MEMRISTOR MODELING IN MATLAB® & PSPICE®

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KEYWORDS
Memristor; modeling; computer simulation; nonlinear dopant drift; window function.

ABSTRACT

The main purpose of the present investigation is to propose a new, modified Joglekar's memristor model and to compare it with the Pickett model using several different window functions. The appropriate value of the exponent in the Joglekar's window function is determined to approach the new model to the Pickett's memristor model, which is based on practical experiments and measurements. The new memristor model is based on both Williams', Joglekar's and BCM models and it has their advantages - considerations of the boundary conditions for hard-switching mode and ability for representation the nonlinear dopant drift. The memristor model proposed here is tunable and correctly expresses the behavior of the memristor element for low- and high-intensity electric fields, and is appropriate for computer simulations of different memristor nanostructures.

INTRODUCTION

The first idea of the theoretical existence of the memristor element is found in the Chua's articles written in 1971 (Chua 1971) and in 1976 (Chua 1976). Since the practical invention of the memristor prototype by Stanley Williams and his team by the Hewlett Packard laboratories in 2008 many research papers about this new element are published (Strukov et al. 2008). In the literature, several basic models appropriate for the memristors are described. Some of these memristor models are: the classical Williams's model with linear dopant drift (Strukov et al. 2008), the Joglekar's model with nonlinear window function (Joglekar and Wolf 2009), the Biolek's model (Biolek et al. 2009), the BCM model (Corinto and Ascoli 2012), and others. The Pickett-Simons model is based on real experiments and measurements of the parameters and characteristics of the memristor element (Abdala and Pickett 2009, Simmons 1963). The Pickett's model is very accurate but it is very hard and non-applicable for computer simulations – several problems with bad convergence occur. The classical Williams model and the Joglekar model use linear drift and non-linear window function, respectively, and they can be used only for soft-switching mode and single-valued state-flux characteristics. The Biolek model uses a special window function appropriate only for multi-valued state-flux characteristics (Biolek et al. 2009). The BCM model is comparatively accurate and suitable for simulations; it uses a linear window function and switched-based algorithm for the boundary conditions. The BCM model is used for presenting both soft-switching and hard-switching modes. By its nature the relationship between the velocity of moving the boundary between the doped and un-doped regions, and the current flowing through the memristor, for high intensity electric fields in the memristor structure is nonlinear and this is the reason for Joglekar to propose the use of nonlinear parabolic window functions. The Joglekar window function has several types that are similar, but they use different exponents of the window function for modeling of the memristors. It is interesting that to this moment the scientists are not sure which exponent of the Joglekar's function is appropriate for computer simulations and for realistic representation of the processes (Prodromakis et al. 2011, Majetta et al. 2012, Zaplatilek 2011). The new idea in this paper is to propose a comparison between the Pickett's model and the modified switch-based Joglekar model for several values of the exponent. After comparison of the results, several values of the exponent in the Joglekar's window function are chosen in accordance to represent the nonlinear ionic drift.

The paper is organized as follows: in Section 2 a description and simulations of the Pickett's model are realized in PSpice; in Section 3 an analytical investigation of the Joglekar's model is made; a pseudo-code algorithm for realizing the modified Joglekar memristor model is illustrated in Section 4; in Section 5 the experimental results from the simulations are presented, and adjusting of the new model with varying the exponent of the window function, with respect to the results of Pickett's model, are realized. In Section 6 the concluding remarks are given.

A BRIEF DESRRIPTION AND SIMULATION OF PICKETT'S MEMRISTOR MODEL

The structure of a memristor cell according to the Pickett's model is shown in Fig. 1. The electrodes are made of platinum and the insulating layer is made of pure titanium dioxide. The conducting channel in the memristor cell is formed by a thin layer of doped with oxygen vacancies titanium dioxide material. There is

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also a thin tunnel barrier with a length of w. The resistance of the conducting layer is approximately equal to R_s = 215 Ω. When the memristor is switched in open state (i>0), the differential equation has to be expressed with (1) (Abdala and Pickett 2009). Formulas (2) – (11) are also taken from (Abdala and Pickett 2009).

