

# STUDY ON STABILISATION TIME AND IMPULSE FORCES OF COMBAT VEHICLES FOR ENHANCEMENT OF FIREPOWER PERFORMANCE

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## KEYWORDS

Stabilisation Time, Hit Probability, Fire Rate, Error Budgets, Impulse Forces, Muzzle Velocity, Standard Deviation, Monte Carlo Method, Engineering Method.

## ABSTRACT

Two measures of firepower performance in relation to guns are fire rate and hit probability. Fire rate is related to the stabilisation time of the combat vehicle. The lower the stabilisation time, the faster the fire rate becomes. Hit probability is a function of many error budgets. In this paper, we simulate two cases. In the first case, when a combat vehicle is driving over a speed bump, the stabilisation time is measured. The second simulation is concerned with the relationship between hit probability and impulse forces applied to the gun mounted on the combat vehicle. The combat vehicle is modelled as a full car with arms. Quantification of stabilisation time and the relationship between impulse forces and hit probability proved to be good sources for enhancing firepower performance through improved fire rate and hit probability. We use an engineering method for error estimates, and we use the Monte Carlo method for hit probability prediction.

## INTRODUCTION

When designing a four-wheeled combat vehicle, four important factors are mobility, firepower, vulnerability and operability. Mobility is concerned with how fast the vehicle drives on the wild road; firepower is related to the fire performance of arms equipped on the combat vehicle; vulnerability is concerned with robustness against enemy firepower; and operability refers to the human factors involved in control of the combat vehicle. Among these four performance measures of combat vehicles, firepower performance plays the most important role in the combat environment. The indices of firepower performance are hit probability and fire rate. Hit probability is a function of error budgets (Weaver, 1990, Groves, 1963). Error budgets come from errors of subsystems, which are divided into five categories: errors from internal ballistics, aiming errors from gun and turret, trajectory errors from external ballistics, meteorological errors of sensors, errors of combat vehicle with road conditions such as gun pitch angle error, gun roll angle

error, and gun vertical displacement error. Table 1 tabulates the subsystems and their related errors for combat vehicles.

Fire rate is a function of stabilisation time for the gun. Waveforms for the gun are determined by rough road conditions such as speed bumps. Hit probability is varied according to impulses which come from repulsion of departing bullets. When a target is located some distance

Table 1: Subsystems and Their Related Errors of Combat Vehicles

subsystems	Kinds of errors
Internal ballistics	error of mass of propellant, muzzle velocity error
gun and turret	aiming error, mechanical error
external ballistics	trajectory error
meteorological sensors	temperature error, density error
Combat vehicles with road conditions	gun pitch angle error, gun roll error, gun vertical displacement error.

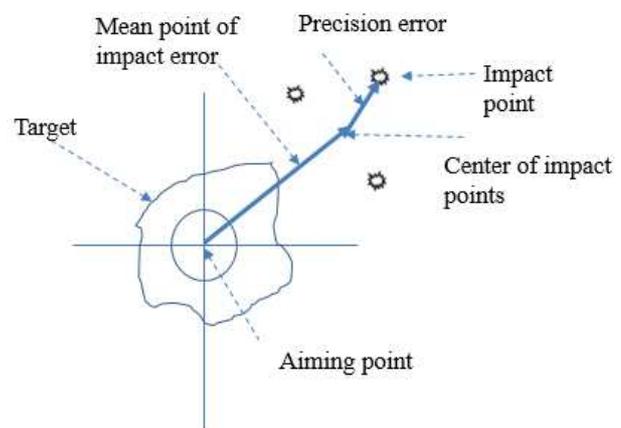


Figure 1: Mean Point of Impact Error and Precision Error

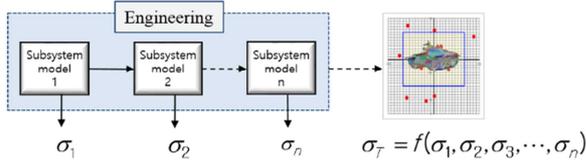
from the position of the combat vehicle, the dispersion of bullets on the target is due to the errors from subsystems of the combat vehicle. The mean point of the impact error (Driels, 2013) is the distance from the aiming point to the mean of the impact points, and the precision error is the distance from the mean point of impact to each

impact point as shown in Figure 1. In this paper, we simulate two cases. The first simulation is that when a combat vehicle driving over a speed bump, the stabilisation time is measured. The second simulation is concerned with the relationship between hit probability and impulse forces applied to the gun mounted on the combat vehicle. The combat vehicle is modelled as a full car with arms. For error estimates, an engineering method is used, and for hit probability prediction, we use the Monte Carlo method.

