CLASSICAL AND NOVEL RISK MEASURES FOR A STOCK INDEX ON A DEVELOPED MARKET

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Abstract

In the present article we conduct an inquiry into several different risk measures, illustrating their advantages and disadvantages, regulatory aspects and apply them on a stock index on a developed market: the DAX index. Specifically we are talking about Value at Risk (VaR), which is now considered a classical measure, its improved version, the Expected Shortfall (ES) and the very novel Entropic Value at Risk (EvaR). The applied computation methods are historic simulation, Monte Carlo simulation and the resampling method, which are all non-parametric methods, yielding robust results. The obtained values are put into the context of the relevant literature, and pertinent conclusions are formulated, especially regarding regulatory applications.

Keywords: Value At Risk, Expected Shortfall, Entropic Value At Risk, simulation, resampling

JEL classification: C14, C15, G11

1. Literature review

In the present chapter we introduce three risk measures: VaR – Value at Risk, ES – Expected Shortfall or otherwise called CVaR – Conditional Value at Risk, and EVaR – Entropic Value at Risk. In the classification given in Albrecht (2003), these measures are all part of the category of absolute risk measures. We examine to what extent these measures can be considered coherent, at the same time we analyse how much these risk measures differ in terms of figures, and highlight which ones were the most realistic. Finally, we examine how the more novel risk measures such as ES and EVaR brought new insights and ameliorated certain aspects of classical VaR.

Because of length considerations we don't have the space to define VaR and ES here, we consider them widely known, we shall only pertain to entropic VaR.

1.1.EVaR (Entropic Value at Risk)

Definition of EVaR (based on Ahmadi-Javid, 2012): Let (Ω, Ƒ, P) be a probability space with Ω a set of all simple events, Ƒ a σ-algebra of subsets of Ω and P a probability measure on Ƒ. Let X be a random variable and $\boldsymbol{L_{M^+}}$ be the set of all Borel measurable functions X: Ω $\rightarrow \boldsymbol{R}$ whose moment-generating function $\boldsymbol{M_X}$(z) exists for all $\boldsymbol{z \geq 0}$. The entropic value-at-risk (EVaR) of $\boldsymbol{X} \in \boldsymbol{L_{M^+}}$ with confidence level (1 – α) is defined as follows:

$$EVaR_{1-\alpha}(X) \coloneqq \inf\{z^{-1}\ln(M_X(\mathrm{z}) / \alpha)\} \qquad \forall z > 0 \tag{1}$$

In finance, the random variable $X \in \boldsymbol{L_{M^+}}$, in the above equation, is used to model the *losses* of a portfolio or stock index. Consider the Chernoff inequality (Chernoff, H. (1981))

$$P(X \geq a) \leq e^{-za} M_X(\mathrm{z}) \qquad \forall z > 0 \tag{2}$$

Solving the equation $e^{-za}M_X(\mathrm{z}) = \alpha$ for a, results in $a_X(\alpha, z) = z^{-1}\ln(M_X(\mathrm{z}) / \alpha)$. By considering the equation (1), we see that $EVaR_{1-\alpha}(X) = \inf\{a_X(\alpha, z)\}$, $\forall z > 0$ which shows the relationship between the EVaR and the Chernoff inequality. Here $a_X(\mathbf{1}, z)$ is the *entropic risk measure* (the term used in finance) or *exponential premium* (the term used in insurance).

In case of the normal distribution ($X \sim N(\mu, \sigma^2)$), this reduces to the closed form formula:

$$EVAR_{1-\alpha}(X) = \mu + \sqrt{-2\ln\alpha}\,\sigma \tag{3}$$

In case of the uniform distribution, $X \sim U(a, b)$:

$$EVAR_{1-\alpha}(X) = inf\left\{t\ln\left(t\frac{e^{t^{-1}b} - e^{t^{-1}a}}{b-a}\right) - tln\alpha\right\}$$

The next chart shows the three types of value at risk in case of the standard normal distribution.

