

# HALF CAR SUSPENSION SYSTEM INTEGRATED WITH PID CONTROLLER

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## KEYWORDS

Half Car Model, PID Controller, Feedback Control, Active Vehicle Suspension System.

## ABSTRACT

In this paper, modeling and simulation of nonlinear, Half-car Active Vehicle Suspension System (HAVSS) reinforced with PID controller is presented. The model has 4 degrees of freedom; comprising of heave movements of the front and rear axle, pitch and heave motions of the unsprung mass of the vehicle. The vehicle is excited by a triangular bump on an otherwise smooth road. The objective of the presented model is to isolate the vehicle body from these road disturbances in order to maximize passenger ride comfort and retain continuous road-wheel contact. PID controller is used to suppress the vibrations generated from the road unevenness. Newton's approach is used in system modeling due its simplicity over D'Alembert's and Lagrange's approach. The simulation, carried out in MATLAB environment, vividly shows the superior performance; in both effectiveness and robustness of the HAVSS over its passive control counterpart.

## INTRODUCTION

Vehicle suspension system is majorly aimed at isolating vehicle body from road disturbances in order to maximize passenger ride comfort and retaining continuous road-wheel contact so as to provide excellent road holding (Changizil and Rouhani 2011). It is therefore a trade-off between conflicting criteria of vehicle ride comfort, quality of vehicle handling and road holding within the limits of suspension travel. It is categorically classified as passive, semi-active and active suspension system (Ghazaly and Moazz 2014). The dynamic behavior of passive (classical) vehicle suspension systems (PVSS) is determined by the selection of the spring stiffness and the damper coefficients. However, fixed values of these components make PVSS to lack enough energy absorption capability (EAC) to sustain the load or road disturbance acted into the vehicle system, hence lack stability if compared with other suspension systems (Dahunsi 2011). The semi-active suspension uses a variable damper or other variable dissipation component in the system.

Typical example of this system is variable dissipater having double tube viscous damper in which the damping coefficient can be varied by changing the diameter of the orifice in a piston.

The active vehicle suspension system, which was first introduced in early 1950's (Ahmed and Svaricek 2013), employed electronic control systems that monitor the operation of the suspension elements. In doing so, it reduce car body accelerations by allowing the suspension to absorb wheel accelerations using an actuator (Pedro and Ekoru 2013).

Many control approaches exist, such as Linear Quadratic Regulator (LQR), Linear Quadratic Gaussian (LQG), Adaptive Sliding Control (ASC), Fuzzy Logic (FL) and Neural Network (NN) methods. However, the most widely used electronic control not only in AVSS, but other industrial motion control is PID controller (Aly 2012). This is sequel to its simplicity, gains tuning ability and it does not require any complicated learning mechanism as in the case of FL and NN (Dukkaphati 2006). This motivated the selection PID in this paper.

(Yildirim 2009) and (Gurel et al. 2010) proposed approaches using NN and Genetic Algorithm respectively, in which the former used over 80 million training samples. In either case, complicated training algorithms are used which may take longer time to converge or even fail to. A very simplified model for quarter car suspension is presented by (Hassan 2014), however it lacks enough EAC since it is PVSS.

This paper proposes a half-car active vehicle suspension system (HAVSS) with a feedback reinforced with PID controller. Careful parametric analysis is carried out prior to the selection of the damper and spring constants in the model. As the paper is structured, Section II presents the mathematical modeling of the HAVSS, Section III gives the PID controller design followed by the simulation results and discussion in section IV. The paper is concluded in section V.

## SUSPENSION SYSTEM MATHEMATICAL MODELLING

The HAVSS consists of unsprung masses modelling the front and rear axle. The entire mass of the vehicle including the chassis, is modeled as unsprung mass resting on the axles as in Figure 1.

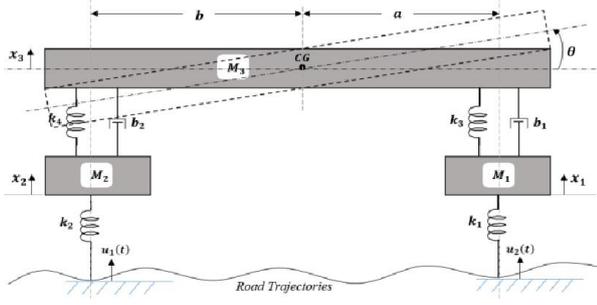


Figure 1: Half Car Suspension System Model

Depending on the input, the vehicle can undergo pitch, heave or both movements concurrently (Woods 1997). Figure 2 shows the free body diagram of the model from which Newton's second law of motion is used to develop the modelling equations using the static equilibrium position as the origin for both the displacement of the Center of Gravity (CG) and angular the displacement of the vehicle body.