## TWO METHODS FOR ERROR ESTIMATES

There are two methods for error estimation: one is an engineering based method and the other is an experiment based method. First, we explain the engineering approach. Errors from each subsystem are extracted from models of each subsystem. For example, the model of the combat vehicle with road conditions is dependent on the vehicle dynamics of the full car including gun and turret. Some errors from the model of each subsystem are fed into the initial conditions for the motion equation of the trajectory of bullets. The error on the target  $\sigma_T$  is given by  $\sigma_T = f(\sigma_1, \sigma_2, \dots, \sigma_n)$ .

Error estimates using engineering approach



Error estimates using experimental approach

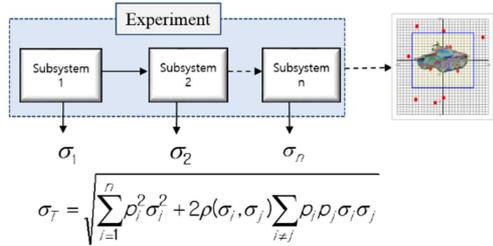


Figure 2: Error Estimates Using Either Engineering or Experimental Approaches

The experimental approach shall now be explained. The error on the target  $\sigma_T$  is a function of some errors  $(\sigma_1, \sigma_2, \dots, \sigma_n)$  from the subsystems. In the case of the experimental approach, each subsystem has its own output  $\sigma_i$  that is obtained by experimentation. The error on the target is then described by

$$\sigma_T^2 = \sum_{i=1}^n p_i^2 \sigma_i^2 + 2 \rho(\sigma_i, \sigma_j) \sum_{i \neq j} p_i p_j \sigma_i \sigma_j \quad (1)$$

where  $p_i$  is a function of shot range and  $\rho(\sigma_i, \sigma_j)$  is a correlation coefficient between  $\sigma_i$  and  $\sigma_j$ . Two methods of error estimates are shown in Figure 2.

## TWO METHODS FOR HIT PROBABILITY PREDICTION

There are two methods of hit probability prediction: one is an analytic method and the other is the Monte Carlo method. First, the analytic method will be explained. Assuming that the target is a rectangle, then the distribution of the bullets on the target should be two normal density functions along the horizontal and vertical directions. Hit probability refers to a single shot hit probability in this paper. Under these conditions, hit probability is written by

$$p_{ss} = \frac{1}{2\pi\sigma_x\sigma_y} \iint_{(x,y) \in target} \exp\left[-\frac{(x-h)^2}{2\sigma_x^2}\right] \exp\left[-\frac{(y-k)^2}{2\sigma_y^2}\right] dx dy \quad (2)$$

where  $\sigma_x$  and  $\sigma_y$  are standard deviations of two normal distributions in the horizontal and vertical directions on the target, respectively.  $h$  and  $k$  are biases in the horizontal and vertical directions on the target, respectively. The width and the height of the rectangular target are respectively set to be  $2a$  and  $2b$ .

The Monte Carlo method employed in this study shall now be explained. At first, supposing that the probability density distributions of muzzle velocity and elevation angles are given, random numbers were generated from two distribution density functions. Using these random but fixed values, we solved the differential equations for motion equations of bullets. On the target, we decided whether the bullet had hit the target or not and, through repetition, we counted the hit probability. Furthermore, we executed the Monte Carlo simulation on the target directly. By selecting two random numbers from the two given probability density functions along the vertical and horizontal directions, we decided whether the coordinates of a randomly selected but fixed number lay inside the target or not. Through repetition, we counted the hit probability.

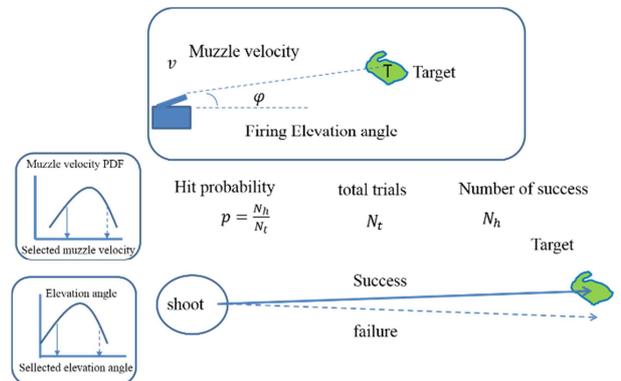


Figure 3: Monte Carlo Simulation for Hit Probability Prediction

In Figure 3, at the position of the gun, we illustrate the Monte Carlo method for prediction of hit probability.