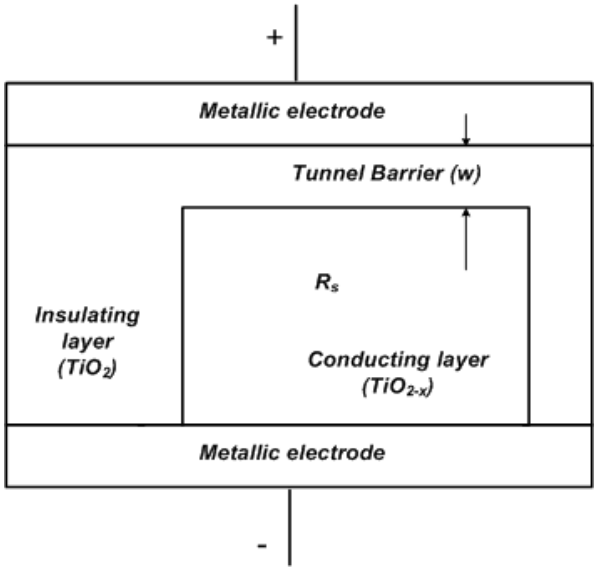

Figure 1. Structure of a memristor element according to the Pickett's memristor model (Abdala and Pickett 2011)

$$\frac{dw}{dt} = f_{off} \sinh\left(\frac{i}{i_{off}}\right) \exp\left[-\exp\left(\frac{w-a_{off}}{w_c} - \frac{|i|}{b}\right) - \frac{w}{w_c}\right] \quad (1)$$

The parameters used in (1) and their corresponding values are given in Table 1.

Table 1. Quantities for open switched memristor

Quantity	f_{off}	i_{off}	a_{off}	b	w_c
Dimension	*μm/s*	*μA*	*nm*	*μA*	*pm*
Value	3.5	115	1.20	500	107

When the memristor is switched in closed state (i<0), the differential equation is:

$$\frac{dw}{dt} = f_{on} \sinh\left(\frac{i}{i_{on}}\right) \exp\left[-\exp\left(\frac{w-a_{on}}{w_c} - \frac{|i|}{b}\right) - \frac{w}{w_c}\right] \quad (2)$$

The parameters and their values are given in Table 2.

Table 2. Quantities for closed switched memristor

Quantity	f_{on}	i_{on}	a_{on}	b	w_c
Dimension	*μm/s*	*μA*	*nm*	*μA*	*pm*
Value	40	8.9	1.80	500	107

The current flowing through the tunnel barrier of the memristor element is :

$$i = \frac{j_0 A}{\Delta w^2}\left[\begin{array}{l}\Phi_1 \exp\left(-B\sqrt{\Phi_1}\right) - \\ -\left(\Phi_1 + |u_g|\right)\exp\left(-B\sqrt{\Phi_1} + |u_g|\right)\end{array}\right] \quad (3)$$

where u_g is the voltage drop over the tunnel barrier.

This voltage drop could be expressed using the Kirchhoff's Voltage Law (KVL) and the voltage u over the memristor:

$$u_g = u - iR_s \quad (4)$$

The constant j_0 could be expressed as follows:

$$j_0 = \frac{e}{2\pi h} = \frac{1.6*10^{-19}}{2*3.14*6.63*10^{-34}} = \\ = 3.84*10^{13},\ [C/(J*s)] \quad (5)$$

where e is the elementary charge of the electron, h is the Planck's constant. The area of the tunnel junction is equal to $A = 10^4$ nm^2.
The deviation of the length of the tunnel junction is:

$$\Delta w = w_2 - w_1,\ nm^2 \quad (6)$$

The second member of (6) is:

$$w_1 = 1.2\frac{\lambda w}{\Phi_0},\ nm^2 \quad (7)$$

where Φ_0 = 0.95 V is the height of the potential barrier. The quantity λ from (7) is obtained as follows:

$$\lambda = \frac{e \ln 2}{8\pi k \varepsilon_0 w*10^{-9}},\ V \quad (8)$$

where k = 5 is the permittivity of the material, ε_0 is the absolute permittivity of the vacuum.

