

# DISCRETE METHOD FOR ESTIMATION OF TIME-DELAY OUTSIDE OF SAMPLING PERIOD

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## KEYWORDS

Time-delay, on-line identification, time-varying delay, model based prediction, MATLAB/SIMULINK.

## ABSTRACT

The aim of this paper is to suggest a new approach in time-delay identification. With the focus on the value of the delay it utilizes known system parameters for model based predictions in order to estimate the most precise value of the time-delay. This method was further extended by applying a modified internal model based on smaller sampling value which enables to receive results with greater precision than a single original sampling step. The suggested procedure therefore grants a deeper insight into the development of the time-delay and influencing effects from outer signals. The identification algorithm was realized in the MATLAB environment, which was also used for simulations verifying precision of the approach.

## INTRODUCTION

Time-delay occurrence in industrial processes brings significant complications. As a cause of desynchronization between input and output signals it makes use of conventional control techniques ineffective. It is necessary to apply special control strategies designed specifically to deal with this kind of behaviour. Disadvantage of these control algorithms is, among others, an increase of demands for amount of recorded data and performed computations, as well as dependence on precise information about the duration of the delay (Normey-Rico and Camacho, 2007). Furthermore, number of control approaches which are sufficiently robust with respect to the time-delay is scarce (Normey-Rico and Camacho, 2008), (Karafyllis and Krstic, 2013). Therefore it is necessary for the value of the delay to be exactly determined and if possible even during the control of a process.

Due to the complexity of the time-delay identification problem and its resistance to conventional approaches, it remains a subject of active research (Richard, 2003). One of basic principles for time-delay estimation is based on calculation of correlation between input and output signals (Knapp and Carter, 2003). A number of

significant studies have also occurred in the area of relay based identification strategy working with oscillations and cycle measurements (Majhi, 2007). Another suitable strategy for adaptive algorithms is a multi-delay approach consisting of a number of possible delays with the goal of identifying the most precise one (Drakunov et al., 2006). Risk of changes in parameters during a control process suggests an application of recursive approach to identification. An extension of established algorithms with recursive delay estimation was already presented in (Elnaggar et al., 1989) In cases where time-delay may change in time, or depend on system parameters it is suitable to apply an adaptive identification technique. For such systems it can be assumed that the time-delay is the only unknown system parameter (Orlov et al., 2003). Adaptive or on-line identification strategies find practical applications by themselves as well as a part of a bigger adaptive system. A method of increased precision beyond limitations of sampling time based on least-squares method and frequency analysis was suggested in (Ferretti et al., 1991). Despite a number of scientific studies, the demand for precise time-delay estimation still remains.

With the intention to develop an on-line identification approach targeted exclusively on time-delay itself, we have proposed a multi-delay strategy derived from the predictive control. It is an identification technique focused purely on time-delay parameter with assumption that systems structure and remaining values are known. As the key mechanics the predictive equation and its defining parameters are analysed in the first chapter. Consequently, the proposed time-delay identification algorithm is described and illustrated step by step. The functionality verification follows in the next section in a form of simulated process with time varying time-delay parameter.

## THEORETICAL BASIS FOR IDENTIFICATION

The goal during time-delay estimation is to find a particular shape of output signal corresponding to specific input signal. In order to enable the on-line application of the algorithm, it is necessary to address the fact that output shape can have almost any form. Even if we don't consider presence of noise, a number of ongoing output changes may exist, especially in systems of high-

er order. To safely assume the shape of the output in relation to the recorded input the knowledge of system parameters is necessary. Therefore, the design of the identification method assumes that the structure of the studied system and its parameters with the exception of time-delay itself are known.

Part of the design was the option to increase the precision through the application of system parameters to derive a mathematical description with lowered sampling period. The existence of multiple models led to suggestion of multi-delay approach with determining the most precise outcome based on qualitative criterion.

### Predictive principle

The mechanism for estimating of future behaviour of the controlled system was chosen as the basic element for our identification technique. Its principle stands on using a mathematical model of the studied system as a simulated representative of the real system's behaviour. In its general form for 2<sup>nd</sup> order discrete system

$$G(z) = \frac{b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}} \quad (1)$$

an equation for a future output estimation in declarative version can be stated (Haber et al., 2011)

$$\hat{y}(k+1) = -a_1 y(k) - a_2 y(k-1) + b_1 u(k) + b_2 u(k-1) \quad (2)$$

Provided that the control input development and the two past values of the output are known, it is possible to estimate the future outputs up to the duration of the control input.

$$\begin{aligned} \hat{y}(k+i) &= -a_1 y(k-1+i) - a_2 y(k-2+i) \\ &\quad + b_1 u(k-1+i) + b_2 u(k-2+i) \end{aligned} \quad (3)$$

$$i = 1, 2, \dots, N$$

The amount of future steps which will be predicted is determined by constant  $N$  called prediction horizon. This value limits the length of predicted output, as well as computing demands of this operation.

