REACHABILITY OF FRACTIONAL CONTINUOUS-TIME LINEAR SYSTEMS USING THE CAPUTO-FABRIZIO DERIVATIVE

Tadeusz Kaczorek
Białystok University of Technology
Faculty of Electrical Engineering
Wiejska 45D, 15-351 Białystok
E-mail: kaczorek@isep.pw.edu.pl

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ABSTRACT
The Caputo-Fabrizio definition of the fractional derivative is applied to analysis of the positivity and reachability of continuous-time linear systems. Necessary and sufficient conditions for the reachability of standard and positive fractional continuous-time linear systems are established.

INTRODUCTION
A dynamical system is called positive if its trajectory starting from any nonnegative initial condition state remains forever in the positive orthant for all nonnegative inputs. An overview of the art in positive system theory is given in the monographs (Farina and Rinaldi 2000; Kaczorek 2001) and in the papers (Kaczorek 1997, 1998, 2011b, 2014a, 2014b, 2015b). Models having positive behavior can be found in engineering, economics, social sciences, biology and medicine, etc.

The positive standard and descriptor systems and their stability have been analyzed in (Kaczorek 1997, 1998, 2001, 2011b, 2014b, 2015b). The positive linear systems with different fractional orders have been addressed in (Kaczorek 2011b, 2012) and the descriptor discrete-time linear systems in (Kaczorek 1998). Descriptor positive discrete-time and continuous-time nonlinear systems have been analyzed in (Kaczorek 2014a) and the positivity and linearization of nonlinear discrete-time systems by state-feedbacks in (Kaczorek 2014b). New stability tests of positive standard and fractional linear systems have been investigated in (Kaczorek 2011a). The stability and robust stabilization of discrete-time switched systems have been analyzed in (Zhang, Xie, Zhang and Wang 2014; Zhang, Han, Wu and Hung 2014). Minimum energy control of 2D systems in Hilbert spaces has been analyzed in (Klakam 1983). Controllability of dynamical systems has been investigated in (Kalman 1960; Klakam 1991, 1997, 1998).

Recently a new definition of the fractional derivative without singular kernel has been proposed in (Caputo and Fabrizio 2015; Losada and Nieto 2015).

In this paper the Caputo-Fabrizio definition of the fractional derivative will be applied to analysis of the reachability of the standard and positive linear systems. The paper is organized as follows. In section 2 necessary and sufficient conditions for the reachability of fractional standard continuous-time linear systems are established. Necessary and sufficient conditions for the positivity of the fractional systems and sufficient conditions for the reachability of the positive systems are proposed in section 3. Concluding remarks are given in section 4.

The following notation will be used: \( \mathbb{R} \) - the set of real numbers, \( \mathbb{R}^{n \times m}_{\text{non}} \) - the set of \( n \times m \) real matrices, \( \mathbb{R}^{n \times m}_{\text{pos}} \) - the set of \( n \times m \) matrices with nonnegative entries and \( \mathbb{R}^{n \times n}_{\text{pos}}, M_{n} \) - the set of \( n \times n \) Metzler matrices, \( I_{n} \) - the \( n \times n \) identity matrix.

REACHABILITY OF STANDARD FRACTIONAL SYSTEMS
The Caputo-Fabrizio definition of fractional derivative of order \( \alpha \) of the function \( f(t) \) for \( 0 < \alpha < 1 \) has the form (Caputo and Fabrizio 2015; Losada and Nieto 2015)

\[
\frac{d^{\alpha} f(t)}{dt^{\alpha}} = \frac{1}{\Gamma(1-\alpha)} \int_{0}^{t} \exp\left(-\frac{x}{1-\alpha}(t-\tau)\right) f'(\tau) d\tau, \tag{1}
\]

\[
\frac{df(t)}{d\tau}, \ t \geq 0.
\]

Consider the fractional differential state equations

\[
\frac{d^{\alpha} x(t)}{dt^{\alpha}} = Ax(t) + Bu(t), \ 0 < \alpha < 1, \tag{2a}
\]

\[
y(t) = Cx(t) + Du(t), \tag{2b}
\]

where \( x(t) \in \mathbb{R}^{n}, u(t) \in \mathbb{R}^{m}, y(t) \in \mathbb{R}^{p} \) are the state, input and output vectors and \( A \in \mathbb{R}^{n \times n}_{\text{non}}, B \in \mathbb{R}^{n \times m}_{\text{non}}, C \in \mathbb{R}^{p \times n}_{\text{non}}, D \in \mathbb{R}^{p \times m}_{\text{non}} \).