After transformation of (7) we have: w_1 = 0.126 nm. Using (6) we obtain:

$$w_2 = w_1 + w\left(1 - \frac{9.2*\lambda}{3\Phi_0 + 4\lambda - 2|u_g|}\right),\ nm \quad (9)$$

The quantity B is expressed as follows:

$$B = \frac{4\pi\Delta w*10^{-9}\sqrt{2me}}{h},\ [V^{-\frac{1}{2}}] \quad (10)$$

where m is the electron weight.
The quantity Φ_1 is expressed as follows:

$$\Phi_1 = \Phi_0 - |u_g| \left(\frac{w_1 + w_2}{w} \right) - \\ - \frac{1.15 * \lambda w}{\Delta w} \ln \left[\frac{w_2 (w - w_1)}{w_1 (w - w_2)} \right], \ V \qquad (11)$$

Using formulas (1) – (11) a PSpice memristor library model is created and a computer simulation is made. It is theoretically possible generation of MATLAB code or SIMULINK scheme also but several convergence problems occur. So our results are obtained by using a PSpice computer simulation (Abdala and Pickett 2009). The time diagrams of the memristor voltage and current for the Pickett's memristor model are presented in Fig. 2. The experiment was made using a sinusoidal voltage with a magnitude of 0.7 *V* and a frequency of 0.5 *Hz*. The same parameters of the voltage source are used also for the other results obtained for this model.

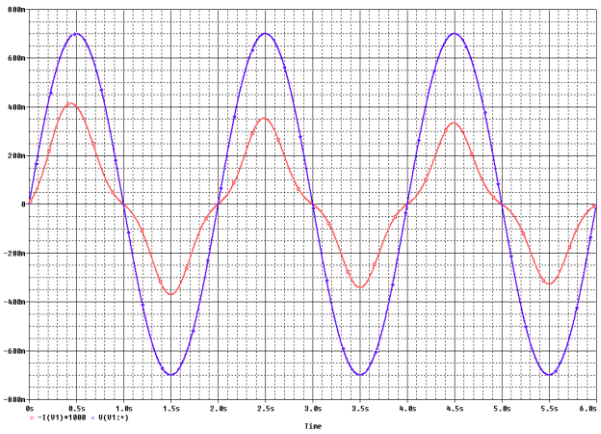

Figure 2: Time Diagrams of the Memristor Voltage and Current for sine voltage with $f = 0.5$ *Hz* and $u_m = 0.7$ *V*

The current-voltage characteristics of the memristor element are given in Fig. 3.

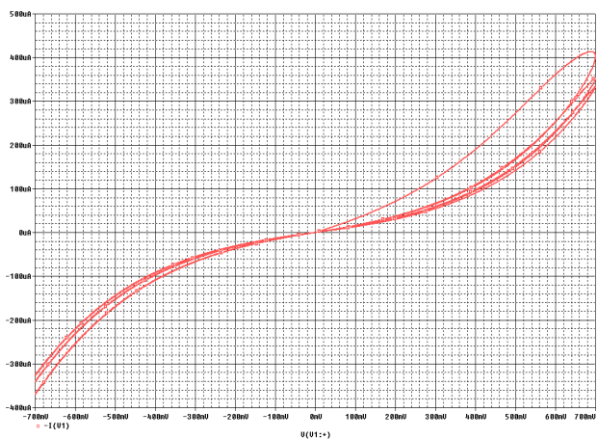

Figure 3: Current-Voltage Characteristics of the Memristor for sine-wave voltage with $f = 0.5$ *Hz* and $u_m = 0.7$ *V*

The state-flux relationship for the memristor element using the Pickett's model is presented in Fig. 4.

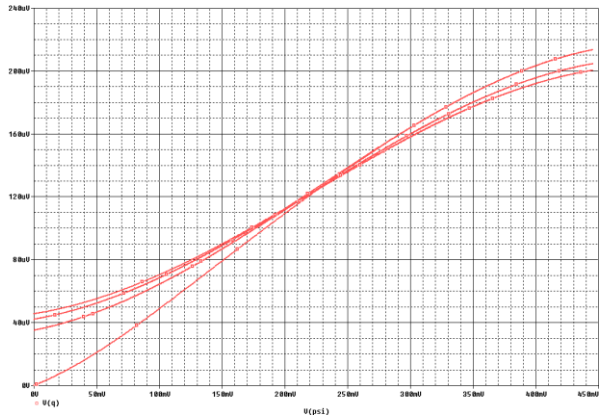

Figure 4: State-Flux Relationship of the Memristor element for sine-wave voltage with $f = 0.5$ *Hz* and $u_m = 0.7$ *V*

ANALYTICAL INVESTIGATION OF JOGLEKAR'S MEMRISTOR MODEL

The structure of a memristor cell according to the Williams-Joglekar's models is presented in Fig. 5. The lengths of the doped and un-doped regions are denoted with *w* and with *D*, respectively. (Strukov et al. 2008). The contacts of the element are made of platinum.

