NONLINEAR GAIN SCHEDULED CONTROLLER FOR A SPHERE LIQUID TANK

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KEYWORDS

Sphere liquid tank, nonlinear model, continuous-time model, parametrized linear model, scheduling variable, polynomial method, gain scheduled controller.

ABSTRACT

In this paper, we develop a gain-scheduled controller for a nonlinear plant of the sphere liquid tank. The proposed strategy is based on linearization of the nonlinear state equation around selected operating points. This methodology allows to apply any linear control design method. In particular, we discuss a polynomial method to achieve desired stability and performance requirements. Following this idea, both 1DOF and 2DOF control configurations are considered. The linear design methods are applied at each operating point in order to arrive at a set of linear control laws. Additionally, the parameters of the resulting family of linear controllers are scheduled as functions of reference variable, resulting in a single controller. Nonlocal performance of gain scheduled controller for the nonlinear model is checked by mathematical simulation.

INTRODUCTION

Realistic models of technological processes are often nonlinear. One consequence is that dynamical behavior of the system to be controlled changes with operating regions. In other words, superposition principle known from linear systems does not hold any longer and we are faced with more complicated situation. However, since linear models are so much more traceable, the first step in analyzing a nonlinear system comes with a trick of linearization. This is very intuitive approach but may run into problems when trying to control system by using classical controller with a fixed parameters. The basic limitation associated with this approach is the fact that a considered controller can only operate in the neighborhood of a single operating point, predicting the local behavior of the nonlinear system. Interestingly enough, in many cases, it is possible to capture how the dynamics of a system change in its equilibrium points by introducing parameterized linear model (PLM). Moreover, it may be even possible to find one or more variables that parameterizes these equilibrium points. In such cases, it is intuitively reasonable to linearize the nonlinear model about selected operating points, capturing key states of a system, design a linear controller at each point, and interpolate the resulting family of linear controllers by monitoring scheduling variables.

However, one difficulty, we may face in control design of the real plants is an appropriate choice of scheduling variable. Several references, including (Rugh 1991; Shamma and Athans 1991; Shamma and Athans 1992) discuss this crucial point of the gain scheduling. Based on this results, it seems that there is no general guideline to follow. In addition, except few standard methods, the scheduling itself appears to be an art. Generally speaking, this difficulty can be overcome by the rule of "schedule on slow variable". As we go over the model of a sphere liquid tank (SLT), we will see that the choice of appropriate scheduling variable is more based on the physics and characteristics of the model.

There has been considerable research in gain scheduling. Most efforts have been devoted to the analytical framework (Shamma and Athans 1990; Rugh 1991) and common principles for gain scheduling and less attention engineering application except few applications in car engines (Jiang 1994; Kaminer et al. 1995). It seems that technological processes have not caught the attention of researches in the context of a gain scheduling.

To address those need, we have stressed to illustrate a gain scheduling strategy for a nonlinear system of the SLT, extending region of validity of linearization approach by designing a controller that is a prescription for moving from one design to another. We refer the interest reader to (Lawrence and Rugh 1995) for deeper and more insightful understanding of the gain scheduling procedure.

Thus, the control problem for a model of the nonlinear system of the SLT has been reduced to a problem of designing a family of linear controllers that are interpreted as a single controller via scheduling variable. Moreover, we have also considered a control system under fixed-gain scheduled controller. Finally, we demonstrate a method of control law synthesis based on polynomial method (Kučera 1993; Mikleš and Fikar 2004) which ensures stability as well as asymptotic tracking of the reference signal. Such a challenge has been met by organizing both 1DOF and 2DOF control system configurations. The use of these structures is based on (Grimble 1993), where detailed description and successful applications can be found.

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MODEL OF THE SLT

A simplified model of the SLT system taken from (Dostál et al. 2008) is shown in Figure 1. Pump with a flow rate of proportional to the voltage, discharges liquid into the spherical tank. Liquid leaves the tank through the opening in the base. There are no reactants or reaction kinetics and stoichiometry to consider. The model also includes hydraulic relationship for the tank outlet stream.

