NONLINEAR SIMULINK MODEL OF MAGNETIC LEVITATION LABORATORY PLANT

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ABSTRACT

The paper deals with modelling of a magnetic levitation laboratory plant. The goal of the work was to create a nonlinear model in a MATLAB / Simulink environment representing behaviour of a real-time CE152 laboratory plant. The CE152 is a magnetic levitation model developed by Humusoft company. From the control point of view, the CE152 magnetic levitation plant is a nonlinear very fast system. The model of the plant is developed using first principle modelling and subsequently made more precise using real-time experiments. The behaviour of the resulting Simulink model is compared with the behaviour real-time plant. The Simulink model can be further used in the process of controller design.

INTRODUCTION

Knowledge of a model a controlled plant is necessary for most of current control algorithms (Bobál et al. 2005). It is obvious that some information about controlled plant is required to allow design of a controller with satisfactory performance. A plant model can be also used to investigate properties and behaviour of the modelled plant without a risk of damage of violating technological constraints of the real plant. Two basic approaches of obtaining plant model exist: the black box approach and the first principles modelling (mathematical-physical analysis of the plant).

The black box approach to the modelling (Liu 2001), (Ljung 1999) is based on analysis of input and output signals of the plant. In this case the knowledge of physical principle of controlled plant is not required but obtained model is generally valid only for signals it was calculated from.

The first principle modelling provides general model valid for whole range of plant inputs and states. The model is created by analysing the modelled plant and combining physical laws (Himmelblau and Riggs 2004). On the other hand, there are usually many unknown constants and relations when performing analysis of a plant. Therefore, modelling by first principle modelling is suitable for simple controlled plants with small number of parameters. First principle modelling can be used for obtaining basic information about controlled plant (rage of gain, rank of suitable sample time, etc.). Some simplifications must be used to obtain reasonable results in more complicated cases. These simplifications must relate with the purpose of the model. The first principle modelling is also referred to as white box modelling.

The paper combines of both methods. Basic relations are derived using first principles. The obtained model is further improved on the basis of measurements. This approach is known as grey box modelling (Tan and Li 2002). The goal of the work was to obtain a mathematical model of the CE152 Magnetic levitation plant (Humusoft 1996) and to design the model in MATLAB-Simulink environment. The CE152 plant was developed by Humusof Ltd. and serves as a real-time model of fast nonlinear system. The major reason for creating the model of this laboratory equipment was usage of the model in control design process.

THE CE152 MAGNETIC LEVITATION SYSTEM SPECIFICATION

A photo of the CE 152 magnetic levitation plant is presented in Figure 1.

Figure 1: Photo of the CE152 plant

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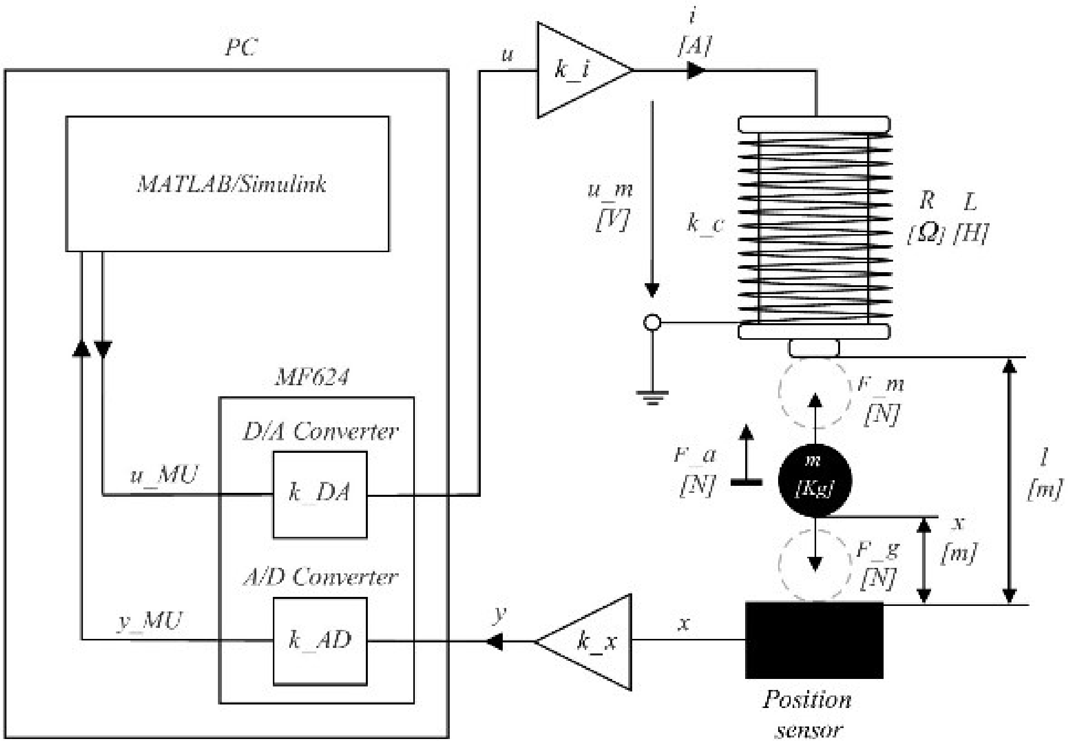