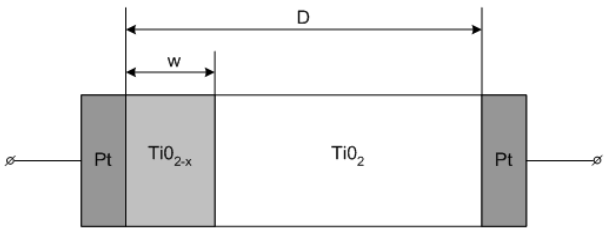

Figure 5. Structure of a memristor element according to the Williams's and Joglekar's models

The state variable of the memristor element *x* is defined as a ratio between *w* and *D* (Strukov et al. 2008):

$$x = \frac{w}{D} \qquad (12)$$

The formulas (12) – (18) are also obtained from (Strukov et al. 2008).

The equivalent resistance of the memristor element R_{eq} is expressed as a resistance of the series connection of the doped and un-doped regions, which maximal resistances for the full length of *D* are $R_{ON} = 100\ \Omega$ and $R_{OFF} = 16\ k\Omega$. The resistances of the regions are proportional to their normalized length:

$$R_{eq} = R_{doped} + R_{undoped} = R_{ON} x + R_{OFF} (1 - x) \qquad (13)$$

The current-voltage relationship of the memristor element is presented with the next formula:

$$u(t) = iR_{eq} = \left[R_{ON} x + R_{OFF} (1 - x) \right] i(t) \qquad (14)$$

The voltage drop over the doped region u_w is expressed with the following formula (15):

$$u_w = iR_{doped} = iR_{ON}x = iR_{ON}\frac{w}{D} \tag{15}$$

The intensity of the electric field in the doped layer is:

$$E = \frac{u_w}{w} = \frac{1}{w}iR_{ON}\frac{w}{D} = \frac{iR_{ON}}{D} \tag{16}$$

The rate of moving of the boundary between the doped and un-doped regions due to the electric field is:

$$v = \frac{dw}{dt} = \frac{d(xD)}{dt} = D\frac{dx}{dt} = \mu E \tag{17}$$

After transforming (17) and using (16) we obtain:

$$\frac{dx}{dt} = \frac{\mu R_{ON}}{D^2}i = ki \tag{18}$$

The relation (18) represents the linear drift model. For expressing the nonlinear ionic drift, formula (18) has to be modified, using the Joglekar's parabolic window function (Joglekar and Wolf 2009):

$$\frac{dx}{dt} = kif(x) = ki\left[1-(2x-1)^{2p}\right] \tag{19}$$

where $p = 1, 2, 3, \ldots, N$.
Using (19) and (14), the basic differential equation of the memristor element is obtained:

$$\frac{1-x}{1-(2x-1)^{2p}}dx = \frac{k}{R_{OFF}}u(t)dt \tag{20}$$

If we choose $p = 1$, then (20) has a closed-form solution:

$$x = x_0 \exp\left[\frac{4k}{R_{OFF}}\int_{t_0}^{t} u d\tau\right] \tag{21}$$

where x_0 is the initial value of the state variable x.

If $p > 1$, then (20) is solvable only in Gaussian hyper-geometric form. In the next section we will solve (20) numerically using transformation from differential equation to difference equation. The Joglekar's window functions for several values of p are presented in Fig. 6. For $p = 1$ it is a parabolic function and the nonlinearity in this case is higher. From Fig. 6 it is obvious that if we increase the exponent p, then the window function obtains a linear region and very steep regions in the left and in the right fields of the function. Theoretically, if we choose the exponent p to be equal to infinity, then the window function will tend to the BCM linearized window function. In the next section, we will use a switch-based algorithm for the window so it would be appropriate for soft- and hard-switching modes.

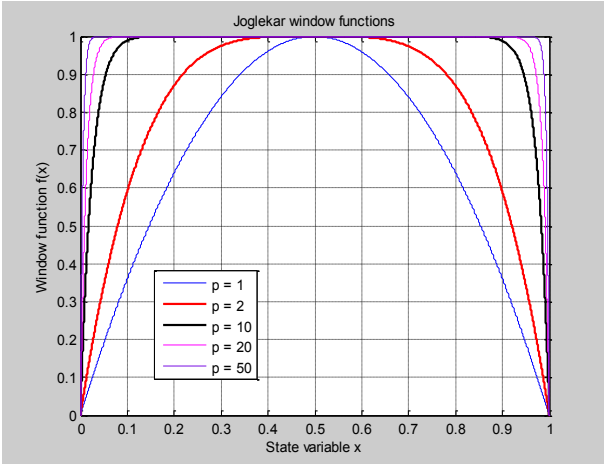

Figure 6. Joglekar's parabolic window functions for different values of the exponent p