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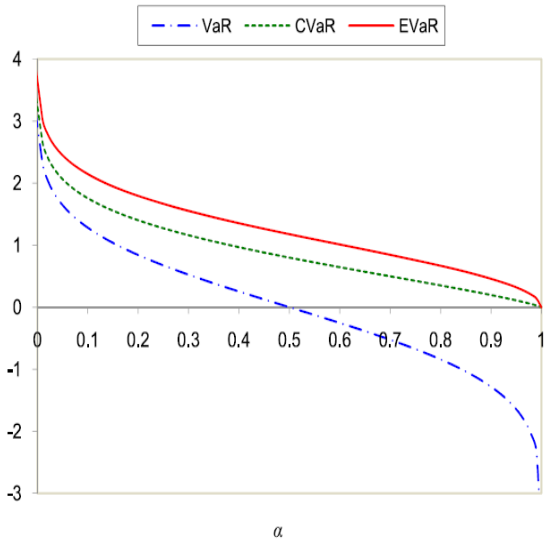
Chart 1: VaR, CVaR and EVaR in case of the standard normal distribution



Source: Ahamdi-Javid A., 2012

Comparing the three types of VaR, it can be seen that EVaR is more conservative (risk averse) than CVaR and VaR. This means that EVaR will prescribe higher capital buffers for financial and insurance institutions (this makes EVaR less attractive for certain institutions). Having a closed form formula in case of certain distributions, EVaR is more computationally tractable than CVaR (which makes it more useful for stochastic optimization problems).

The main advantage of simple VaR is that it performs a full characterization of the distribution returns, leading also to improved performance. The main disadvantage of it is that in reality the market returns are not normally distributed. The main advantage of ES is that it captures the average losses exceeding VaR, the main disadvantage being the fact that it is not easily computationally tractable.

1.2. Review of some empirical results about value at risk

We report the following interesting results in the literature regarding entropic VaR:

Rudloff et al (2008) consider the optimal selection of portfolios for utility maximizing investors under both budget and risk constraints. The risk is measured in terms of entropic risk. They find that even though the entropic risk (ER)-optimal portfolio and the pure stock portfolio coincide w.r.t. entropic risk (since the risk bound was chosen in this way), the ER-optimal portfolio clearly outperforms the stock portfolio w.r.t. expected utility. Long and Qi (2014) study the discrete optimization problem by optimizing the Entropic Value-at-Risk, and propose an efficient approximation algorithm to resolve the problem via solving a sequence of nominal problems. Zheng and Chen (2012) propose a new coherent risk measure called iso-entropic risk measure. Comparing with several current important risk measures, it turns out that this new risk measure has advantages over the others. It is coherent, it is a smooth measure, and it is a more prudent risk measure than CVaR.

Dargiri M. N. et al (2013) compared VaR and CVaR in Malaysian sectors and based on the KLCI (Kuala Lumpur Composite Index) benchmark stock index. They used daily data covering the 2002-2012 period. They calculated VaR with the parametric, delta-normal, ARCH (Autoregressive Conditional Heteroskedasticity) -based, historic and Monte Carlo simulation methods and CVaR with delta normal and historical simulation. The study analysed eight sectors and according to the VaR calculated with all the methods listed above, the technology sector proved to be the most risky, in contrast with the construction sector (which was the most risky according to CVaR). Using a confidence level of 95% and a one day holding period, VaR reached the biggest values in case of the Monte Carlo simulation, and CVaR proved to be more conservative than VaR, having bigger values for all sectors and indices.

Čorkalo, S. (2011) used the variance-covariance method, historic and Monte Carlo simulation with bootstrapping for calculating VaR. Bootstrapping is an alternative to random number generation which relies on random sampling with replacement. The article studied 5 stocks from the Zagreb Stock Exchange and data between 2008-2010: the biggest values were obtained in the case of historic simulation, followed closely by the Monte Carlo with bootstrapping method.

Iqbal J et al (2010) used parametric (EQWMA[1], EWMA[2] and ARCH) and non-parametric (historic simulation and bootstrapping) methods for calculating VaR for the KSE100[3] index. They used daily log-returns between June 1992 and June 2008. At the 95% confidence level, the VaR calculated with historical simulation was 2.524%, whereas the VaR obtained with bootstrapping was 2.547%. Similarly, at 99% confidence level the VaR obtained with bootstrapping was bigger than the one obtained with historic simulation.

Dutta, D. & Bhattacharya, B. (2008) used historic simulation and bootstrap historic simulation for VaR in the case of the S&P CNX Nifty Indian stock index, using data between April 2000 and march 2007 at a confidence level of 95%. The authors find a bigger VaR in case of the bootstrap method and at the same time favour this method compared to the simple historic simulation based on its small-sample behaviour.

[1] Equally Weighted Moving Average
[2] Exponentially Weighted Moving Average
[3] Karachi Stock Index, Karachi, Pakistan

2. Data and methodology

In this study we calculate for the German DAX stock index the three risk measures that we presented (VaR, CVaR and EVaR) applying three methods: historical simulation, Monte Carlo simulation and resampling. We are aiming at comparing the values obtained and the methods.