Theorem 1. The solution \( x(t) \) of the equation (2a) for a given initial condition \( x(0) = x_{0} \) and input \( u(t) \) has the form
The standard fractional system (2) is reachable in time $t \in [0,t_f]$ if and only if the fractional system

$$\frac{d\hat{x}(t)}{dt^\alpha} = A\hat{x}(t) + \hat{B}u(t)$$

is reachable in time $t \in [0,t_f]$. The solution of the differential equation (6) for $u_0 = u(0) = 0$ has the form

$$u(t) = \int_0^t e^{-\beta(t-\tau)}\hat{u}(\tau)d\tau.$$  \hspace{1cm} (8)

To show that the input

$$\hat{u}(t) = \hat{B}^T e^{\hat{A}(t-t)} \hat{R}_f^{-1} x_f, \quad t \in [0,t_f]$$

steers the state from $x_0 = 0$ to $x_f$ in time $t \in [0,t_f]$ we substitute (9) into (7) and we obtain

$$x(t_f) = \int_0^{t_f} e^{\hat{A}(t_f-t)} \hat{R}_f^{-1} e^{\hat{A}t} x_f dt.$$  \hspace{1cm} (10)

Substituting (9) into (10) we obtain (5). \hspace{1cm} \Box

From Theorem 1 and its proof follows the corollary.

**Corollary 1.** The fractional system (2) is reachable in time $t \in [0,t_f]$ if and only if the fractional system

$$\frac{d^\alpha x(t)}{dt^\alpha} = A x(t) + Bu(t)$$

is reachable in time $t \in [0,t_f]$. The input $\hat{u}(t)$ steers the state $x(t)$ from $x_0 = 0$ to $x_f$ in time $t \in [0,t_f]$ of the system (11) if and only if the input (8) steers the state from $x_0 = 0$ to $x_f$ in time $t \in [0,t_f]$ of the system (2a).

**Example 1.** Consider the fractional system described by the equation (2a) with $\alpha = 0.5$, zero initial condition $x_0 = 0$, $u_0 = 0$ and

$$A = \begin{bmatrix} -1 & a \\ 0 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} 2 \\ 2 \end{bmatrix},$$

$a$ - parameter. (12)

Compute the input $u(t)$ which steers the system from $x_0 = 0$ to $x_f = [1 \quad 1]^T$ ($T$ denotes transpose) in time $t \in [0,1]$. Using (3b) and (12) we obtain

$$\hat{A} = \alpha A - (1-\alpha)A^{-1} A$$

$$\hat{B} = [I_n - (1-\alpha)A]^{-1} (1-\alpha)B, \quad \beta = \frac{\alpha}{1-\alpha}.$$  \hspace{1cm} (13a)

$$\hat{x}_0 = [I_n - (1-\alpha)A]^{-1} x_0, \quad e^{\hat{A}t} = \mathcal{L}^{-1}([I_n, s - \hat{A}]^{-1}).$$

$$\hat{u}(\tau) = \frac{du(\tau)}{d\tau}, \quad u(0) = u_0.$$  \hspace{1cm} (13b)
\[ \dot{B} = [I_2 - (1 - \alpha)A]^{-1}(1 - \alpha)B \]
\[ = \begin{bmatrix} 1.5 & -0.5a \\ 0 & 2 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{1}{6} a + 4 \]  
(13b)

Taking into account that the eigenvalues of the matrix (13a) are \( \lambda_1 = \frac{1}{2}, \lambda_2 = \frac{1}{3} \) and using the Sylvester formula we obtain
\[ e^{\lambda t} = \frac{\hat{\lambda}_2 - I_2}{\hat{\lambda}_1 - \hat{\lambda}_2} e^{\hat{\lambda}_2 t} + \frac{\hat{\lambda}_1 - I_2}{\hat{\lambda}_2 - \hat{\lambda}_1} e^{\hat{\lambda}_1 t} \]
\[ = \begin{bmatrix} 0 -a \\ 0 1 \end{bmatrix} e^{-t/2} + \begin{bmatrix} 1/a \\ 0 0 \end{bmatrix} e^{-t/3} \]
\[ = \begin{bmatrix} e^{-t/3} + \frac{1}{a} (e^{-t/3} - e^{-t/2}) \\ 0 \end{bmatrix} \]  
(14)

