
G(s) = \mathbf{A}^{-1}(s) \mathbf{B}(s)
\]  
(19)
To aid insight into controllers
\[
G_u(s) = Q(s) \mathbf{P}^{-1}(s), \quad G_y(s) = R(s) \mathbf{P}^{-1}(s)
\]  
(20)
we will work with both reference signals and disturbance signals as follows
\[
w(s) = \begin{bmatrix} w_{10} & w_{20} \end{bmatrix} \begin{bmatrix} s \end{bmatrix}
\]  
(21)
\[
v(s) = \begin{bmatrix} v_{10} & v_{20} \end{bmatrix} \begin{bmatrix} s \end{bmatrix}
\]  
(22)

where \( w(s) \) stands for the references and \( v(s) \) is a vector of disturbances.

To proceed with the design of the controllers, we now restrict our attention to the question of deriving signals in control configuration. We leave it as an exercise for the reader to verify that (omit the dependence on \( s \) for simplifying notation)
\[
y(s) = \mathbf{A}^{-1} \mathbf{B}[u(s) + v(s)]
\]  
(23)
\[
u_u(s) = R \mathbf{P}^{-1} \left[ w(s) - y(s) \right] - \mathbf{Q} \mathbf{P}^{-1} y(s)
\]  
(24)
\[
y(s) = \mathbf{P} \mathbf{D}^{-1} \left[ (\mathbf{A} \mathbf{P} + \mathbf{B} \mathbf{Q}) \mathbf{P}^{-1} w(s) - \mathbf{B} v(s) \right]
\]  
(25)
\[
e(s) = \mathbf{P} \mathbf{D}^{-1} \left[ (\mathbf{A} \mathbf{P} + \mathbf{B} \mathbf{Q}) \mathbf{P}^{-1} w(s) - \mathbf{B} v(s) \right]
\]  
(26)

Looking at the substitution
\[
D = \mathbf{A} \mathbf{P} + \mathbf{B} \left( \mathbf{R} + \mathbf{Q} \right)
\]  
(27)
given in equations (25), (26) it can be seen that the control system is stable if we design \( D \) to be Hurwitz.

From this point on, we concentrate our attention on showing how to ensure zero steady-state tracking error in the presence of uncertainties.

As is well known we have to use integral control such that
\[
P(s) = s \breve{P}(s)
\]  
(28)
\[
Q(s) = s \breve{Q}(s)
\]  
(29)
Substitution of these expressions back into the matrix Diophantine equation (27) yields
\[
A(s) \breve{P}(s) + B(s) T(s) = D(s)
\]  
(30)
where the polynomial matrix \( T(s) \) on the left hand side actually has the form
\[
T(s) = R(s) + s \breve{Q}(s)
\]  
(31)

Another question that comes to our mind immediately is: What are the degrees of polynomial matrices \( T \), \( R \) and \( \breve{Q} \)?

The rough answer to this question obviously takes the form
\[
\deg T(s) = \deg R(s), \quad \deg T(s) = \deg R(s)
\]  
(32)
In order to give the reader sense of excitement let us uncover the partial solution of matrix Diophantine equation. Since the first term on the left hand side of (30) does not contain constant matrix of \( \delta \), we can observe that
\[
B, T_e = D_e, \quad R_e = T_e
\]  
(33)

Toward the goal, suppose we have succeeded in finding polynomial matrices
\[
P(s) = s \breve{P}(s) = \begin{bmatrix} s \alpha_1 & s \alpha_2 \end{bmatrix}
\]  
(34)
\[
T(s) = \begin{bmatrix} t_1 s + t_{11} & t_1 s + t_{12} \\ t_2 s + t_{22} & t_2 s + t_{22} \end{bmatrix}
\]  
(35)

that satisfy (30) for the diagonal matrix
\[
D(s) = \begin{bmatrix} (s + \alpha)^2 & 0 \\ 0 & (s + \alpha)^2 \end{bmatrix}
\]  
(36)

However, unfortunately this solution does not capture relationship that exist between matrices \( R \) and \( Q \). As a guideline in searching for the relationship, the weighting matrix \( \Gamma \) is chosen. A large amount of practical experience tells us that in many cases it is reasonable to have weighting matrix in diagonal form.
\[
\Gamma(s) = \begin{bmatrix} \gamma_{11} & 0 \\ 0 & \gamma_{12} \end{bmatrix}
\]  
(37)
Recalling the expansion of the polynomial matrix, see e.g. (Rosenwasser and Lampe 2006; Blomberg and Ylinen 2006), we can state the solution of the feedback control problem by polynomial matrices of the form

\[ R(s) = \begin{bmatrix} y_1 f_1 s + t_{a1} & y_1 f_2 s \\ y_2 f_1 s & y_2 f_2 s + t_{a2} \end{bmatrix} \]  \tag{38}

\[ Q(s) = \begin{bmatrix} (1 - y_1 f_1 s) & (1 - y_1 f_2 s) \\ (1 - y_2 f_1 s) & (1 - y_2 f_2 s) \end{bmatrix} \]  \tag{39}

So far, we have formed the basic idea of the control problem. All that remains now is to show that matrix feedback controllers take the form

\[ G_e(s) = \begin{bmatrix} 1 - y_1 f_1 s & 1 - y_1 f_2 s \\ 1 - y_2 f_1 s & 1 - y_2 f_2 s \end{bmatrix} \]  \tag{40}

\[ G_s(s) = \begin{bmatrix} y_1 f_1 s + \frac{t_{a1}}{s} & y_1 f_2 s \\ y_2 f_1 s & y_2 f_2 s + \frac{t_{a2}}{s} \end{bmatrix} \]  \tag{41}

### SIMULATION OF THE TWO FUNNEL LIQUID TANKS

We have developed a simulator that simulates adequately behavior of two funnel liquid tanks in series. Idealistic model has been implemented according to equations (1)-(4). The simulator has been coded in C# and can be used in different operation modes either to generate simulation data or run fast simulation of simplified model of two funnel liquid tanks. There are three essential components through which one can interact: a control initialization pane, an identification library pane, a chart pane.