### Prediction with time-delay

In case of our method, we intend to estimate the output of a system containing time-delay, which the model may express in the following form

$$G(z) = \frac{b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}} z^{-d} \quad (4)$$

where  $d$  represents the number of sampling steps of time-delay. The previous relation (3) needs to be extended as

$$\begin{aligned} \hat{y}(k+i) &= -a_1 y(k-1+i) - a_2 y(k-2+i) \\ &\quad + b_1 u(k-1+i-d) + b_2 u(k-2+i-d) \end{aligned} \quad (5)$$

$$i = 1, 2, \dots, N$$

Data from the record of the input signal are then shifted by the length determined by the delay duration.

### METHOD OF ESTIMATION

Based on knowledge of remaining system parameters, it is possible to determine the output values in the future sampling steps. When we apply this procedure for a series of internal models with a value of time-delay varying in a specified interval a number of possible outcomes is obtained. Consequently, these results are compared with measured output and the most precise set gives the most probable value of the time-delay.

In order to eliminate the necessity to wait the length of the prediction horizon to receive all the data for comparison the whole procedure was transformed to work only with data already measured. Initial values for the prediction are therefore extracted from the output data measured at horizons length in the past, as well as control input values.

$$\begin{aligned} \hat{y}(k-i) &= -a_1 y(k-1-i) - a_2 y(k-2-i) \\ &\quad + b_1 u(k-1-i-d) + b_2 u(k-2-i-d) \end{aligned} \quad (6)$$

$$i = N-1, N-2, \dots, 0$$

For illustration purposes the following data are from simulated identification of a 2<sup>nd</sup> order linear system with parameters

$$\frac{2}{4s^2 + 5s + 1} \quad (7)$$

in its discrete form

$$\frac{0,4728z^{-1} + 0,2076z^{-2}}{1 - 0,7419z^{-1} + 0,0821z^{-2}} \quad (8)$$

with sampling period  $T_0 = 2$ s and time-delay  $d$  set to 4.5s. The prediction horizon size was set to 10 sampling steps.

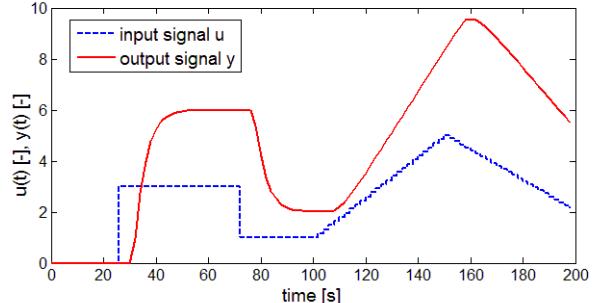


Figure 1: Controlled process as a source of input and output values for identification

Applying the identification procedure at time of 90s with maximal expected delay set to 12s provides the following range of possible outcomes.

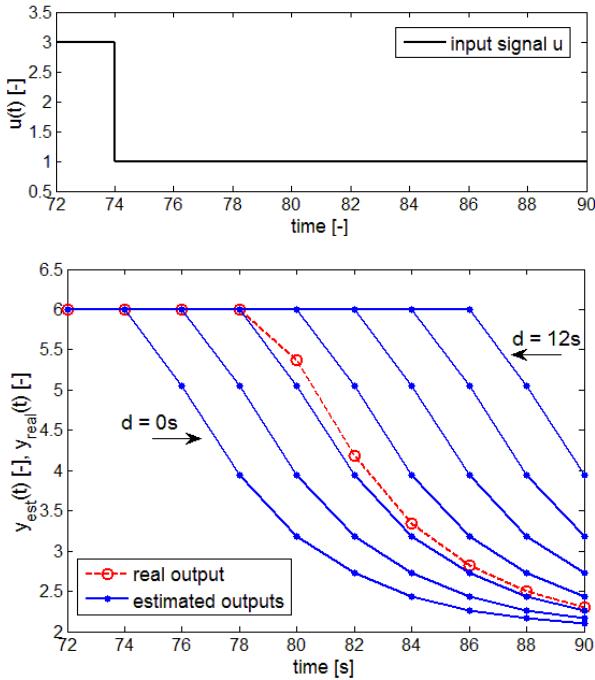


Figure 2: Set of possible system outputs based on different time-delay values

Estimated outputs are compared with the real one and using a discrete version of a qualitative criterion ISE its dependency on the duration of the time-delay is obtained.

$$ISE(k, d) = \sum_{i=0}^{N-1} [y_d(k-i) - y(k-i)]^2 \quad (9)$$

where  $k$  represents the current step. Therefore we obtain a set of values representing divergences from delay estimations in every sampling step.