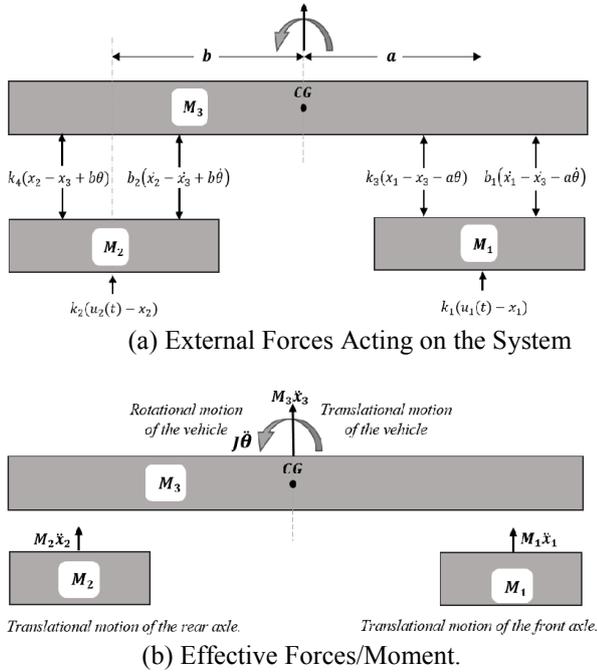


Figure 2: Free-body Diagram of the Model

Equating the external and effective forces and moments, the equations of motion for the front axle, rear axle, chassis translation and rotation are given by;

### Front Axle

$$M_1 \ddot{x}_1 + b_1 \dot{x}_1 + (k_1 + k_3)x_1 - b_1 \dot{x}_3 - k_3 x_3 - ab_1 \dot{\theta} - ak_3 \theta = k_1 u_1(t) \quad (1)$$

### Rear Axle

$$M_2 \ddot{x}_2 + b_2 \dot{x}_2 + (k_2 + k_4)x_2 - b_2 \dot{x}_3 - k_4 x_3 + bb_2 \dot{\theta} + bk_4 \theta = k_2 u_2(t) \quad (2)$$

### Chassis (Translational)

$$M_3 \ddot{x}_3 + (b_1 + b_2) \dot{x}_3 + (k_3 + k_4)x_2 - b_1 \dot{x}_1 - k_3 x_1 - b_2 \dot{x}_2 - k_4 x_2 - (bb_2 - ab_1) \dot{\theta} - (bk_4 - ak_3) \theta = 0 \quad (3)$$

### Chassis (Rotational)

$$J \ddot{\theta} + (b^2 b_2 + a^2 b_1) \dot{\theta} + (a^2 k_3 + b^2 k_4) \theta - ab_1 \dot{x}_1 - ak_3 x_1 + bb_2 \dot{x}_2 + bk_4 x_2 - (bb_2 - ab_1) \dot{x}_3 - (bk_4 - ak_3) x_3 = 0 \quad (4)$$

In order to verify the proposed algorithm, a numerical simulation for the performance of the HAVSS is carried out as a passive and an active suspension system for comparison. The model parameters used are described in Table 1 below.

Table 1: Half-Car Modelling Components

| Parameter                             | Symbol     | Values  | Unit    |
|---------------------------------------|------------|---------|---------|
| <b>Constants</b>                      |            |         |         |
| Sprung mass of the vehicle Chassis    | $M_3$      | 1795    | Kg      |
| Moment of inertia of the vehicle      | $J$        | 3443.05 | $Kgm^2$ |
| Unsprung mass of the front axle       | $M_1$      | 87.15   | Kg      |
| Unsprung mass of the rear axle        | $M_2$      | 140.4   | Kg      |
| Stiffness of the front tire material  | $k_1$      | 190     | $kN/m$  |
| Stiffness of the rear tire material   | $k_2$      | 190     | $kN/m$  |
| Spring constant of the front axle     | $k_3$      | 36350   | $N/m$   |
| Spring constant of the rear axle      | $k_4$      | 26530   | $N/m$   |
| Damping coefficient of the front axle | $b_1$      | 1200    | $Ns/m$  |
| Damping coefficient of the rear axle  | $b_2$      | 1100    | $Ns/m$  |
| Front body length from the CG         | $a$        | 1.32    | $m$     |
| Rear body length from the CG          | $b$        | 1.46    | $m$     |
| <b>State Variables</b>                |            |         |         |
| Vehicle vertical displacement         | $x_3$      | ---     | $m$     |
| Vehicle rotational movement           | $\theta$   | ---     | $rad$   |
| Front axle vertical displacement      | $x_1$      | ---     | $m$     |
| Rear axle vertical displacement       | $x_2$      | ---     | $m$     |
| <b>Inputs (road excitation)</b>       |            |         |         |
| Road excitation at the front axle     | $u(t)$     | ---     | $m$     |
| Road excitation at the rear axle      | $u(t+t_d)$ | ---     | $m$     |