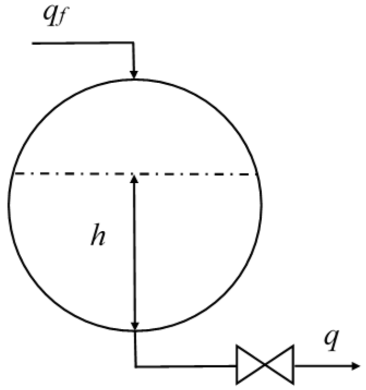

Figure 1: Sphere liquid tank

The only step needed to develop the model of SLT is to write conservation equation (Richardson 1989), representing material balance for a single material. Recall that the general form of a mass balance is given

INPUT = OUTPUT + ACCUMULATION

It is easy to see that the simplified model of SLT can be modeled by

$$\pi h(d-h)\frac{dh}{dt}+q=q_f \tag{1}$$

where h is the liquid level of tank and d is the diameter of the tank. As the liquid moves through the valves, we see dependence of q on liquid level as

$$q=k\sqrt{h} \tag{2}$$

where k is positive valve constant.

NONLINEAR MODEL AND LINEARIZATION

In this section, we shall deal with single-input single-output linearizable nonlinear system represented

$$\dot{x}=f(x,u) \tag{3}$$

$$y=h(x) \tag{4}$$

where $\dot{x}$ denotes derivative of x with respect to time variable and u is specified input variable. We call the variable x the state variable and y the output variable. We shall refer to (3) and (4) together as the state-space model. To obtain a state-space model of the SLT, let us take $x=h$ as state variable and $u=q_f$ as control input. Then the state equation is

$$\frac{dx}{dt}=\frac{1}{\pi x(d-x)}(u-k\sqrt{x}) \tag{5}$$

and the output equation takes the form

$$y=x \tag{6}$$

The equilibrium points of the system are determined by setting $\dot{x}=0$ and solving for x. Therefore the equilibrium points correspond to the solution of

$$0=\frac{1}{\pi x(d-x)}(u-k\sqrt{x}) \tag{7}$$

Having calculated equilibrium points of state equation, our goal now is to approximate (5) about selected single operating point. Suppose $x\neq 0$ and $u\neq 0$, and consider the change of variables

$$x_\delta(t)=x(t)-\bar{x} \tag{8}$$

$$u_\delta(t)=u(t)-\bar{u} \tag{9}$$

$$y_\delta(t)=y(t)-\bar{y} \tag{10}$$

It should be noted that in the new variables system has equilibrium in origin.

Linearization of the nonlinear state equation (5) about operating point yields

$$\dot{x}_\delta=ax_\delta+bu_\delta \tag{11}$$

where

$$a=\left.\frac{\partial f}{\partial x}\right|_{x=\bar{x},u=\bar{u}}=\left.\frac{-k\sqrt{x}}{2x^2\pi(d-x)}\right|_{x=\bar{x},u=\bar{u}} \tag{12}$$

$$b=\left.\frac{\partial f}{\partial x}\right|_{x=\bar{x},u=\bar{u}}=\left.\frac{1}{\pi x(d-x)}\right|_{x=\bar{x},u=\bar{u}} \tag{13}$$

GAIN SCHEDULED CONTROLLER DESIGN

A schematic block diagram representation of the gain scheduled control system is captured in Figure 2. From the figure, it can be easily seen that controller parameters are automatically changed in open loop fashion by monitoring operating conditions. From this point of view, presented gain scheduled control system can be understand as a feedback control system in which the feedback gains are adjusted using feedforward gain scheduler.