The DAX index comprises the 30 biggest (in terms of capitalization and liquidity) German stocks on the Frankfurt Stock Exchange. The DAX, introduced in 1987, is a total return index, i.e. it reflects the dividends paid out by the listed companies, at the same time being well-diversified (ranging from vehicle to chemical industries, banking etc).

We used daily index quotations from January 2 2009 to 17 April 2014[4], calculating their logarithmic return for 1357 days. Apart from the descriptive statistics we also tested the normality of the distribution of returns. The conducted normality tests were: Jarque-Bera, Doornik-Hansen, Shapiro-Wilk and Lilliefors. At a 95% confidence level all tests were significant (they all had low "p-levels", thus the null hypothesis of normality can be safely rejected (this was already hinted by the skewness and kurtosis of the distribution).

2.1. The applied methodology

Historical simulation

Historical simulation, being one of the simplest methods for calculating VaR, is obtained by sorting ascendingly the returns of the historical period, and then taking the quantile for the given confidence level. On the other hand, CVaR (ES) can be calculated by sorting ascendingly the returns, and then taking the average of the returns exceeding VaR at the given confidence level. We calculated VaR in Microsoft Excel and ES in the "R" statistical software. We fed the data into the vector of returns, saved the length of the vector in variable „N", and the 5% quantile into variable M. We performed iterative operations to sort descending the vector of returns. In another iteration ("for" cycle) we saved these values into a vector containing averages and calculated the average of the average vector. This gave us the ES at 95% confidence level.

An advantage of this method is that it doesn't necessitate the normality of the distribution of returns. A disadvantage is that it is based on historical data, hence it assumes that the past repeats itself.

Monte Carlo simulation

The method consists of the following steps: first we generate a big amount of random numbers (r_t), then we generate returns out of them, finally arriving at the index prices with the following formula:

$$P_t = P_{t-1}\exp(r)$$

The next step is to sort descending the index prices, into different bins, construct their histogram, then calculate the middle of their intervals, which we multiply by their probability. Then we calculate the price for a given confidence level with the NORMDIST() function, then we sum up the negative values beginning from this value. This will give the value of VaR.

The advantage of this method is that it is also suitable when we want to calculate VaR for a longer period, and in this case the historical simulation would be too volatile.

Resampling (bootstrap historical simulation) method[5]

The method's main idea is that using historical data we can generate thousands of samples, reusing the historical data assuming the repetitivity of data. We once again used the "R" statistical software: In case of VaR we saved the returns into a vector, saved its length and the number of samples into other two variables. Next we defined an iteration up until the number of samples, and determined the value of VaR for each sample, similarly to the historic simulation, i.e. the quantile at 95% confidence level, from the descending sorted returns. In the case of ES the commands were slightly modified. Here we saved the number of data in an „M" variable, and because ES is a sort of an "average of VaR's", we had to apply a double iteration (a "for" cycle inside another) to the descending sorted returns and take the average of the vector of data for each sample.

EVaR was calculated using formula (3) from Ahmadi-Javid A. (2012) so we only needed the mean and the standard deviation of returns for each sample which we fed into the formula. It can be seen that this resampling method is a hybrid between historical and Monte Carlo simulation. Its advantage is that it doesn't assume anything about the distribution of data, and also that one can generate arbitrarily large samples which is very useful when there's very little historical data.

3. Empirical results

3.1 Historical simulation

Classical VaR was calculated in Microsoft Excel: for the available data we calculated its 95% quantile, (68 in our case), then with the aid of the SMALL() function we calculated the corresponding return and this gave us the 95% VaR. The value of CVaR (ES) was more easily calculated in the R statistical programme.

EVaR was determined with the Ahmadi-Javid A. (2012) formula employing the daily average return and standard deviation, at a 95% confidence level.

In the next table we present the risk measures calculated with historical simulation.

[4] Source: http://www.dax-indices.com

[5] Also called resampling with replacement

Table 1: Risk measures calculated with historical simulation at 95% confidence level for the DAX index between 2009.01.02 and 2014.04.17 (%)

Measures	VaR(95%)	ES(95%)	EVaR(95%)
Values	-2.29%	-3.29%	3.46%

Source: authors' calculations

The figure for classical VaR bears the following interpretation: under normal market conditions there is a 5% likelihood of a daily loss exceeding 2.29%.
ES can be interpreted the following way (in the pessimistic view): there is 5% chance of facing losses greater than VaR, i.e. in this 5% case the average loss of the index will be 3.29%.
As shown in the table, EVaR is always non-negative (but it's still regarded as a loss), and it gives the most stringent (conservative) risk measure compared to the other two, its interpretation being that there's a 95% chance that the daily loss will not exceed 3.46%.