### TRAJECTORY EQUATIONS

The trajectory differential equations (McCoy, 1999) are written by

$$\begin{aligned} \dot{v}_x &= -C_d^* v(v_x - w_x) \\ \dot{v}_y &= -C_d^* v(v_y - w_y) - g \\ \dot{v}_z &= -C_d^* v(v_z - w_z) \end{aligned} \quad (3)$$

where  $v_x, v_y,$  and  $v_z$  are range, upward, and deflection velocity, respectively, and where  $w_x, w_y,$  and  $w_z$  are range, upward and deflection velocity of winds, respectively. The constant value  $C_d^*$  is a modified drag coefficient value and the value is assumed to be  $1.8253 \times 10^{-4} \text{m}^{-1}$ . The value  $g$  is the gravitational acceleration. The value  $v$  is the scalar value of the velocity vector and is expressible as  $\sqrt{v_x^2 + v_y^2 + v_z^2}$  m/sec. The above standard differential equations are solved by numerical analysis. On the other hand, the firing angle is assumed to be very low, and so under this condition we obtained the algebraic solution (Kang, 2015).

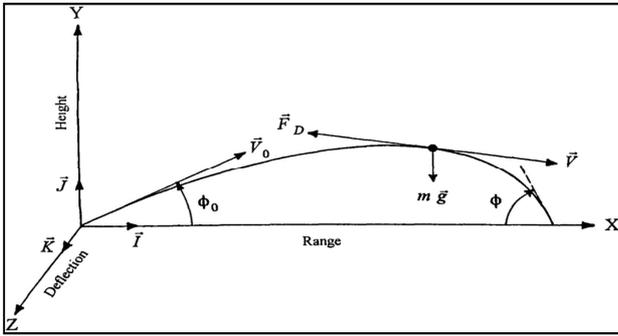


Figure 4: Bullet Trajectory (McCoy, 1999)

### MODELLING OF COMBAT VEHICLE

The combat vehicle is modelled as a full car with a gun and turret as shown in Figure 6. Guns and turrets are herein referred to as arms. The diagram of arms is depicted in Figure 5. All parameters are tabulated in Table 2.  $r_1$  denotes a longitudinal distance between the gun center of gravity and the body center of gravity.  $r_2$  denotes a transversal distance between the gun center of gravity and the body center of gravity.

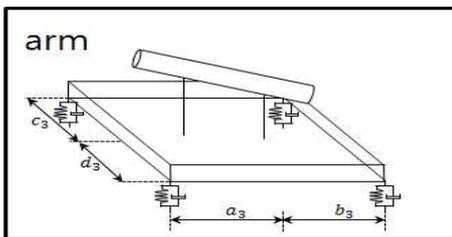


Figure 5: Diagram of Arms

A full car model with arms is used investigate the dynamic responses of a combat vehicle when crossing a speed bump and with the pressure of impulse forces.