Using (4) for \( t_f = 1 \) and (14), (13b) we obtain
\[ R_f = \int_0^{t_f} e^{\hat{\lambda}_i t} \hat{B} B^T e^{\hat{\lambda}_i t} dt = \int_0^{t_f} (e^{\hat{\lambda}_i t} \hat{B})(e^{\hat{\lambda}_i t} \hat{B})^T dt \]
\[ = \int_0^{t_f} \begin{bmatrix} 2 \frac{2}{3} e^{-t/3} - \frac{1}{2} e^{-t/2} \\ \frac{1}{2} e^{-t/2} \end{bmatrix} + \frac{2}{3} e^{-t/3} \begin{bmatrix} 2 e^{-t/2} \\ 1 \end{bmatrix} \]
\[ \times \begin{bmatrix} 2 e^{-t/3} - \frac{1}{2} e^{-t/2} \\ \frac{1}{2} e^{-t/2} \end{bmatrix} \begin{bmatrix} 2 \frac{2}{3} e^{-t/3} - \frac{1}{2} e^{-t/2} \\ \frac{1}{2} e^{-t/2} \end{bmatrix} dt \]
\[ = \int_0^{t_f} \begin{bmatrix} 2 \frac{2}{3} e^{-t/3} - \frac{1}{2} e^{-t/2} \\ \frac{1}{2} e^{-t/2} \end{bmatrix} \begin{bmatrix} 2 \frac{2}{3} e^{-t/3} - \frac{1}{2} e^{-t/2} \\ \frac{1}{2} e^{-t/2} \end{bmatrix} dt \]
\[ = \begin{bmatrix} 2 \frac{2}{3} e^{-t/3} - \frac{1}{2} e^{-t/2} \\ \frac{1}{2} e^{-t/2} \end{bmatrix} \begin{bmatrix} 2 \frac{2}{3} e^{-t/3} - \frac{1}{2} e^{-t/2} \\ \frac{1}{2} e^{-t/2} \end{bmatrix} dt \]
\[ = \begin{bmatrix} 2 \frac{2}{3} e^{-t/3} - \frac{1}{2} e^{-t/2} \\ \frac{1}{2} e^{-t/2} \end{bmatrix} \begin{bmatrix} 2 \frac{2}{3} e^{-t/3} - \frac{1}{2} e^{-t/2} \\ \frac{1}{2} e^{-t/2} \end{bmatrix} dt \]
\[ = \begin{bmatrix} 0.0301a^2 + 0.1965a + 0.3244 \\ 0.0681a + 0.2262 \\ 0.0681a + 0.2262 \\ 0.158 \end{bmatrix} \]  
(15)

The matrix (15) is nonsingular since
\[ \det R_f = 0.0001a^2 + 0.0002a + 0.0001 \neq 0 \]  
for \( a \neq -1 \).

The input steering the system from \( x_0 = 0 \) to \( x_f = [1 \ 1]^T \) in time \( t \in [0, 1] \) is given by
\[ u(t) = \int_0^t e^{-\beta(t-t_f)} \hat{B}^T e^{\hat{\lambda}_f (t-t_f)} d\tau R_f^{-1} x_f \]
\[ = e^{-t} \int_0^t e^{-\tau} \hat{B}^T e^{\hat{\lambda}_f \tau} d\tau R_f^{-1} x_f \]
\[ = e^{-t} \int_0^t e^{-\tau} \begin{bmatrix} 0.11a & 0.6065 \\ 0.0301a^2 + 0.1965a + 0.3244 & 0.0681a + 0.2262 \\ 0.0681a + 0.2262 & 0.158 \end{bmatrix} \begin{bmatrix} 0.7165 \\ 0 \end{bmatrix} \\ e^{-\tau} \begin{bmatrix} 0 \\ 0.11a \\ 0.0301a^2 + 0.1965a + 0.3244 \\ 0.0681a + 0.2262 \\ 0.0681a + 0.2262 \\ 0.158 \end{bmatrix} \]  
(16)

For example for \( a = 1 \) we obtain
\[ u(t) = e^{-t} \int_0^t e^{-\tau} \left( \begin{bmatrix} 0.4777a + 0.4777 \end{bmatrix} - 0.333e^{3\tau} \right) \]
\[ = -244.1644e^{3\tau} + 198.6615e^{3\tau} + 45.2029e^{-\tau}. \]  
(17)
REACHABILITY OF POSITIVE FRACTIONAL SYSTEMS

Consider the fractional system (2).

