

# RESILIENCY ON ORDER-DRIVEN MARKETS

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## KEYWORDS

Resiliency, market liquidity, limit order book, vector-autoregressive model

## ABSTRACT

Market liquidity has an important role in trading on stock markets, since on illiquid markets the implicit cost of trading can cause notable losses for the investors. Therefore market participants should always measure the liquidity of the markets, which they can carry out in two ways, in a static and in a dynamic form. The most commonly used liquidity measures – bid-ask spread and the turnover – quantify liquidity statically, and there are only a few ways to measure liquidity dynamically. In this paper we will introduce a method which enables the market participants to analyse the liquidity of a market in a dynamic framework. We will use a vector-autoregressive estimation and simulation method to show how the liquidity of the market recovers after a shock happens on the market, namely we will measure the resiliency of the market.

## INTRODUCTION

In this paper we empirically investigate the resiliency of the order-driven markets concentrating on the structure of the limit order book. We define resiliency as the feature of the market in which new orders flow quickly to correct liquidity of the market after a shock. In other words, the recovery process of the order book in response to a temporary order imbalance. A temporary order imbalance can be the result of an aggressive market order or series of market orders which are being fulfilled against large number of limit orders in a very short period.

Market resiliency is known as one of the characters of market liquidity. While numerous studies concern with the static dimensions of liquidity namely tightness, depth or breadth, (e.g. BIS (1999) or Sarr and Lybek (2002)) relatively low numbers of papers discuss the dynamic dimensions of liquidity, the resiliency and

immediacy. We will focus on resiliency in this paper, and show how the order book recovers after limit orders disappeared due to market transactions.

When a trader executes a market order, the transactional price may divert from the best available price (best bid price for a market sell order or the best ask price for the market buy order) in the limit order book. This can happen in the case, if the market order is larger, than the amount of orders available on the best limit price level in the book. In this case the market order will be fulfilled on more than one price levels. If a market is illiquid, it means that the number of orders in the limit order book is low, so a large market order will be fulfilled on several price levels in the limit order book, which will cause the implicit cost of trading to be high. Implicit cost of trading refers to the margin between the immediately executed transactional price and the mid price (the price halfway between the best bid, and best ask price). We will focus on how fast the implicit cost of trading reverts back to the level describes the normal times after a shock on the market. It is crucial for the market players to know the duration of the correction and the possible long term effects of this kind of shocks.

Our paper is built up as following: first, we introduce the theoretical background of market resiliency, secondly we show with a descriptive statistical analysis the data we use in our estimation then we describe the statistical method, the vector-autoregressive (VAR) model we have used in the next chapter. The fourth part will show our results. Finally, the last section summaries our results and conclusions.

## THEORETICAL BACKGROUND

We briefly review the most important theoretical models related to market resiliency, focusing on the different definitions researchers give for resiliency and the main empirical contributions and findings regarding resiliency, especially the VAR approach used for studying resiliency and the shape of the limit order book.

## Market resiliency definitions

The first researcher who was dealing with defining resiliency was Garbade (1982). He defined a market resilient, if new orders pour in promptly after a temporary order imbalance. In contrast Kyle (1985) and Harris (2003) emphasized analyzing the market price instead of the order flow. Kyle (1985) stated that the resiliency is the speed with which pricing errors caused by uninformative order flow shocks are corrected or neutralized in the market. While Harris (2003) defined resiliency nearly the same way, since he said that resiliency shows how quickly prices revert to former levels after they change in response to large order flow initiated by uninformed traders.

Finally, we highlight two more notable definitions, which are quite different from the earlier. Foucault et al. (2005) defined resiliency with a probability. They stated that resiliency is the probability that after a liquidity shock, the spread reverts to its former level before the next transaction. While Dong et al. (2007) have observed a mean-reversion parameter in the stock's pricing-error process, and defined resiliency with this parameter.

Based on the literature, we give a new definition on resiliency, since in our research partly our goal was to complete the notion of resiliency of the order driven market. We defined resiliency as the recovery process of the limit order book in response to temporary order imbalance, by quantifying how fast the implicit cost of trading reverts back to the level describes the normal times.

## Empirical findings

VAR models are often used in market microstructure research to model resiliency. Among the pioneers, Hasbrouck (1991) analyses the impact of large trades. He finds that the impact arrives with a lag and it is higher when the asset is not frequently traded, or if the trade size is large, or the market spread is wide. Later, Dufour and Engle (2000) extend the model of Hasbrouck (1991), by incorporating the duration between the consecutive trades to exploit information for modelling the price and trade dynamics. Their empirical model proposes that the market is more active when increased ratio of informed traders present in the market. When frequency of the transactions increases the price impact of trades and the speed of price adjustment to trade related information also increase. Moreover, the speed-up positively affects the positive autocorrelation of signed trades.