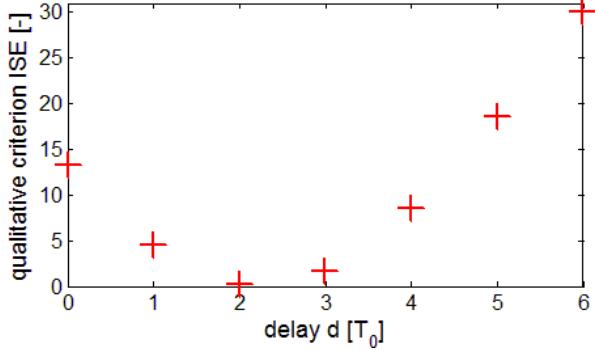


Figure 3: ISE criterion dependency on the estimated value of time-delay

Considering the content of Figure 3 we may assume, that data interpolation followed by determining its minimal value may produce even more precise estimation. Nonetheless, interpolation cannot guarantee notably precise information about the development of the system dynamics in reference to the time-delay. Therefore, the internal model is modified to a form with decreased sampling period. This model is again applied in the identification algorithm. The control input signal has to be shaped accordingly to fit the new sampling time. In

order to determine initial values of the output an interpolation between two recorded values was used.

To decrease unnecessary computations, this time the predictions are performed only in the interval between two smallest values gained by the criterion in Figure 3.

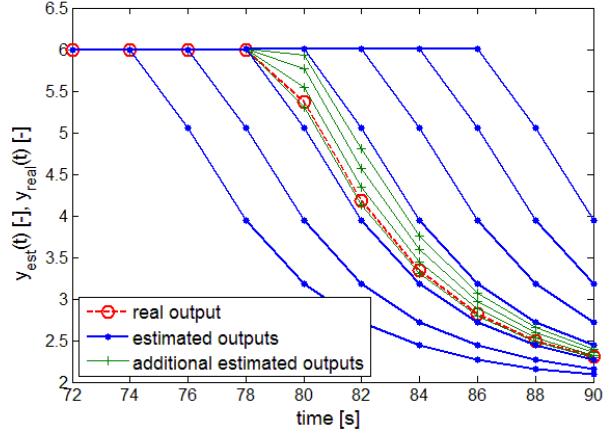


Figure 4: Extended set of possible system outputs

Repeated evaluation of the precision by applying the identical criterion (9) for the newly received estimates offers the result with a greater amount of data in the critical area.

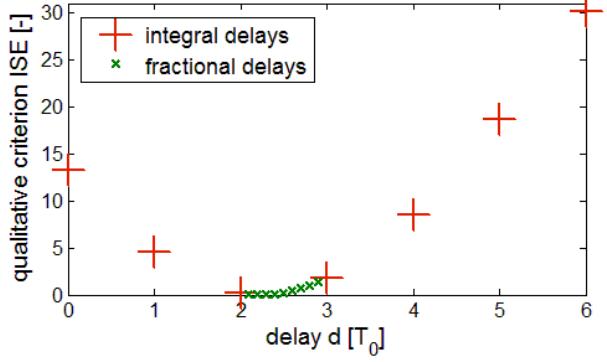


Figure 5: Extended ISE criterion dependency on the estimated value of time-delay

Figure 5 demonstrates how additional data received from the modified internal model with sampling time 0.2s fill in the missing part of the crucial segment.