In state-space format, the modeling equations; (1), (2), (3) and (4), are represented as;

$$\begin{aligned} \dot{y}(t) &= Ay(t) + Bu(t) \\ x(t) &= Cy(t) + Du(t) \end{aligned} \quad (5)$$

Where  $y(t)$  is the state vector describing the front and rear axle vertical displacements and velocities, translational and rotational displacements of the unsprung mass of the vehicle.

$$y(t) = \begin{pmatrix} x_1 \\ \dot{x}_1 \\ x_2 \\ \dot{x}_2 \\ x_3 \\ \dot{x}_3 \\ \theta \\ \dot{\theta} \end{pmatrix}, A = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{(k_1+k_3)}{M_1} & \frac{b_1}{M_1} & \frac{0}{M_1} & \frac{0}{M_1} & \frac{k_3}{M_1} & \frac{b_1}{M_1} & \frac{ak_3}{M_1} & \frac{ab_1}{M_1} \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{(k_2+k_4)}{M_2} & \frac{b_2}{M_2} & \frac{k_4}{M_2} & \frac{b_2}{M_2} & -\frac{bk_4}{M_2} & -\frac{bb_2}{M_2} \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ \frac{k_3}{M_3} & \frac{b_1}{M_3} & \frac{k_4}{M_3} & \frac{b_2}{M_3} & -\frac{(k_3+k_4)}{M_3} & \frac{(b_1+b_2)}{M_3} & \frac{(bk_4-ak_3)}{M_3} & \frac{(bb_2-ab_1)}{M_3} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ \frac{ak_3}{J} & \frac{ab_1}{J} & -\frac{bk_4}{J} & -\frac{bb_2(bk_4-ak_3)(bb_2-ab_1)}{J} & \frac{(a^2k_3+a^2k_4)}{J} & \frac{(b^2b_2+a^2b_1)}{J} & 0 & 0 \end{pmatrix} \text{ and } B = \begin{pmatrix} 0 & 0 \\ \frac{k_1}{M_1} & \frac{0}{M_1} \\ 0 & 0 \\ \frac{0}{M_2} & \frac{k_2}{M_2} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \quad (6)$$

$u(t)$  is the input to the system which is more-or-less the disturbance from the road excitation. Many factors that can be considered as the input, like (Ekoru 2012);

1. Wind gusts, gale and vehicle aerodynamics.
2. Poor damped engine vibrations.
3. Vehicle maneuvering (corner or accelerating).
4. Road surface conditions.

The last of the aforementioned factors appeared to be more distorting to vehicle stability than the rest. Hence the vehicle is excited by a triangular bump (triangular impulse) on an otherwise smooth road. This is illustrated in Figure 3 and expressed in (7), where  $A$  is the bump amplitude,  $t_d$  is the time delay between the front and rear wheel while hitting the bump. It depends on the speed of the vehicle.

$$u(t) = \begin{cases} At, & t_1 \leq t \leq t_2 \\ 0 & \text{Otherwise} \end{cases} \quad (7)$$

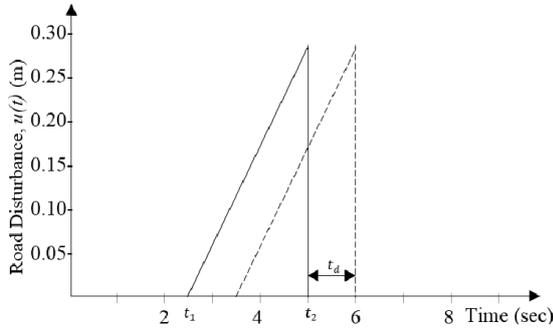


Figure 3: Typical Road Profile

## PID CONTROLLER DESIGN

The model represented in its state-space format is serially coupled with a PID controller. Two of the outputs (displacements of the axles) are fed back to the input to form double close-loops as shown in Figure 4. The two PID controllers' output,  $e_o$  in relation to their respective inputs and control gains is expressed as (Talib, Darus and Hussain 2013);

$$e_o(t) = K_p e_i(t) + K_i \int e_i(t) dt + K_d \dot{e}_i(t) \quad (8)$$

$$\text{Front axle: } e_{if}(t) = u(t) - x_1(t) \quad (9)$$

$$\text{Rear axle: } e_{ir}(t) = u(t + t_d) - x_2(t) \quad (10)$$

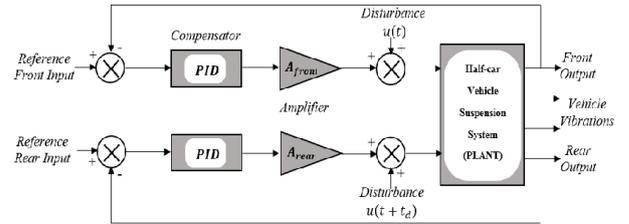


Figure 4: System Control Architecture.