**Definition 2.** The fractional system (2) is called (internally) positive if the state vector \( x(t) \in \mathbb{R}^n_+ \) and the output vector \( y(t) \in \mathbb{R}^p_+ \), \( t \geq 0 \) for all initial conditions \( x_0 \in \mathbb{R}^n_+ \) and all inputs \( u(t) \in \mathbb{R}^m_+ \), \( \dot{u}(t) \in \mathbb{R}^m_+ \), \( t \geq 0 \).

**Definition 3.** A real matrix \( A = [a_{ij}] \in \mathbb{R}^{m \times n}_+ \) is called a Metzler matrix if its off-diagonal entries are nonnegative, i.e., \( a_{ij} \geq 0 \) for \( i \neq j \), \( i, j = 1, \ldots, n \).

**Lemma 1.** Let \( \hat{A} \in M_n \) and \( 0 < \alpha < 1 \). Then

\[
e^{\hat{A}t} \in \mathbb{R}^{m \times n}_+ \quad \text{for} \quad t \geq 0.
\]

**Proof.** The proof is similar to the one given in (Kaczorek 2001).

**Theorem 3.** The fractional system (2) is positive if and only if

\[
\hat{A} \in M_n, \quad \hat{B} \in \mathbb{R}^{m \times n}_+, \quad C \in \mathbb{R}^{p \times n}_+, \quad D \in \mathbb{R}^{p \times m}_+.
\]

**Proof.** Sufficiently, if \( \hat{A} \in M_n \) and \( \hat{B} \in \mathbb{R}^{m \times n}_+ \) then from (3) we have \( x(t) \in \mathbb{R}^n_+ \), \( t \geq 0 \) since by Lemma 1 \( e^{\hat{A}t} \in \mathbb{R}^{m \times n}_+ \) and \( x_0 \in \mathbb{R}^n_+ \), \( u(t) \in \mathbb{R}^m_+ \), \( \dot{u}(t) \in \mathbb{R}^m_+ \), \( t \geq 0 \).

**Necessity.** Let \( u(t) = 0 \), \( t \geq 0 \) and \( x_0 = e_i \) (\( i \)-th column of the identity matrix \( I_n \)). The trajectory does not leave the orthant \( \mathbb{R}^n_+ \) only if \( \alpha^f \hat{A} x(0) = \hat{A} e_i \geq 0 \) what implies \( \hat{a}_{ij} \geq 0 \) for \( i \neq j \), \( i, j = 1, \ldots, n \) and \( \hat{A} \in M_n \). If \( x_0 = 0 \) then \( \alpha^f \hat{A} x(0) = Bu(0) \geq 0 \) and this implies \( B \in \mathbb{R}^{m \times n}_+ \) since \( u(0) \in \mathbb{R}^m_+ \) is arbitrary. From (2b) for \( u(t) = 0 \), \( t \geq 0 \) we have \( y(0) = Cx(0) \) and \( C \in \mathbb{R}^{p \times n}_+ \) since \( x(0) = x_0 \in \mathbb{R}^n_+ \) is arbitrary. Assuming \( x_0 = 0 \) from (2b) we have \( y(0) = Du(0) \) and \( D \in \mathbb{R}^{p \times m}_+ \) since \( u(0) \in \mathbb{R}^m_+ \) is arbitrary. □

**Lemma 2.** If \( \lambda_k \), \( k = 1, \ldots, n \) are the eigenvalues of the matrix \( A \) then the eigenvalues of the matrix \( \hat{A} = \alpha[I_n - (1 - \alpha)A]^{-1}A \) are given by

\[
\hat{\lambda}_k = \alpha[1-(1-\alpha)\lambda_k]^{-1} \lambda_k.
\]

**Proof.** It is well-known (Gantmacher 1959) that if \( f(\lambda_k) \) is well-defined on the spectrum \( \lambda_k \), \( k = 1, \ldots, n \) of the matrix \( A \) then the eigenvalues of the matrix \( f(A) \) are given by \( f(\lambda_k) \), \( k = 1, \ldots, n \). In this case \( f(A) = \alpha[I_n - (1 - \alpha)A]^{-1}A \). □

**Lemma 3.** The matrix \( \overline{A} = (1 - \alpha)A \in \mathbb{R}^{m \times n}_+ \) for \( 0 < \alpha < 1 \) is asymptotically stable if and only if the matrix \( A \) is asymptotically stable.