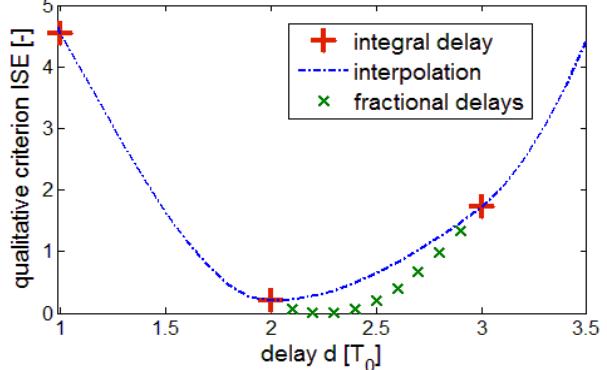


Figure 6: Comparison between identification results and interpolation

The distinction from the interpolated data is displayed in Figure 6, where piecewise cubic Hermite interpolation

and our estimation results were compared. A significant difference appears between both minimal values. Length of the horizon determines the amount of data used in prediction and therefore criterion calculations. Larger area of identification can more accurately determine the correct outcome especially if noise is present. Nevertheless, during a sudden change in time-delay, in theory, approximately half of the horizon length of the new data needs to be processed in order to establish a new minimum in the criterion set. Therefore, changing the horizon value shifts the balance between precision and speed of the identification.

## ALGORITHM VERIFICATION RESULTS

The identification algorithm performance was tested by a simulated system with time-varying delay. System parameters were kept identical to (7) (8), only the value of the delay was set to change. The length of the prediction horizon was set to 10 sampling steps which were previously verified to provide acceptable results. The procedure was applied in every sampling step, however several first steps could not be analysed as the amount of data was not sufficient to fill the prediction horizon. Ideal conditions for identification require constant changes in recorded signals. Therefore, the shape of control input consists of ramps and step changes.

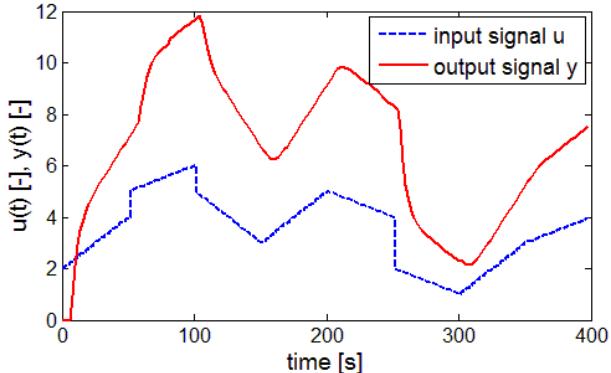


Figure 7: Controlled process as a source of input and output values for identification

Figure 7 shows recorded input and output signals which were used in the identification algorithm.

To test the option of on-line application a time-varying output signal delay was suggested. Its final form contains areas of constant value, sudden steps and ramps.

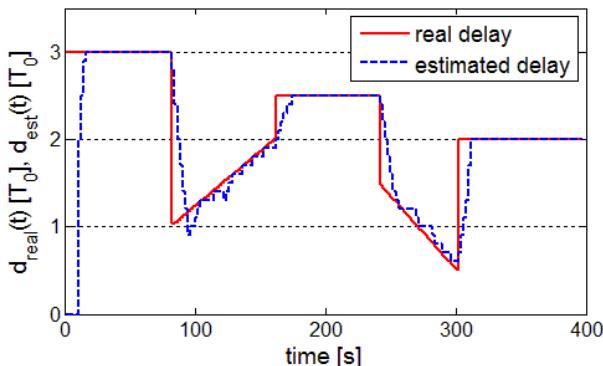


Figure 8: Time-delay on-line estimation

As can be seen in the Figure 8 delay estimation results are able to determine the correct value of the system time-delay while it is static. To increase the precision during large step changes of the time-delay, the maximal change in results has been limited to a half of the sampling period. Consequently, the development of the estimations is slower, but on the other hand outcomes are closer to real values. Constant changes in the form of ramps have proven to be the most difficult to estimate, for the mere fact that the bigger is the delay the longer it takes to get the correct result, which tends to distort results in the whole section of the process.

## Noise compensation

From the principle of the method is clear, that the outcome precision depends on how the real process behaviour is close to the internal model description. We have selected noise as a tool to simulate inaccuracies of a real system.

To demonstrate the negative influence of noise, the previous model (7) was extended with a source of white noise. The input signal trajectory was kept identical, as well as the time-delay development.