PSEUDO-CODE ALGORITHM OF THE MODIFIED JOGLEKAR'S MEMRISTOR MODEL FOR SOFT-SWITCHING AND HARD-SWITCHING MODES

The basic differential equation of the memristor model (9) could be numerically solved. For this purpose we transform (9) in difference equation substituting the differentials of the quantities with finite differences. Follows the pseudo-code used for the computer simulation of the modified Joglekar's model (Joglekar and Wolf 2009, Corinto and Ascoli 2012).

```
Procedure: A modified memristor model
           with nonlinear dopant drift
Given: Voltage source with instantaneous value v(t)
Given: Duration of the simulation: T = t_max – t_min
Given: The number of samples n
Given: The sample step time: deltat = T / n; p = 5;
Given: Memristor parameters: R_on = 100;
  R_off = 16e3; mu = 1e-14; D = 10e-9; x_min = 0.05;
  x_max = 0.9999; k = mu*R_on/(D^2); x_0 = 0.2;
set x = x0  for t = 0
  for t = tmin:deltat:tmax
      psi = int(v(t)dt,-∞,t);
  for n=1,  x_1=x_0;
  for n=2:1:n+1,
    x_1(n)=x_1(n-1)+deltat*u(n-1)*(1/(1-x_1(n-
        -1)))*(k/R_off)*(1-((2*x_1(n-1)-1)^(2*p)));
  if x_1(n)<=xmin & u(n)<=0, x_1(n)=x_min;
  elseif x_1(n)>=x_max & u(n)>=0, x_1(n)=x_max;
  else x_1(n)=x_1(n-1)+deltat*u(n-1)*(1/(1-
  x_1(n-1)))*(k/Roff)*(1-((2*x_1(n-1)-1)^(2*p)));
  end
  end cycle
  return u,i,psi, x, M;
end procedure
```

EXPERIMENTAL RESULTS FROM THE COMPUTER SIMULATION REALIZED IN MATLAB FOR SOFT-SWITCHING AND HARD-SWITCHING MODES

The state-flux relationships of the modified Joglekar's memristor model for several different values of the exponent *p* are given in Fig. 7. It is evident that when the exponent *p* is from 1 to 10, the characteristics really differ from one to another. When *p* has higher values, then the state-flux relations almost coincide to each other. This phenomenon is also valid for the current-voltage relationships which are obtained for the same conditions. The current-voltage relationships of the memristor are illustrated in Fig. 8. Both the state-flux and the current-voltage relationships are obtained for the so called soft-switching mode. For this mode the values of magnitude and frequency of voltage are the same as these used for simulation of Pickett's model.

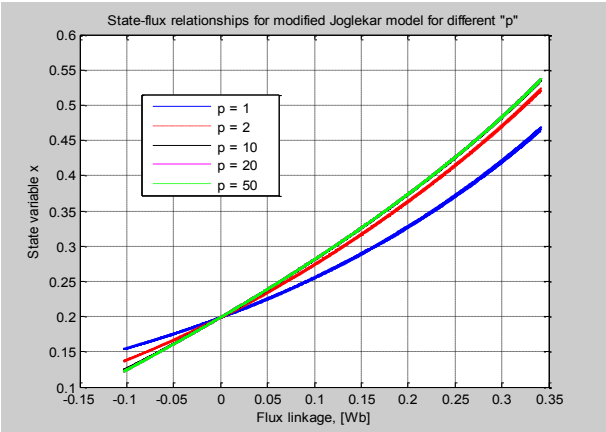

Figure 7. State-flux relationships for the modified Joglekar memristor model for different exponents *p* and soft-switching operation

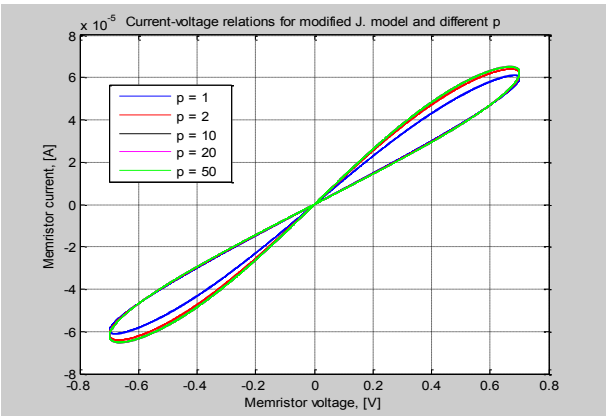

Figure 8. Current-voltage characteristics of the Joglekar modified memristor model for different exponents *p* and soft-switching operation

It is interesting to compare the results obtained for the Pickett's memristor model and the Joglekar's modified model – Fig. 3, Fig. 4 and Fig. 7, Fig. 8. Obviously, these characteristics are similar to each other.