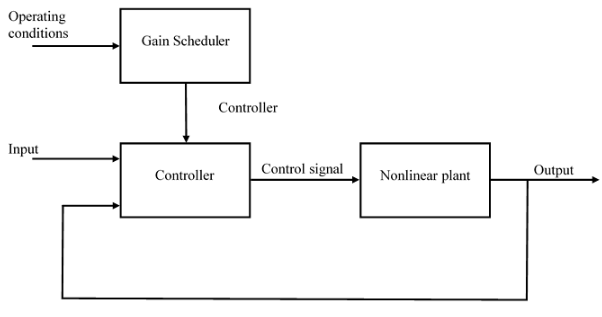

Figure 2: Gain scheduled control

In order to understand the idea behind the gain scheduling let us first consider the nonlinear system

$$\dot{x} = f(x,u,\alpha) \tag{14}$$

$$y = h(x,\alpha) \tag{15}$$

The nonlinear system is basically same as the system that we have introduced in the previous section by equations (3) and (4). The only difference here is that both state and output equations are parameterized by a new scheduling variable α representing the operating conditions.
In view of our SLT model, we can describe the development of a gain scheduled controller for tracking control of nonlinear system by the following steps.

Selection of scheduling variable

The first step in constructing a gain scheduled controller is to find an appropriate scheduling variable. From this point on, we will shortly concentrate our attention on a problem of selection of scheduling variable.

It is not difficult to notice that if we want to track reference signal *w*, it is necessary to maintain the input, at the value $\overline{u} = k\sqrt{w}$. This implies that for every value of *w* in operating range, we can define the desired operating point by $x = w$ and $u = k\sqrt{w}$

The foregoing discussion gives us a clue on how to choose a scheduling variable. Thus, it should come as no surprise that is more than reasonable define $\alpha = w$ as the appropriate scheduling variable.

Construction of parametrized linear models

Once the identification of scheduling variable is clear, we can construct a family of parameterized linear models (PLM) by rewriting (11) into

$$\dot{x}_\delta = a(\alpha)x_\delta + b(\alpha)u_\delta \tag{16}$$

Intuitively speaking, the parameters *a*, *b* of PLM are scheduled as functions of the scheduling variable α. In other words, the key how to move from one operating point to another is given by

$$a(\alpha) = \frac{-k\sqrt{\alpha}}{2\alpha^2\pi(d-\alpha)} \tag{17}$$

$$b(\alpha) = \frac{1}{\pi\alpha(d-\alpha)} \tag{18}$$

Design of linear controller

Since the basis for the construction of family of parametrized linear models has been previously explored, we would like to look more closely at derivation of the linear controller at each operating point that is a prescription for designing *u* such that *y* asymptotically tracks *w* with all generated signals remaining bounded. In order to achieve this control objective, we analyze both 1DOF and 2DOF control configuration. The first case is shown in Figure 3 and includes one degree of freedom that is represented by feedback controller G_Q. In the second case, we have considered two degrees of freedom. As can be seen from Figure 4, desired controller consists of the feedback part G_Q and the feedforward part G_R.
In both control configurations *w* represents the reference signal, *v* is the load disturbance, *y* is the controlled output and *u* is the control input

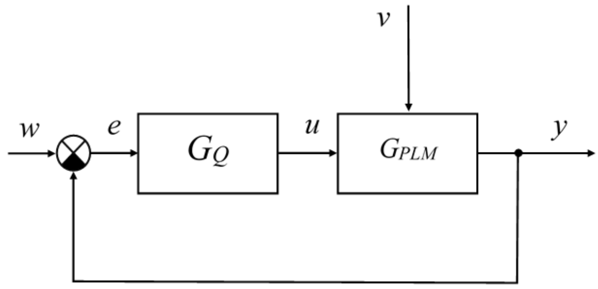

Figure 3: 1DOF control system configuration

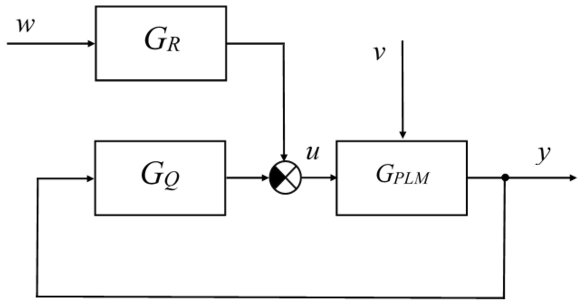

Figure 4: 2DOF control system configuration

Consider now the single-input single output linearized system described by equation (11) or, equivalently, by the transfer function model

$$Y(s) = G_{PLM}(s)U(s) = \frac{b(s)}{a(s)}U(s) \tag{19}$$

where *U*(*s*) and *Y*(*s*) are Laplace transforms of the control input *u*(*t*) and measured output *y*(*t*)**,** respectively.