3.2 Monte Carlo simulation

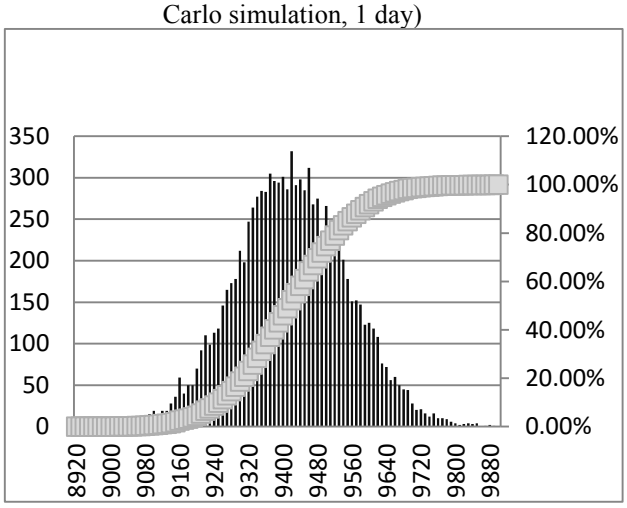
In this case we generated 10000 normally distributed random numbers and calculated the log-returns for them, using the daily, historical mean return and standard deviation. Then we arrived at the simulated index prices the following way:

$$r\log(i) = (\alpha * t) + (\sigma * t) * rand(i)$$
$$S(i) = r\log(i) * S_0$$

Where S(i) = the simulated price at moment „i",
r = simulated return
α = mean return
σ = standard deviation
t = daily period
i=1→10000 (index)

In the next step we divided the simulated index prices from the minimal (8923 euro) to the maximal (9889 euro) value and constructed the division of prices on a 10 euro scale. The next chart shows the obtained histogram and distribution function.

Chart 2.: Histogram of the simulated prices (Monte Carlo simulation, 1 day)



Runs of the script	VaR (95%)
First	-2.266%
Second	-2.267%
Third	-2.267%
Fourth	-2.266%
Fifth	-2.266%

Table 2: VaR for different runs of the resampling script Source: authors' calculations in "R"

Next, using the NORMDIST() function we calculated the return corresponding to the 95% probability which gave us the value of VaR (2.24% which means that under normal market conditions the daily loss should not be higher than 2.24% with a probability of 95%).
We also calculated VaR in absolute index values. We calculated the interval mean points for the divisions and their probability. Next we filled in zeroes until the target index price (because it is unknown what happens beyond VaR), then we subtracted today's price from the interval mean, and multiplied the result by its probability. Finally beginning with the target price we summed up the negative values which gave us the VaR in absolute figures. According to this under normal market conditions the daily loss on the DAX index should not be higher than 37.33 euro with a probability of 95% (the finer the division of the intervals, the more accurate VaR will be).
CVaR (ES) was once again determined with the "R" script already presented in the case of historical simulation. The average of ES turned out to be 2.80%, which means that under normal market conditions, if the 5% probability event occurs, then the average loss will be 2.80% daily.
For EVaR we used the Ahmadi-Javid A. (2012) formula, but this time the daily average return and standard deviation were obtained from 10000 simulated returns. This gave us an EVaR of 3.42% meaning that the daily expected loss with 95% probability is 3.42%.

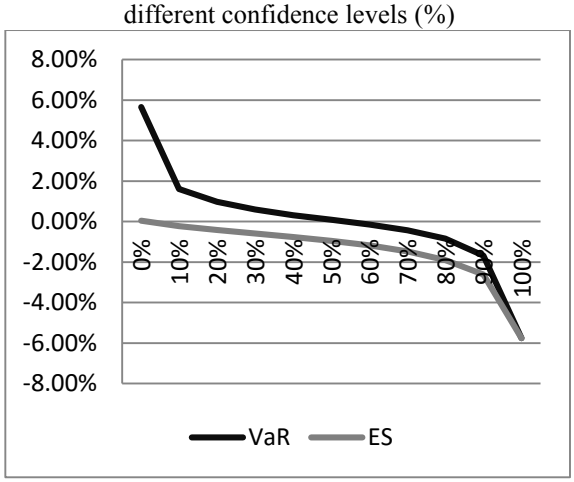
3.3 Resampling (Bootstrap historical simulation)