One of the advantages of the new modified Joglekar memristor model is the possibility for adjusting the window function so that the model is able to represent the processes in the memristor structure for low and high intensities of the internal electric field. The next paragraph illustrates the results obtained for high magnitude voltage, for example 3 *V*. The state-flux characteristics are given in Fig. 9. The current-voltage characteristics are presented in Fig. 10, respectively. They are both obtained for the hard-switching mode, using high voltage and low frequency signal.

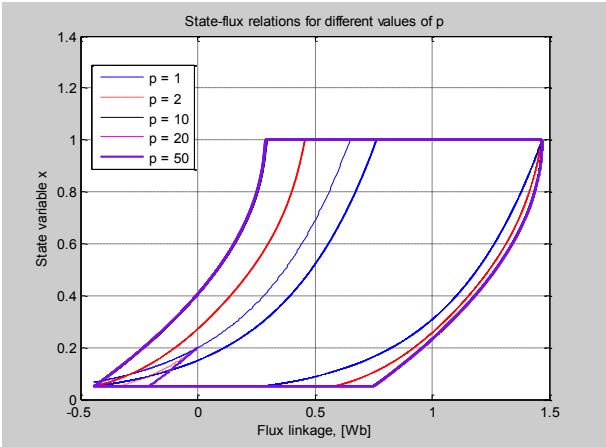

Figure 9. State-flux relationships for the modified Joglekar memristor model for different exponents *p* and hard-switching mode

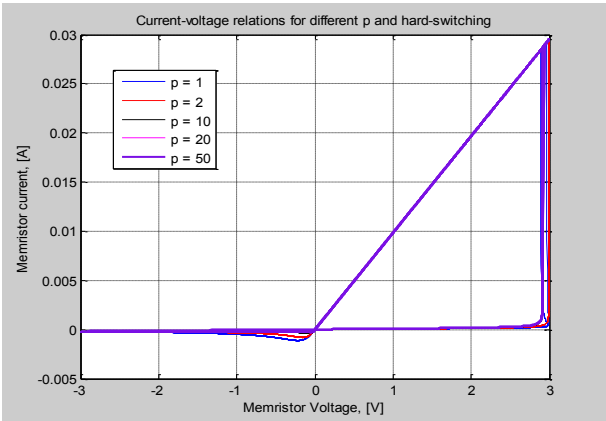

Figure 10. Current-voltage characteristics of the Joglekar modified model for different exponents *p* and hard-switching

After a careful consideration we can conclude that the new memristor model is a good-quality one because it characterizes the processes in memristor structures for low and high intensities of the electric field. The new algorithm is also a tunable model – it is adjusted with varying the exponent *p*. The new model correctly reproduces the boundary conditions for soft-switching and hard-switching modes. In other words, the new pseudo-code presented above illustrates successfully the behavior of memristors based on titanium dioxide. It includes the advantages both of the BCM and Joglekar's memristor models.

CONCLUSION

After comparison of the results obtained by the computer simulations of the Pickett's memristor model and the Joglekar modified model, it is clear that the exponent have to be selected between 2 and 10. If we choose $p = 1$, the model is strongly nonlinear and then this model is appropriate for high intensity electric fields. If we increase the exponent, the window function of the modified Joglekar's model tends to coincide with the BCM window function, which is appropriate for linear ionic drift, or – for low intensity electric fields. Another advantage of the modified Joglekar's model is its ability to model both soft-switching and hard-switching modes. This memristor model is also appropriate for computer simulations unlike Pickett's model, which has many convergence problems and is not very appropriate for simulations. In the end, we can say that the new memristor model is a good one because it represents the processes in memristor structures for low and high voltages. It is also a tunable model and it contains the advantages of the BCM and Joglekar's models. The new model is adjusted by varying of the exponent p, and it represents the behavior of the memristor structure for different modes.

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