![Two liquid tanks simulator](image)

The simulator contains also several identification algorithms presented in the paper of (Kulhavý and Kárný 1984) which could be utilized for parameter estimation. The reference trajectory for each liquid level is entered in the simulator as a sequence of points representing combination of time values and liquid levels at corresponding sample times. The actual liquid levels of the tanks are then computed according two feedback controllers (40), (41).

### SIMULATIONS AND RESULTS

In this section, we simulate the adaptive control of two funnel liquid tanks in series with the help of simulation tool, which was discussed in the previous section. The parameters of the tanks and valves are \( k_1 = 0.316 \text{m}^3/\text{min}, \ k_2 = 0.296 \text{m}^3/\text{min}, \ D = 1.5 \text{m}, \ H = 2.5 \text{m} \). The initial conditions we started with are \( h_i = 1.8 \text{m}, \ h_o = 1.4 \text{m}, \ q_1 = 0.2 \text{m}, \ q_2 = 0.15 \text{m} \).

The results are illustrated in Figures 4-10 for different values of \( \gamma_{11} \) and \( \gamma_{12} \) as well as for different values \( \alpha_1 \) and \( \alpha_2 \). Each figure shows the line plot of the control responses to step change in reference trajectories. In all cases, the recursive estimation of model parameters was performed with the constant sampling period \( T_s = 0.1 \text{min} \). Figure 4 and 5 illustrate that there is an insignificant difference between two identification approaches.

![Controlled liquid levels – delta IOLM](image)

![Controlled liquid levels – CT IOLM](image)

This claim is also supported by Figure 6 in which parameter evolution of continuous-time and delta IOLM is captured. As can be seen from this example, we have found such a combination of parameters that gave a reasonably great responses with no overshoots.
Figure 6: Parameter evolution during control

Figure 7 illustrates the significant and rapid effect of diagonal elements of weighting matrix $\Gamma$. Having higher values of $\gamma_{11}$ and $\gamma_{12}$, we can accelerate the control. Unfortunately, their higher values greatly affect control input responses as is shown in Figure 8. This finding plays an important role in control design of real plants where the control inputs are physically limited.

Next simulation in Figure 9 shows the effect of double real pole $\alpha$ on the controlled output responses. It is easy to see that if we select lower values of poles in the open left-half plan, we speed up responses. However, one should be careful about experimenting with poles, because too small poles usually correspond to overshoots. In other words, the system may go unstable before we have time to react. If we make a change in either the first pole or the second pole, then this will generally affect all the outputs, that is, there is interaction between the inputs and outputs. However, since we manipulate weights, there is minimal interaction between them. This important feature could be seen in Figure 10.

CONCLUSION

In this paper we illustrated a promising adaptive strategy to control nonlinear system of two funnel liquid tanks in series. The main motivation for our approach is to enable the use of design via linearization. We have demonstrated that if one uses linear model with some possibly time-varying parameters, control of nonlinear system may works suprisingly well. We have detailed identification problem of IOLM in two possible directions. In the first place, the direct estimation has been discussed in the sense of filtred variables. In the second place, we have applied alternative delta model approach to the estimation problem. The application has led to insight into the parameters of IOLM. As our results show, the algebraic procedure, based on the solution of the matrix Diophantine equation, is well suited for the controller synthesis. An adaptive control approach parameterizes the uncertainty in terms of certain unknown parameters of the IOLM matrices and tries to use feedback to learn...
these parameters on line, that is, during the control of the nonlinear system. Tuning parameters of controller have a direct and major effect on feedback performance. However, finding the right combinations of tuning parameters to get prescribed behavior may be difficult. Much effort has been spent on the code development of reusable model of nonlinear system of two funnel liquid tanks in series and special care has been taken towards its implementation into simulation editor. We are open to any suggestion and recommendation to improve our proposal.

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ADAM KRHOVJÁK studied at the Tomas Bata University in Zlín, Czech Republic, where he obtained his master degree in Automatic Control and Informatics in 2013. He now attends PhD. study in the Department of Process Control, Faculty of Applied Informatics of the Tomas Bata University in Zlín. His research interest focus on modeling and simulation of continuous time technological processes, adaptive and nonlinear control.

PETR DOSTÁL studied at the Technical University of Pardubice, where he obtained his master degree in 1968 and PhD. degree in Technical Cybernetics in 1979. In the year 2000 he became professor in Process Control. He is now Professor in the Department of Process Control, Faculty of Applied Informatics of the Tomas Bata University in Zlín. His research interest are modeling and simulation of continuous-time chemical processes, polynomial methods, optimal and adaptive control.

STANISLAV TALAŠ studied at the Tomas Bata University in Zlín, Czech Republic, where he obtained his master degree in Automatic Control and Informatics in 2013. He now attends PhD. study in the Department of Process Control, Faculty of Applied Informatics of the Tomas Bata University in Zlín. His e-mail address is talas@fai.utb.cz.