Engle and Patton (2004) take also on the approach of Hasbrouck (1991) by modeling the dynamics of bid and ask prices rather as a system than a single mid-quote variable. According to them, short duration and medium volume trades have the largest impact on quote prices, the spread is mean-reverting, and traders have a greater impact on quotes in both the short and long run for the infrequently traded stocks. They also find evidence for a strong asymmetric impact of a trade

on bid and ask prices in the short run. Similarly, Escribano and Pascual (2006) conclude that an unexpected buy order has a bigger effect on average on the ask quotes, than an unexpected sell trade on the bid quote, since the buyer initiated trades are more informative. They also emphasize that it is worth to model the bid and ask prices together instead of modeling only the mid-price, because it causes serious loss of important information.

Coopejans et al. (2003) analyzes the stochastic dynamics of liquidity and its relation to returns and volatility in a VAR model. They find that the volume and liquidity is concentrated in certain points in time, so strategic order placement has a value, moreover they have shown also, that liquidity is clustering, which means that if liquidity increases on one side of the book, it will increase on the other side as well. Another interesting result was that they have pointed out, that shocks in liquidity elapses quickly, so resiliency is high during shocks, while the shock in volatility has a contemporaneous and persistent effect on liquidity.

Hmaied et al. (2006) estimated also a VAR model to analyze resiliency. By their results, there is a dynamic relationship between spread, depth and volatility; buyers are likely to be more information motivated than sellers, and they behave differently; and finally, the impulse response analysis shows that shocks are absorbed more quickly for frequently traded stocks than for infrequently traded ones.

Another notable research on the field of resiliency that uses a VAR model is carried out by Wuyts (2011). In his VAR model he incorporates different dimensions of liquidity, by having various liquidity indicators in the model. He proposes that in a market, where there isn't a market maker, who would ensure the proper liquidity of the market, the market is resilient. He confirms this statement by showing that each liquidity indicator he analyzes (depth at the best prices, spread, order book imbalances) reverts to a steady-state value within 15 orders after a shock. His other interesting result in respect to our research is the effect of the shock on the ask and the bid side of the order book is not the same. He states that the effect of the shock on the ask side is stronger.

## DATA

Our dataset is provided by the Budapest Stock Exchange, and contains the intraday event-based – but aggregated if more than one event has occurred within one second – data of the most liquid Hungarian stock, the OTP for 2013. OTP is from the banking sector with a capitalization of 4.4 billion EUR. The average daily traded volume for the OTP is 16.5 million EUR. The dataset contains the Adverse Price Movement (more details about the Adverse Price Movement: Gomber and Schweikert, (2002)) data for the ask and bid sides of the limit order book, for eleven different order sizes, for those seconds of trading, when there was any kind of change in the order book. It means that each row in the dataset is reported, when there is a market buy/sell,

limit bid/ask, or a cancellation on the ask/bid side of the book. Therefore we are able to trace back from the dataset what has happened on the market, since we had turnover data, and we had information also about the order book, namely that what is the total value of order on the ask/bid side.

In Table 1 we show the daily number of orders in 2013 for the OTP. The mean and median daily number of orders can be seen, also the standard deviation of the orders, and those two data, that had the maximum and minimum number of a certain order at one day.

Table 1: Number of orders per day

OTP, 2013					
	mean	median	sd	min	max
Market orders	1,233	1,077	694	324	6,608
Limit bid	1,289	1,155	610	393	4,939
Limit ask	1,177	1,114	436	306	3,098
Market buy	615	519	401	138	3,950
Market sell	620	552	311	188	2,666
Cancel bid	542	501	240	173	1,692
Cancel ask	440	421	161	118	1,101

In Table 1 it can be seen, that the number of limit orders has the highest number of orders compared to the market orders, and cancellations. Another interesting fact is presented in Table 1, namely that the number of orders are higher on the bid side of the book, than on the ask side in case of all the indicators, and also the number of market sell orders (which are fulfilled against the bid side of the book) are higher than the buy market orders. Although the number of orders are higher on the bid side of the limit order book, and market sell order side, the value of the orders shows the opposite relationship, which can be the consequence of the fact, that the prices on the ask side of the limit order book is higher. The value of orders is shown in Table 2.