The reference inputs are zeros (smooth road surface), which together with the disturbance form the input to the plant. The amplifier is placed to boost the effect of the PID. It is imperative to note that (Craig 1986);

1. Proportional control action is generated based on the present error.
2. Integral control action is generated based on the past error.
3. Derivative control action is generated based on the anticipated future error.
4. PID deals with the present, past and anticipated error.

The excellent performance of PID controller depends on the optimal values and proper combination of the controller gains;  $K_p$ ,  $K_i$  and  $K_d$  in (8). Iterative learning algorithm is an interesting approach of tuning PID parameters, however, it is complex. Ziegler-Nichols tuning criterion (Talib, Darus and Hussain 2013) is used in obtaining the values of the gains used in the model.  $K_i$  and  $K_d$  are initially set to zero and  $K_p$  is increased from zero to 725 and 650 for front and rear respectively, the values at which the outputs oscillates at constant amplitude. Other gains are later updated as shown in Table 2 below.

Table 2: PID Tuning Parameters

| Parameter           | Symbol | PIDGains values |           |
|---------------------|--------|-----------------|-----------|
|                     |        | Front Axle      | Rear Axle |
| <b>Proportional</b> | $K_p$  | 725             | 650       |
| <b>Integral</b>     | $K_i$  | 392             | 347       |
| <b>Derivative</b>   | $K_d$  | 195             | 162       |

The compensator measures the 'error' between the reference inputs and the measured output, as expressed in (9) and (10), at the front and rear axles;  $e_{if}(t)$  and  $e_{ir}(t)$  respectively. It then produce signals to the AHSS that minimize the error or even drive it to zero.

## SIMULATION RESULTS AND DISCUSSION

The simulation is carried out in state-of-the-art MATLAB environment within a total simulation time of 10.00 s. The car is assumed to be moving with a linear velocity of 20km/hr ( $v=5.56m/s$ ) at the time it went over the triangular bump of 0.25m amplitude. Considering  $a$  and  $b$ , the time delay,  $t_d$  can be computed as 0.50s using (11);

$$t_d = \frac{(a+b)}{v} \quad (11)$$

The system response is analyzed considering both active (with the PID) and the passive system simultaneously in order to ascertain the performance of the HAVSS over PVSS. Settling time and maximum overshoot are used as the parameters in establishing the effectiveness and robustness of the models. From Figure 5, which depicts the response of the front axle vertical displacement, it can be deduced that the maximum suspension travels occurred at the maximum heights of the road disturbance input for both models.

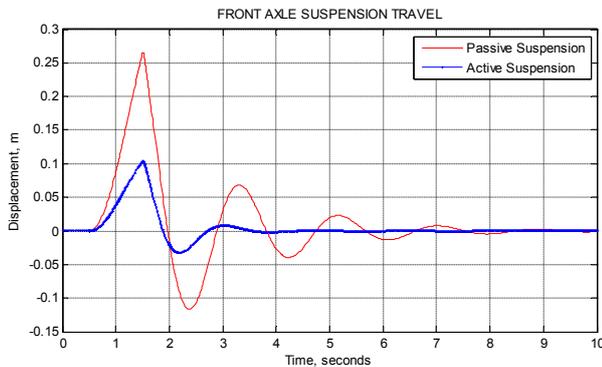


Figure 5: Front Axle Vertical Displacement.

It is worth-noting that the HAVSS not only lessens the maximum suspension travel (maximum overshoot) from 0.2661m of PVSS to 0.1039m, but hasten the settling time by almost 4.96 s. The maximum overshoot occurred at 1.5 s, some 0.50 s earlier than that of the rear suspension travel shown in Figure 6 below. This is due to the time delay between the front and rear wheel when hitting the bump.