**Proof.** The eigenvalues \( \overline{\lambda}_k \), \( k = 1, \ldots, n \) of the matrix \( \overline{A} \) are related with the eigenvalues \( \lambda_k \), \( k = 1, \ldots, n \) of the matrix \( A \) by

\[
\overline{\lambda}_k = (1 - \alpha)\lambda_k, \quad k = 1, \ldots, n.
\]

since the characteristic polynomials of the matrices are related by the equality

\[
det[I_n \overline{\lambda}_k - \overline{A}] = det[I_n \lambda_k - (1 - \alpha)A] = (1 - \alpha)^n det[I_n \overline{\lambda}_k - A] = (1 - \alpha)^n det[I_n \lambda_k - A].
\]

Therefore, from (21) it follows that \( \Re \overline{\lambda}_k < 0 \), \( k = 1, \ldots, n \) if and only if \( \Re \lambda_k < 0 \), \( k = 1, \ldots, n \). □

**Lemma 4.** The matrix

\[
\hat{A} = \alpha[I_n - (1 - \alpha)A]^{-1}A \in M_n
\]

is asymptotically stable if and only if the eigenvalues

\[
\hat{\lambda}_k = -(1-\alpha) \lambda_k + j\beta_k, \quad k = 1, \ldots, n
\]

of the matrix \( A \) satisfy the condition...
\[ 1 + (1 - \alpha) \alpha_k \alpha_k + (1 - \alpha) \beta_k^2 = n(k) > 0. \]

**Proof.** From (20) for \( \hat{\lambda}_k = \hat{-\alpha}_k + j \hat{\beta}_k \) and \( \lambda_k = -\alpha_k + j \beta_k \), \( k = 1, \ldots, n \) we have

\[
\hat{\lambda}_k = -\hat{-\alpha}_k + j \hat{\beta}_k = \alpha (1 - \alpha) \lambda_k^{-1} \lambda_k
\]

\[
= \alpha (1 - \alpha)(-\alpha_k + j \beta_k)^{-1} (-\alpha_k + j \beta_k)
\]

\[
= \alpha 1 + (1 - \alpha) \alpha_k + j (1 - \alpha) \beta_k
\]

\[
[1 + (1 - \alpha) \alpha_k]^2 + [1 - (1 - \alpha) \beta_k]^2
\]

\[
\alpha \left( \frac{1 + (1 - \alpha) \alpha_k}{1 + (1 - \alpha) \alpha_k^2 + [1 - (1 - \alpha) \beta_k]^2} \right)
\]

\[
\hat{\alpha}_k = \alpha \left( \frac{1 + (1 - \alpha) \alpha_k}{1 + (1 - \alpha) \alpha_k^2 + [1 - (1 - \alpha) \beta_k]^2} \right) \]

\[
= \alpha \frac{n(k)}{d(k)}, \quad k = 1, \ldots, n.
\]

From (25) it follows that \( \hat{\alpha}_k > 0, \quad k = 1, \ldots, n \) if and only if \( \alpha(n(k)) > 0, \quad k = 1, \ldots, n \). □

**Lemma 5.** The matrices

\[
\hat{A} = \alpha [I_n - (1 - \alpha) A]^{-1} A \in M_n,
\]

\[
\hat{B} = [I_n - (1 - \alpha) A]^{-1} (1 - \alpha) B \in \mathbb{R}^{n \times p}_+.
\]

if \( A \in M_n \) is asymptotically stable and \( B \in \mathbb{R}^{n \times p}_+ \).

**Proof.** The matrix \( [I_n - (1 - \alpha) A]^{-1} \in \mathbb{R}^{nP}_+ \) if the matrix \( A \in M_n \) is asymptotically stable (Kaczorek 2001). Therefore, by Lemma 3 and \( (1 - \alpha) B \in \mathbb{R}^{n \times p}_+ \) for \( 0 < \alpha < 1 \) holds if \( A \in M_n \) is asymptotically stable. □

From Lemma 4 and Theorem 3 we have the following.

**Theorem 4.** The fractional system (2) is positive if \( A \in M_n \) is asymptotically stable and \( B \in \mathbb{R}^{n \times p}_+ \).