Table 2: Value of orders per event

OTP, 2013 (Thousand HUF)					
	mean	median	sd	min	max
Limit bid	3,044	1,344	6,362	0.06	510,786
Limit ask	3,125	1,392	7,129	0.20	696,150
Cancel bid	4,170	1,390	21,897	0.10	1,689,201
Cancel ask	4,557	1,437	30,201	0.75	1,694,071
Market buy	3,922	1,490	11,268	4.00	1,187,500
Market sell	3,767	1,405	10,582	4.00	898,900

Table 3 shows the average time elapses between two same types of orders. It can be seen that in every 30 seconds there is at least one market buy and sell, one limit bid and ask order and one cancellation on both sides of the book. Based on the data of Table 3, the average duration between two consecutive events is around 2-3 seconds. This means that it is appropriate to use our database for our analysis, since there are only a few cases in our database when there was more than one event within one second.

Table 3: Time elapsed between the same types of orders

OTP, 2013 (seconds)		
	Mean	Median
Market buy	46.6	15.0
Market sell	26.2	16.0
Limit bid	22.2	10.0
Limit ask	24.4	12.0
Cancel bid	53.0	23.0
Cancel ask	65.2	29.0

In Table 4 the two most commonly used liquidity indicators can be seen and the mid price for the OTP in 2013. The bid-ask spread shows that what would be the implicit cost of trading in case of buying and selling a security at the same time, if a trader could fulfill a market order on the best price levels. This means that the lower the bid-ask spread the implicit cost would be lower, so the market is more liquid. In case of the trading volume, the higher value is, the market is considered as more liquid.

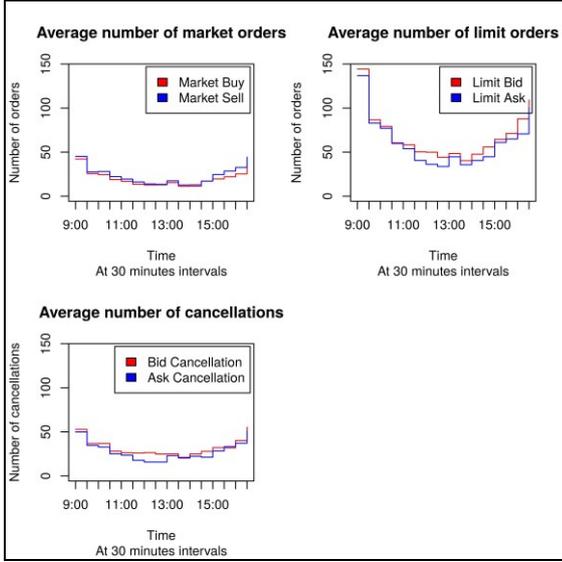
Table 4: Spreads and trading volume

OTP, 2013			
	mean	median	sd
midprice (HUF)	4,581	4,564	265
bid-ask spread (bp)	9.6	8.4	6.6
trading volume (THUF)	3,831	1,446	10,900

Based on the literature we have also analyzed the seasonal patterns of the different order types. In the analysis we have considered the seasonality of the number of orders instead of the total volume of the orders, since in our analysis the dynamics of the different order submissions, not the volume of the trades is important. On Figure 1 we show the daily patterns of the different orders.

In case of the market orders, the most intense order submission can be seen in the last 1-2 hours of trading, which can be the consequence that traders try to close their positions, so they are getting more aggressive by

the end of the day. The same is true for the cancellations as well, since those limit orders that were not fulfilled during the trading day, the traders are cancelling as we are closer to the end of the trade. The variation is the highest in case of the limit orders. At the beginning of the trading day, the submission of limit orders is very high, because the traders are building up the limit order book that time. The lowest limit order submission can be seen in the middle of the day, while at the end of the day the activity increases again.



Figures 1: Seasonal patterns of the order-flow

Based on Figure 1, we will use in our VAR model dummy variables for the time, to handle the problem caused by the changing trading during the day.

## METHODOLOGY

For analyzing the order book dynamics we estimate a vector-autoregressive system similar to Hasbrouck (1991)'s logic. Hasbrouck built a basic model to capture how the information reveals through market transactions. According to this approach, market returns are driven by the public information and market trades are induced by the earlier returns and new price information that traders may have. Considering only one time period lag, the main equations are shown in Equation 1 and Equation 2:

$$r_t = \beta_{11}r_{t-1} + \beta_{12}x_{t-1}^0 + \varepsilon_t \quad (1)$$

$$x_t^0 = \beta_{21}r_{t-1} + \beta_{22}x_{t-1}^0 + \vartheta_t \quad (2)$$

where  $r_t$  is the return,  $x_t^0$  is the sign of the trade,  $\varepsilon$  denotes the public information and  $\vartheta$  indicates the