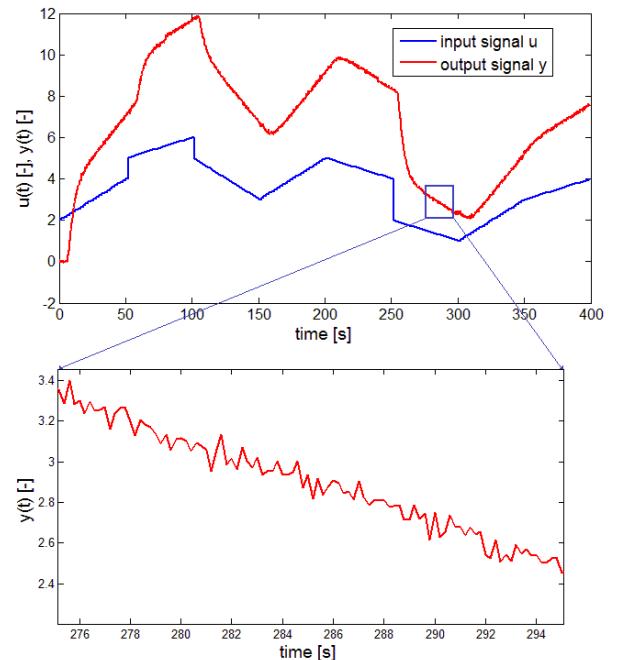


Figure 9: Controlled process with added noise

Figure 9 displays the system input signal and delayed output burdened by noise. This process was initially identified with algorithm parameters identical to the previous case.

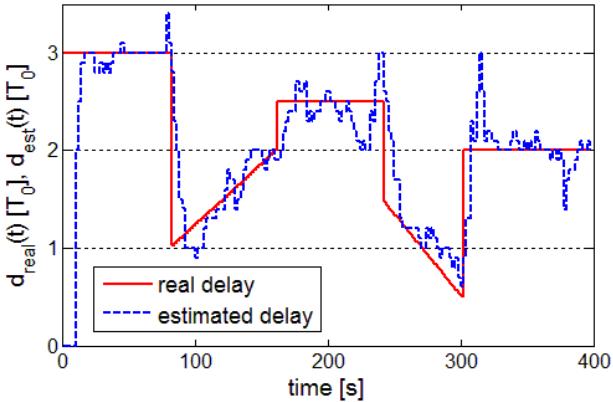


Figure 10: Time-delay online estimation from process with added noise

Figure 10 shows that noise can significantly influence the value of qualitative criterion and consequently entire identification result. Noise influence heavily depends on the ratio between noise and system gain, as well as chosen precision of the method.

In order to compensate above mentioned negative effects, we have expanded the prediction horizon, which has enlarged the area involved in determining of the estimation precision. Therefore, the prediction horizon  $N$  in the identification algorithm was set to 15 sampling steps in order to decrease the influence of the noise, which has extended the area studied in every step by 10 seconds of recorded data.

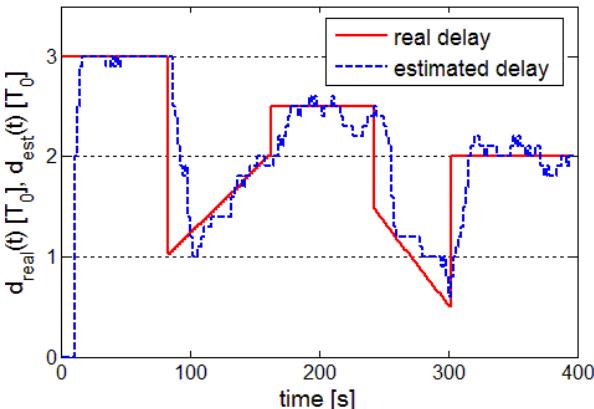


Figure 11: Time-delay on-line estimation from process with added noise and greater value of the prediction horizon

Figure 11 demonstrates the effectiveness of widening of the prediction horizon as a significant amount of considerably differing results has been cleared away. On the other hand, due to the higher value of the prediction horizon, sections with constant change of time-delay tend to be processed less accurately and expressed in small periodic steps. Nevertheless, the overall difference between true and estimated delay has been diminished, therefore it seems safe to claim that the bigger prediction horizon is able to a certain extent negate effects of imprecision.

## CONCLUSION

In this paper we have suggested a new discrete method for time-delay estimation, enabling to determine its value even in cases when it is not an integer. Extending of the multi-delay approach with a version of the studied system with a smaller sampling step made possible to predict output values of system with a fractional time-delay. Functionality of the designed method was verified in simulations and has proven the correctness of its results.

Periodic measurements during a process control can provide an opportunity to measure changes in time-delay effect during the process that would stay otherwise hidden. Additionally, they provide information about a possible influence of system states on time-delay. The estimation concept can be applied both online and outside of a controlled process. It is also suitable for any type of system which can be described in a form of internal model.

Further research involves application of the method on a real system and increase in robustness of the identification algorithm.

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