**Definition 4.** A state \( x_f \in \mathbb{R}^n_+ \) of the positive system (2) is called reachable in time \( t \in [0, t_f] \) if there exists an input \( u(t) \in \mathbb{R}^m_+ \) for \( t \in [0, t_f] \) which steers the state of the system from zero initial condition \( x_0 = 0 \) to the final state \( x_f \in \mathbb{R}^n_+ \). If every state \( x_f \in \mathbb{R}^n_+ \) is reachable in time \( t \in [0, t_f] \) then the system is called reachable in time \( t \in [0, t_f] \). The positive system (2) is called reachable if for every \( x_f \in \mathbb{R}^n_+ \) there exists \( t_f \) and an input \( u(t) \in \mathbb{R}^m_+ \) for \( t \in [0, t_f] \) which steers the state of the system from \( x_0 = 0 \) to \( x_f \).

**Definition 5.** A matrix \( A \in \mathbb{R}^{m \times n}_+ \) is called monomial if in each row and in each column only one entry is positive and the remaining entries are zero.

**Theorem 5.** The positive fractional system (2) is reachable in time \( t \in [0, t_f] \) if the matrix

\[
R_f = R(t_f) = \int_0^{t_f} e^{-\beta t} \hat{A} \hat{B} e^{\hat{A} T} \hat{C} dt
\]

is monomial.

The input which steers the state of the system from \( x_0 = 0 \) to \( x_f \) is given by

\[
u(t) = \int_0^t e^{t \beta} \hat{B} e^{A T} \hat{C} \hat{A}^{-1} x_f dt
\]

**Proof.** It is well-known (Kaczorek 2001) that \( R_f \in \mathbb{R}^{m \times m}_+ \) if and only if the matrix \( R_f \in \mathbb{R}^{m \times m}_+ \) is monomial. In a similar way as in proof of Theorem 1 it can be shown that the input (28) steers the state of positive system from \( x_0 = 0 \) to \( x_f \in \mathbb{R}^n_+ \) in time \( t \in [0, t_f] \). From (28) it follows that \( u(t) \in \mathbb{R}^m_+ \) since \( e^{t \beta} > 0 \) for \( \beta = \frac{\alpha}{1 - \alpha} > 0 \), \( 0 < \alpha < 1 \), \( \hat{B} e^{A T} \hat{C} \in \mathbb{R}^{m \times n}_+ \) and \( \hat{A}^{-1} x_f \in \mathbb{R}^n_+ \). □

**Example 2.** (Continuation of Example 1) Note that the matrix \( R_f \) given by (15) is monomial only for \( a = -3.3216 \). Therefore, we cannot say anything about the reachability of the positive system with (12) in time \( t \in [0, 1] \) for \( a \geq 0 \).

**CONCLUDING REMARKS**

The Caputo-Fabrizio definition of the fractional derivative has been applied to analysis of the positivity and reachability of continuous-time linear systems. Necessary and sufficient conditions for the reachability of standard continuous-time linear systems have been established (Theorem 1). Necessary and sufficient conditions for the positivity of the fractional linear systems have been given (Theorems 3 and 4). Sufficient conditions for the reachability of the fractional positive linear systems have been established (Theorem 5). The considerations are illustrated by numerical examples of standard and positive fractional linear systems.
The considerations can be extended to continuous-discrete linear systems.

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AUTHOR BIOGRAPHIES

TADEUSZ KACZOREK received the M.Sc., Ph.D. and D.Sc. degrees in electrical engineering from the Warsaw University of Technology in 1956, 1962 and 1964, respectively. In the years 1968–69 he was the dean of the Electrical Engineering Faculty, and in the period of 1970–73 he was a deputy rector of the Warsaw University of Technology. In 1971 he became a professor and in 1974 a full professor at the same university. Since 2003 he has been a professor at the Białystok University of Technology. In 1986 he was elected a corresponding member and in 1996 a full member of the Polish Academy of Sciences. In the years 1988–1991 he was the director of the Research Centre of the Polish Academy of Sciences in Rome. In 2004 he was elected an honorary member of the Hungarian Academy of Sciences. He was granted honorary doctorates by 13 universities. His research interests cover systems theory, especially singular multidimensional systems, positive multidimensional systems, singular positive 1D and 2D systems, as well as positive fractional 1D and 2D systems. He initiated research in the field of singular 2D, positive 2D and positive fractional linear systems. He published 28 books (8 in English) and over 1000 scientific papers. He also supervised 69 Ph.D. theses. He is the editor-in-chief of the Bulletin of the Polish Academy of Sciences: Technical Sciences and a member of editorial boards of ten international journals.