private information of the traders, while  $\beta$ -s are the coefficients. This equation system can be extended with more lagged periods. Hasbrouck (1991) used this approach to show that middle-sized trades and trades, when spread is high have more intense effects on the future prices. Because the trading frequency may divert in the different sessions of a day, the estimation above can be enhanced by dealing with the arrival time of order submissions. Dufour and Engle (2000) suggested to extend both  $\beta_2$  coefficients with other variable components according to Equation 3:

$$\beta_{*0} = \gamma_{*0} + \sum_{j=1}^J \gamma_{*j} D_j + \sum_{k=1}^K \delta_k \ln(T_{t-k}) + \sum_{k=1}^K \varphi_k z_{t-k} \quad (3)$$

where  $D$  is a time dummy for a certain time period of a day,  $T$  denotes the duration between market transactions, and  $\gamma$ ,  $\delta$  and  $\varphi$  are model coefficients,  $z$  signs other explanatory variables. The time dummies exogenous, durations are endogenous variables in the VAR estimation.

This framework is able to do further explorations about price impacts and detection of order book dynamics. By our first target, we are interested in the limit order submission responses on a market trade or a bunch of consecutive trades. Hence, we extend the analysis with order book variables that go after the market transactions. We look for and explore what happens after a market trade in the limit order book. Our approach combines the models above and use additional variables. The general equation of the vector-autoregressive system can be formulated according to Equation 4:

$$(*)_t = \sum_{k=1}^K \beta_{*1k} r_{t-k} + \sum_{k=1}^K \beta_{*2k} x_{t-k}^0 + \sum_{k=1}^K \beta'_{*3k} y_{t-k} + \varepsilon_{*t} \quad (4)$$

where  $y$  vector collects all of the variables that describes the main characters of the order book. This variable may consist of the spread, the increments on the numbers of bid (ask) orders, the increments on the numbers of price levels (both sides, separately), bid and ask depth and other structural characteristics.

In contrast to the conventional 'transaction time' approach, we estimate these models in 'limit order submission time'. That means we aggregate the consecutive market trades and price variations. This procedure helps us to handle rapid series of market orders as a liquidity shock. Moreover, we look for the direct limit order submission responses by events after such liquidity shocks. However 'limit order time' approach is not able to reveal appropriately the price impacts of trades, but it captures the structural impacts of liquidity runs.

## VAR MODEL OF THE BOOK

Our research question is that how fast the liquidity – described by the factors of the limit order book – of the market recovers after a shock. We estimate the VAR model for OTP in the period of 02.09.2013-30.10.2013. For the simpler interpretation of our results, we only plot our impulse-response function (IRF) simulations, which show how the values of the different indicators evolve after the shock. In Figure 2 and 3 the IRF can be seen in case of some specified shocks.

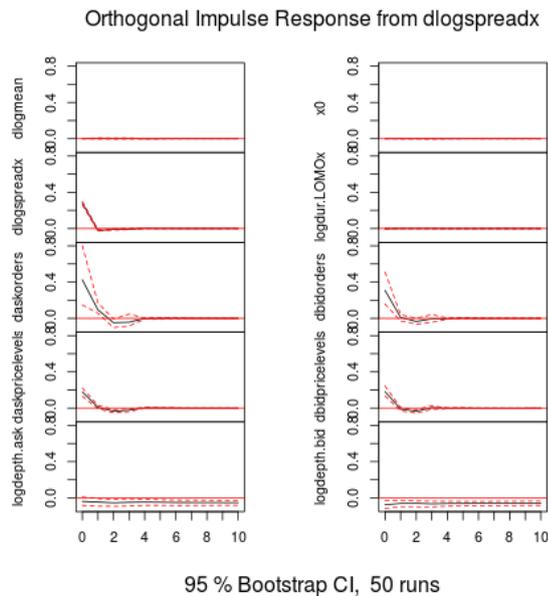


Figure 2: IRF-s when the shock is an incoming market trade after a decreasing spread

In Rosu (2009)'s market microstructure theory, limit orders dominantly are submitted to with a better quote than best ask or bid quotes. This narrows the spread. Here we focus on how the limit orders arrive in this period. Figure 2 depicts the dynamics of the order book when spread is decreasing. In Figure 2 the variables are the following:

- dlogmean: variation of the logarithmic mid-price
- x0: sign of a market transaction
- dlogspreadx: variation of the spread (times sign of the trade)
- logdur.LOMOX: difference between limit and market order durations (proxy of intensity)
- daskorders/dbidorders: change in the number of ask/bid orders
- daskpricelevels/dbidpricelevels: variation of the number of ask/bid price levels
- logdepth.ask/ logdepth.bid: logarithm of the depth volume on the ask/bid side of the book

We introduced a variable  $ds_{spread} = - (spread_t - spread_{t-1})$  and shock the system with this to describe what happens in narrowing. In this case the market order or the series of market orders arrive, while the spread was decreasing on the market due to several

limit orders. Since spread can narrow, if patient traders submit limit orders below the best ask or above the best bid. This process shows some persistency in terms of the consecutive limit orders that increase the number of limit orders in the book. According to Figure 2, one can see that after the market orders have arrived, the traders didn't stop giving limit orders. It can be seen on the increasing number of bid and ask orders, and also on the increasing number of bid and ask order levels in the limit order book. Also the market depth reduces because there are less limit order volume on the new best quote. Later, the process reverts, and after 5-6 periods, the spread stops to decrease, and the number of limit orders decline as well.

Figure 3 captures the case when a market buy order arrives in an actively traded period. Activity is defined by Rosu (2014) as the speed of market order submission related to the limit order submission and limit cancellations. In this period, a market buy shock first bite the book and the number of ask orders get lower. Later, ask orders are refilled by patient traders. Similar, but less intense process can be documented on the bid side. The market depth also increases because spread widen as a result of high market intensity

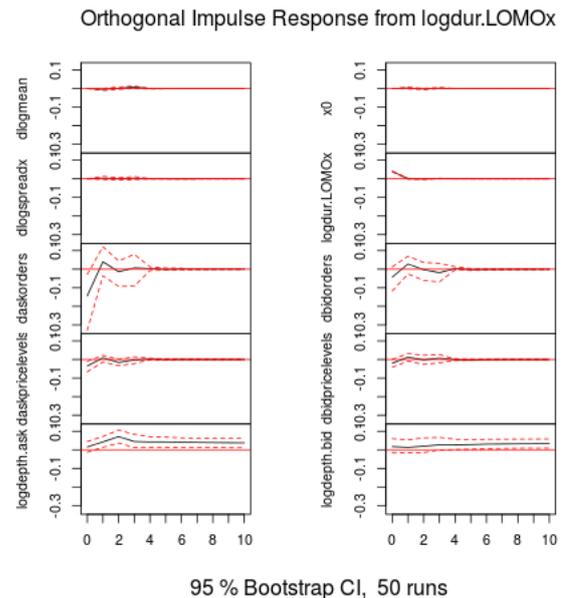


Figure 3: IRF-s when the shock is a market buy in an active market

In contrast, Figure 3 captures the case when a market buy order arrives in an actively traded period, which means that the number of limit order submission is not as active, as it was in the previous case, but the emphasis of trading is based on the market orders. Activity is defined by Rosu (2014) as the speed of market order submission related to the limit order submission and limit cancellations. In this period, a market buy shock first bite the limit order book, and the number of ask orders get lower. Later, ask orders are refilled by patient traders. Similar, but less intense

process can be documented on the bid side. The market depth also increases because spread widens as a result of high market intensity.

## CONCLUSION

In our paper we have analysed the resiliency of order-driven markets. We have built our analysis on a vector-autoregressive model, which is the most commonly used method in the literature to analyse and estimate the resiliency of the market. We have highlighted two special market situations, and showed by the impulse-response functions, that what is the short and long term effect of the situations to the structure of the limit order book. This is important because the liquidity of the market is defined by the structure of the order book, mainly the smaller the bid-ask spread, or the deeper the limit order book is, the more liquid is the market. One of the situations we have analysed was a market state, when the market order, or the series of market orders – shock is defined by the submission of a market order – arrives when the spread was decreasing on the market, since the limit order submission were frequent. We found, that the market orders didn't prevent the submission of the limit orders, or the bid-ask spread to decrease, so the liquidity of the market didn't get worse. The market order had an effect only after 5-6 periods after the submission. The other situation we have analysed was nearly the opposite situation, when the limit orders were not frequent on the market, so the effect of the market order could decrease the bid-ask spread, and the overall liquidity of the market. Our main contribution to the literature was to take into account the whole structure of the limit order book to analyse the resiliency of the market. In the future researches it is worth to analyse further and more complex market situations, and their effect on the structure of the order book, and to the market liquidity.

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## ACKNOWLEDGEMENTS

Dániel Havran is participant of the Hungarian Academy of Science Postdoctoral Fellowship Programme, in the period of 2013/09-2015/09.