In this case we determined all three risk measures with the R statistical software, and we employed a sample size of 10000 observations.
For classical VaR we calculated its value for each sample at 95% confidence level, and the final value with resampling was given by the average of the sample VaR's. Also, each time we ran the script, we arrived at a different average VaR (see the table below, where we applied the script five times, with very small differences between the runs). We will use further the first value for comparisons between the different VaR methods.
Overall, according to the resampled VaR, under normal market conditions, the average daily loss on the DAX will not be higher than 2.26% with 95% probability.
The sample size for ES was also 10000, and we saved the ES values into a vector, the average of which gave us the final value for ES. This way the resulting ES

obtained with resampling was 3.29%. This means that under normal market conditions, if the 5% probability event occurs, then the average daily loss will be 3.29%.

The next chart shows VaR and ES for different confidence levels: it can be seen that whereas VaR is both positive and negative, ES is always negative. This is because VaR gives a pointwise estimate of the maximal loss, while ES stems from an averaging of the losses beyond VaR.

Chart 3.: VaR and ES values for the DAX index at different confidence levels (%)



Source: authors' calculations in "R"

In the case of EVaR we applied the resampling method the following way: we calculated the average return and standard deviation for each sample, and then we used the Ahmadi-Javid formula. Lastly the average of these values gave us the final EVaR (3.46% in this case).

3.4 Comparison of the risk measures calculated with the different methods

Let us now compare the risk measures calculated with the three different methods presented in the previous subchapters (see the table below).

	Historical simulation	Monte Carlo simulation	Resampling
VaR (95%)	-2.292%	-2.239%	-2.266%
ES (95%)	-3.293%	-2.796%	-3.290%
EVaR (95%)	3.462%	3.418%	3.460%

Table 3.: Risk measures calculated with the three different methods for the DAX index

Source: authors' calculations in Microsoft Excel

We determined VaR using all three methods. Evidently, the results differ at 95%-os confidence level and daily horizon, due primarily to the fact that the historical data was not normally distributed. The biggest (in absolute value) VaR figure was obtained for historical simulation (2.29% on a daily horizon and 95% confidence level).

ES and EVaR were also calculated with all three methods. As table 3 shows the values obtained from historical simulation and resampling were very close. The inequality established by Ahmadi-Javid A. (2012) also holds for our data: EVaR > ES > VaR. Once again it is evident that EVaR gives the upper bound on the potential losses: daily 3.46% under normal market conditions at a 95% confidence level.

Conclusions

Finally, we must formulate our conclusions, comparing our results with the ones obtained in the literature, and highlighting our contribution to the field of study.

The fact that historical simulation gave us the biggest VaR, ES and EVaR figures is in accordance with the 5-stock portfolio of Čorkalo S. (2011). Also in line with the other sources, we found bigger values for ES than for VaR (for a given confidence level and holding period). Moreover, similar to Dargiri M. N. et al (2013), the fact that we obtained different values for the three calculation methods, was attributable to the fact that the returns of the DAX index were not normally distributed.

Among the three non-parametric methods (historical simulation, Monte Carlo simulation and resampling), we incline towards the resampling method mainly because we worked with relatively small amounts of historical data, but still were able to generate thousands of samples out of them in order to calculate the risk measures. Another advantage of the resampling method is that it permits the conservation of the properties of the original distribution, the new samples are "grown from the same DNS" as their parent distributions, which in our case means that the small generated samples are also non-normally distributed, rendering them more realistic.

We also showed that the Ahmadi-Javid A. (2012) inequality also holds for the DAX index: EVaR > ES > VaR (for a given confidence level, time horizon and calculation method). The most important feature that arises from this is that EVaR is the upper bound on these risk measures, it is the most conservative and the strictest measure, applying it leads to the most cautious portfolio investment strategies. Therefore EVaR is more suitable for the more risk averse investors (it may be suboptimal for less risk averse ones) or those institutional investors that are constrained by regulation to hold more conservative and less risky positions, such as pension funds, insurance funds etc.

Finally, in this respect we may formulate a recommendation for the Basel Committee on Banking Supervision (BCBS), namely that among the Basel 3 guidelines they should consider the usage of Entropic Value at Risk especially for the institutional investors we've indicated above (thus far there is no mentioning of EVaR among the Basel 3 guidelines).

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