From the equation (11) it can be seen that the polynomials *a* and *b* are monic having following structure

$$a(s) = s + a_0 \tag{20}$$

$$b(s) = b_0 \tag{21}$$

To aid insight into controllers

$$G_Q(s) = \frac{q(s)}{p(s)}, \quad G_R(s) = \frac{r(s)}{p(s)} \tag{23}$$

where *q*, *r* and *p* represents polynomials in *s*, we will work with both reference signal and disturbance signal as follows

$$W(s) = \frac{w_0}{s}, \ V(s) = \frac{v_0}{s} \tag{24}$$

As is well known we have to use integral control such that polynomial *p* takes the form

$$p(s) = s\tilde{p}(s) \tag{25}$$

To proceed with the design of the controllers, we leave it as an exercise for the reader to verify that both closed-loop linear systems has the characteristic equation

$$a(s)\,p(s)+b(s)\,q(s)=d(s) \tag{26}$$

One can intuitively expect that the control system is stable if we design d to be Hurwitz polynomial. More specifically, in 2DOF control configuration we have to associate with (26) another equation

$$s\,t(s)+b(s)r(s)=d(s) \tag{27}$$

to satisfy the condition of asymptotic tracking of the reference signal.

Toward the goal suppose we have succeeded in finding polynomials of the transfer functions

$$G_Q(s)=\frac{q(s)}{s\tilde{p}(s)}=\frac{q_1 s+q_0}{sp_0} \tag{28}$$

$$G_R(s)=\frac{r(s)}{s\tilde{p}(s)}=\frac{r_0}{sp_0} \tag{29}$$

that satisfy (26), (27) for the stable polynomial

$$d(s)=(s+\beta_1)(s+\beta_2) \tag{30}$$

Recalling the basis of linear algebra, we can obtain the controller parameters from the solution of the matrix equation

$$\begin{bmatrix} 1 & 0 & 0 \\ a_0 & b_0 & 0 \\ 0 & 0 & b_0 \end{bmatrix}\begin{bmatrix} p_0 \\ q_1 \\ q_0 \end{bmatrix}=\begin{bmatrix} d_2 \\ d_1 \\ d_0 \end{bmatrix} \tag{31}$$

where the coefficients of polynomial d are given by

$$d_2=1,\, d_2=\beta_1+\beta_2,\, d_0=\beta_1\beta_2 \tag{32}$$

It is important to emphasize that selectable poles β_1 and β_2 are the only parameters through which the controller parameters can be adjusted.

Construction of gain scheduled controller

The resulting gain scheduled controller can be obtained by scheduling coefficients of $q(s)$ and $r(s)$ as functions of α; that is, α is replaced by w, so that the gains vary directly with the desired height.

From this, the linear control law, which is prescribed by controllers (28), (29) can be rewritten in terms of scheduling variables as

$$u_{\mathrm{1DOF}}=q_1(\alpha)e+q_0(\alpha)\sigma \tag{33}$$

$$u_{\mathrm{2DOF}}=r_0(\alpha)\tau-q_1(\alpha)y-q_0(\alpha)\upsilon \tag{34}$$

where

$$e=\dot{\sigma}\,,\;\; w=\dot{\tau}\;\; \text{and}\;\; y=\dot{\upsilon} \tag{35}$$