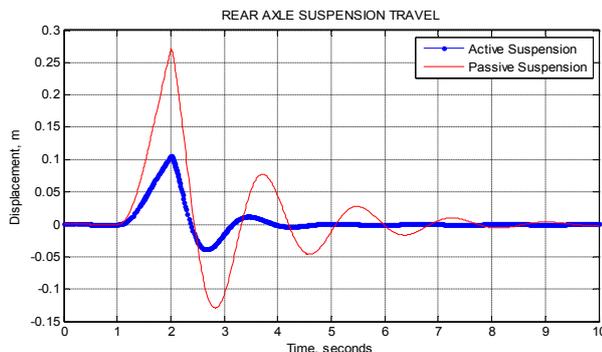


Figure 6: Rear Axle Vertical Displacement.

Similarly, HAVSS has decreased the maximum suspension travel by almost 0.17m compared to its PVSS counterpart. This is in addition to speeding up the system stability as it reached its steady state much earlier than that of PVSS as summarized in Table 3. Figures 4 and 5 describe the vehicle's road holding capability. Failure for the wheels to settle faster leads to skidding and ineffective braking. This is more applicable in sport cars that usually have stiff and harsh suspensions just to improve road holding, however with poor passenger comfort.

The translational displacement of the unsprung mass of the vehicle, shown in Figure 7, shows the effectiveness of HAVSS against PVSS in settling the system response. This effect is more applicable to luxury cars that are more concerned with passengers' comfort.

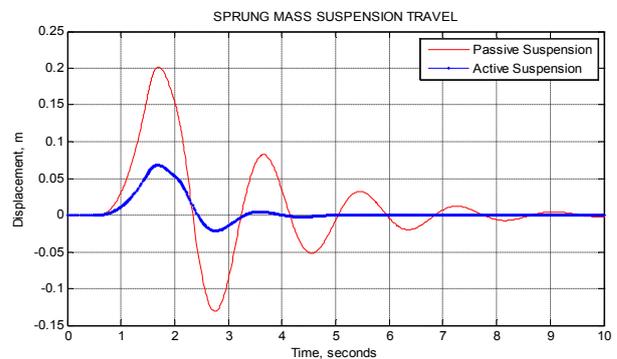


Figure 7: Vehicle Body Vertical Displacement.

The response of the rotational displacement of the unsprung mass of the vehicle, for both HAVSS and PVSS are shown in Figure 8. However, unlike the previous responses, it has an undershoot instead of overshoot, hence represented with negative sign in Table 3. The PID controller tends to restore the system to its steady state faster and with less undershoot.

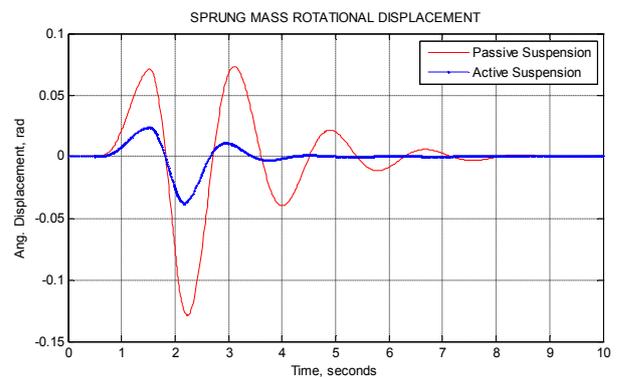


Figure 8: Vehicle Body Rotational Displacement.

Table 3 summarizes the superior performance of the HAVSS over PVSS in restoring the system to original steady state faster.

Table3: Summary of System Responses

| Displacement   | Max. Overshoot (m) |         | Settling Time(s) |        |
|----------------|--------------------|---------|------------------|--------|
|                | Passive            | Active  | Passive          | Active |
| ---            |                    |         |                  |        |
| Front          | 0.2661             | 0.1039  | 8.9133           | 3.9502 |
| Rear           | 0.2709             | 0.1055  | 8.7940           | 4.3170 |
| Vehicle Trans. | 0.2010             | 0.0679  | 9.3623           | 4.8106 |
| Vehicle Ang.   | -0.1287            | -0.0377 | 8.5482           | 4.4907 |

## CONCLUSION

Modeling and simulation of classical half-car suspension system is carried out in this paper. Newton's second law of motion is used to develop the modelling equations. PID controller has been successfully implemented to make the suspension system active. Ziegler-Nichols tuning criterion used in obtaining the PID controller gains used in the model. Road surface condition is considered while modeling the input to the system, in which the vehicle is excited by a triangular bump on an otherwise smooth road. The performance characteristics and the robustness of both passive and active suspension system are evaluated by considering the maximum overshoot and settling time of the responses. From the simulation result, it has been established that the proposed active suspension system proved to be more effective in controlling the vehicle oscillation and more robust in resoring the system to its steady state as compared to the passive suspension system.

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