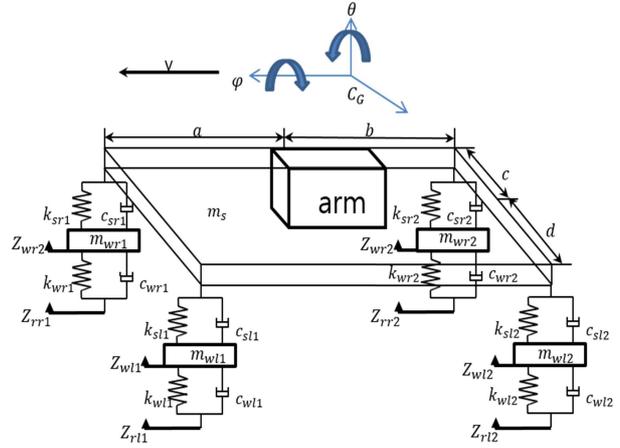


Figure 6: Full car with Arms

Table 2: Parameters of Full Car and Arms

Parameter	Value
Mass of gun center ( $M_g$ )	91.5 Kg
Front & rear inertia ( $I_{yyg}$ )	50 Kg · m <sup>2</sup>
Right & left inertia ( $I_{xxg}$ )	200 Kg · m <sup>2</sup>
Gun damping coefficient ( $C_{gr1}, C_{gl1}, C_{gr2}, C_{gl2}$ )	1000,1000,200,200 N · s/m
Gun stiffness coefficient ( $K_{gr1}, K_{gl1}, K_{gr2}, K_{gl2}$ )	200,200,400,400 N/m
Mass of body center ( $M_b$ )	1200 Kg
Front & rear Inertia ( $I_{yyb}$ )	950 Kg · m <sup>2</sup>
Right & left Inertia ( $I_{xxb}$ )	4000 Kg · m <sup>2</sup>
Length ( $a, b, c, d$ )	1.5,1.5,1,1 m
Length ( $a_3, b_3, c_3, d_3, r_1, r_2$ )	.4,.4,3,2,.1,.1 m
Body damping coefficient ( $C_{sr1}, C_{sl1}, C_{sr2}, C_{sl2}$ )	28030 N · s/m
Body stiffness coefficient ( $K_{sr1}, K_{sl1}, K_{sr2}, K_{sl2}$ )	28030 N/m
Tire mass ( $M_{r1}, M_{l1}, M_{r2}, M_{l2}$ )	60 Kg
Tire damping coefficient ( $C_{wr1}, C_{wl1}, C_{wr2}, C_{wl2}$ )	0 N · s/m
Tire stiffness coefficient ( $K_{wr1}, K_{wl1}, K_{wr2}, K_{wl2}$ )	30000 N/m

### SIMULATION FOR STABILISATION TIME

In consideration of a speed bump (elliptic type bump) located on the road when a combat vehicle is driving, the road condition affects the variation of the position of the gun mounted on the combat vehicle. Using the three waveforms of vertical displacement, pitch angle and roll angle, we obtained a stabilisation time.

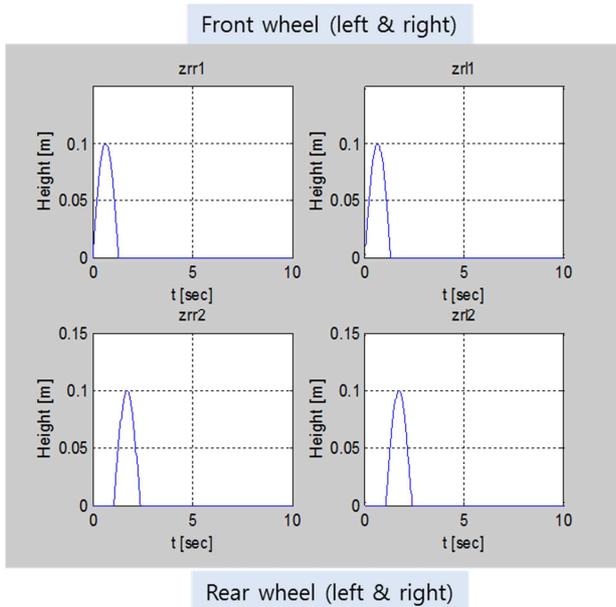


Figure 7: Waveforms Applied to Four Wheels of Combat Vehicle

Stabilisation time is the time duration during which a combat vehicle is stabilised. The stabilisation of a combat vehicle means that the vertical displacement, pitch angle, and roll angle for the gun mounted on the combat vehicle has no transient responses. Specifically, the stabilisation time for vertical displacement refers to time elapsed from the touch of the speed bump by a combat vehicle to the time at which the waveform of the vertical displacement has entered and remained within a 5% of the maximum value of the vertical displacement. After the stabilisation time, the waveform should remain within 5% of the maximum value of the vertical displacement. The stabilisation time for a combat vehicle is defined as the maximum of three stabilisation times for vertical displacement, pitch angle and roll angle.

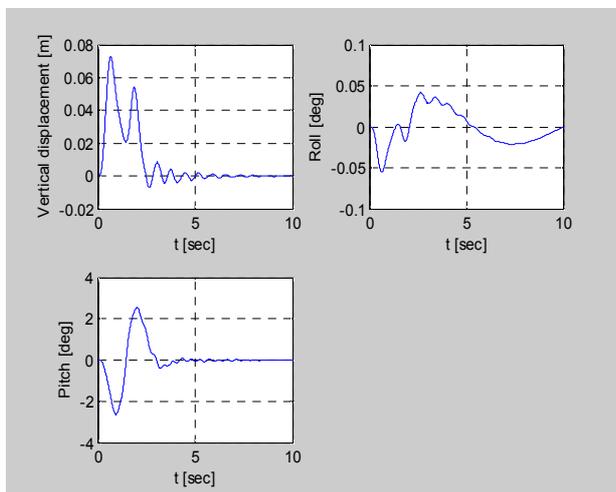


Figure 8: Vertical Displacement, Pitch and Roll Angles at the Gun Position of a Combat Vehicle

The height of the speed bump is set to 10cm and the width of the speed bump is set to 3.7m. The speed of the vehicle is set to 10Km/h. The waveforms applied to the front and rear wheels of the vehicle are shown in Figure 7. The stabilisation time for vertical displacement is 4.15sec, the stabilisation time for pitch angle is 4.12 sec, and the stabilisation time for roll angle is negligible. The total stabilisation for the combat vehicle is 4.15 sec as shown in Figure 8.