So far, we have formed the basic idea of construction of gain scheduled control law. All that remains now is to show, that for a desired Hurwitz polynomial d, the gains are taken as

$$\begin{aligned} q_1(\alpha)&=\frac{\beta_1+\beta_2-a_0(\alpha)}{b_0(\alpha)} \\ q_0(\alpha)&=r_0(\alpha)=\frac{\beta_1\beta_2}{b_0(\alpha)} \end{aligned} \tag{36}$$

When the control (33) and (34) is applied to the nonlinear state equation (5) it results in the closed-loop system

$$\dot{x}=\frac{1}{\pi x(d-x)}\left[q_1\left(e+\frac{q_0}{q_1}\sigma\right)-k\sqrt{x}\right] \tag{37}$$

and

$$\dot{x}=\frac{1}{\pi x(d-x)}\left[r_0\tau-q_1\left(y+\frac{q_0}{q_1}\upsilon\right)-k\sqrt{x}\right], \tag{38}$$

respectively.

In view of the procedure that we have just described, one can notice that three main issues are involved in the development of gain scheduled controller; namely linearization of SLT about the family of operating regions, design of a parametrized family of linear matrix feedback controllers for the parametrized family of linear systems and construction of gain scheduled controller.

SIMULATIONS AND RESULTS

In this section, we simulate the gain scheduled control of SLT. We have developed a custom MATLAB function based on simulator introduced by (Krhovják et al. 2014) that simulates adequately the behavior of SLT. Idealistic model has been implemented according to equations (1) and (2). The popular ODE solver using Euler method (Hairer et al. 1993) was considered to calculate numerical solution of the closed-loop 1DOF and 2DOF system, represented by the equations (37) and (38), respectively. The parameters of the tank and initial conditions we started with are d = 2m, k = 0.8m$^{2.5}$/min, h = 0.25m, q_f = 0.25m.

The simulation results of gain scheduled control are presented in Figures 5-9. Figure 5 clearly illustrates how the linearized plant dynamics vary with the operating conditions that are given by scheduling variable α.

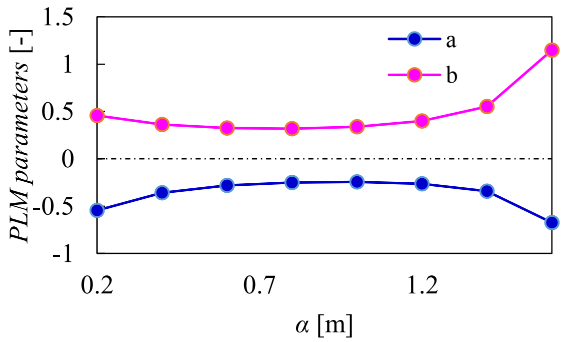

Figure 5: Family of PLM

Figure 6 shows the responses of the control system to the sequence of step changes in reference signal. As can be seen from Figure 6, we have found such a combination of parameters β_1 and β_2 that gave a reasonably good responses. Notice, that better performance is achieved in the case of configuration with two degrees of freedom.

From a gain-scheduling viewpoint, a step change in reference signals causes a new calculation of the equilibrium point of the system. This claim is also supported by Figure 7 in which the gain adjustment is captured. It is important to notice that the change of controller parameters always comes with the step change in reference trajectory.

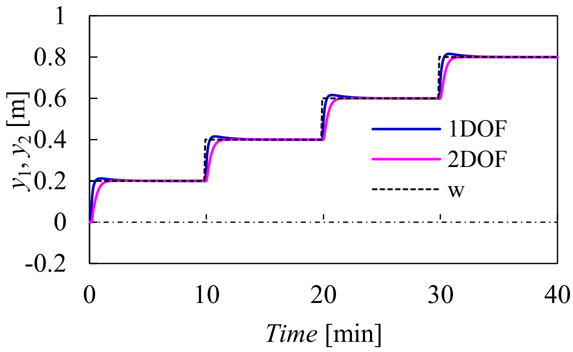

Figure 6: The responses of the closed-loop system to a sequence of step changes

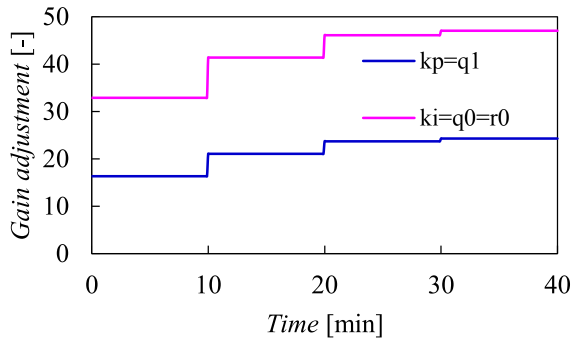

Figure 7: Gain adjustment during control

To appreciate what we gain by gain scheduling, Figure 8 and Figure 9 illustrates responses of the closed-loop system to the same sequence of changes. In the first case, a gain scheduled controller is applied, while in the second case a fixed-gain controller evaluated at α = [1.7] is used. Since our controller is based on linearization, it guarantees only local stabilization. Therefore, one should be careful about big step changes in references. As the reference signals is far from operating point, the performance deteriorates and system may go unstable. In some situations, it may be possible to overcome this difficulty by sequence of small step changes as is shown in Figure 5. In other words, it is necessary to move slowly from one point to another. These observations are consistent with a common gain scheduling rule-of-thumb about the behavior of gain scheduled controller under slowly varying scheduling variable.

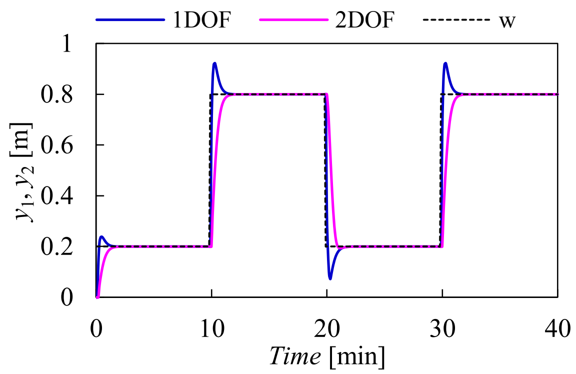

Figure 8: The reference and output signal of the gain scheduled control

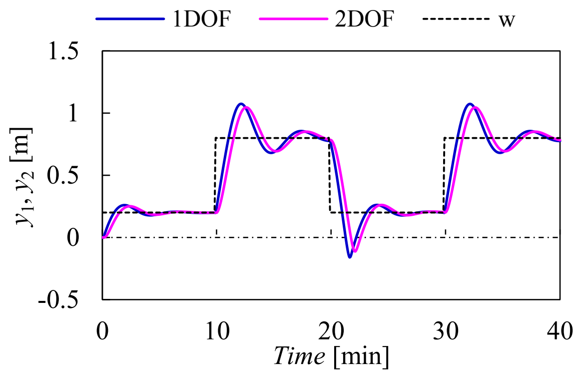

Figure 9: The reference and output signal of the fixed gain control

CONCLUSION

In this paper, a promising gain scheduling strategy to the control of a nonlinear system of a sphere liquid tank has been presented. First, we have detailed studied the simplified model of the technological process. Based on the model, we have followed a general analytical framework for gain scheduling. We have also pointed out that selection of scheduling variable critically depends on particular characteristics of the model. This observation has critical importance and leads us to the conclusion that rule of scheduling on reference variable can be applied for other technological processes. The main advantage of this approach is that linear design methods can be applied to the linearized system at each operating point. Thanks to this feature, the presented procedure leaves room for many other linear control methods. As our results show, the algebraic procedure, based on the solution of the matrix Diophantine equation, is well suited for the controller synthesis. We have demonstrated that a gain scheduled control system has the potential to respond rapidly changing operating conditions. Finally, tuning parameters of controller have a direct and major effect on feedback performance.

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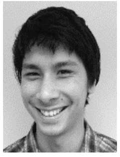

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