### SIMULATION OF HIT PROBABILITY AND IMPULSE FORCES

When impulse force is applied to the gun and variation of muzzle velocity exists, there will be a hitting pattern at the target. The width of the target is set to 4m and its height is set to 4m. The distance between the position of the gun and the target is set to 700m. For error estimates, we use the engineering approach as explained in the section titled Two Methods of Error Estimates. For hit probability prediction, we use the Monte Carlo method as shown in Figure 3. As shown in Figure 9, the crosswind is set to 1m/sec, variance of muzzle velocity is set to 4m/sec and no impulse force to the gun is applied. By using the trajectory of the bullet in the Trajectory Equations section, we obtain a hitting pattern with a hit probability of 51%. Suppose that an impulse force ranges from 0 N to 280 N during 0.5sec, then the hit probability reduces to 24% as shown in Figure 10. A comparison of hit probability with impulse forces and without impulse forces is tabulated in Table 3.

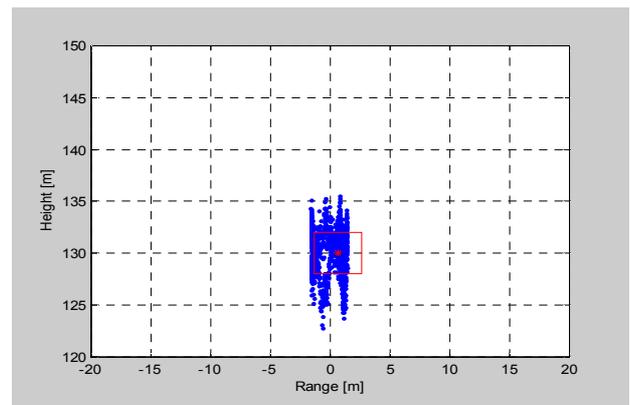


Figure 9: Hit probability Due to Crosswind & Variance of Muzzle Velocity. Hit Probability: 51%

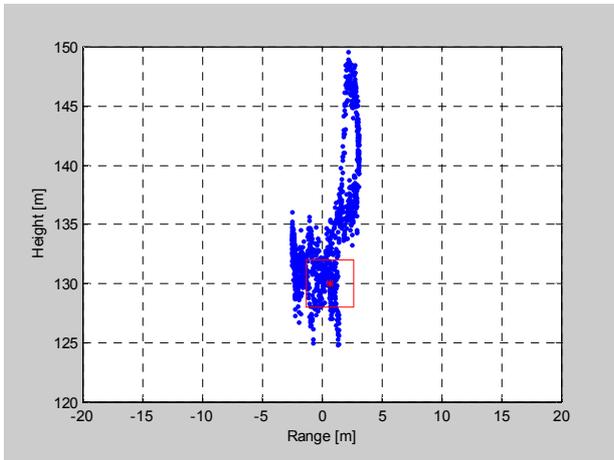


Figure 10: Hit probability Due to Crosswind, Variance of Muzzle Velocity & Impulse Force. Hit Probability: 24%

Table 3: Comparison of Hit Probability with Impulse Forces and without Impulse Forces

Impulse Force	Hit Probability (%)
0 Newton	51
280 Newton during 0.5 sec	24

## CONCLUSION

Two important factors in determining the firepower performance of guns are fire rate and hit probability. In this paper, two cases have been simulated. The first case simulates the situation where a combat vehicle is driving over a speed bump, which we used to measure stabilisation time. The second simulation was developed to investigate the relationship between hit probability and impulse forces applied to the gun mounted on a combat vehicle. The combat vehicle was modelled as a full car with arms. To enhance the hit probability, both the stabilisation time and impulse forces should be reduced. Quantification of stabilisation time, and the relationship between impulse forces and hit probability, are proven here to be good sources for the enhancement of firepower performance in terms of fire rate and hit probability.

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## AUTHOR BIOGRAPHIES



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