Figure 2: Principal scheme of the magnetic levitation model

The CE152 Magnetic levitation system is a nonlinear unstable dynamic system with one input and one output. Input signal is the control signal and output signal gives information about position of a steel ball. Both signal values are converted and scaled to the specific range of the machine unit [MU].

Structure of the CE152 Magnetic levitation system

The system consists of a model of the magnetic levitation system, power supply and a universal data acquisition card MF624. MF624 is a standard PCI card with A/D, D/A converters, analogue/digital inputs and outputs, counters, timers and appropriate drivers. The model is connected to the PC via this card.
Following parts are considered for a modelling of the plant:

- D/A converter,
- A/D converter,
- the position sensor,
- the power amplifier,
- the ball and coil subsystem

Simplified inner structure is shown in Figure 2. A steel ball levitates in magnetic field of the coil driven by power amplifier connected to D/A converter. Position of the steel ball is measured by inductive linear position sensor connected to A/D converter. Both control and measured parameters are sent and received by Simulink.

System behaviour

In this part system behaviour is discussed. As mentioned before the CE152 Magnetic levitation system is a nonlinear unstable dynamic system with one input and one output. When an input control signal of certain value is sent to the system, the ball is lifted upwards to the magnetic core and it stops when it hits the core. This behaviour is caused by electromagnetic force of the magnetic core which overcame the force of gravity. As the ball getting closer to the coli core, accelerating force grows. Because of an obstacle in the form of magnetic core, both the ball and accelerating force stops. Higher input signal means higher electromagnetic force and much more rapidly increasing acceleration. If input control value is decreased under the certain value, the ball falls down. That is happening, because the electromagnetic force is too low to overcome gravitational force. Ball fell down to the head of the sensor, which stops the ball. When the ball hits the sensor it bounces a several times. Presented behaviour is the system basic behaviour as a reaction to a constant input signal. Figure 3 shows the step response on different input values and Figure 4 presents the reaction, when the input value has changed to zero. If input value is not exactly zero, but still too low to hold up the ball, then the ball still falls, but his bouncing behaviour is slightly different due to the non-zero force of attraction.

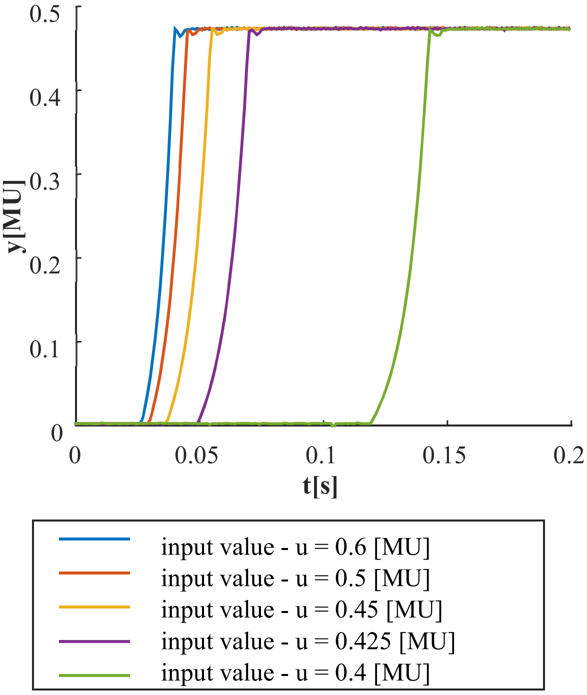

Figure 3: Step response on different input value

Tested input signal was step signal because other input signals like ramp or sinus signal don't have adequate information value, thanks to the described system behaviour, when once the electromagnetic force is strong enough, the ball is attracted to magnetic core, despite increasing input signal. In opposite case, when input signal decreasing, it is the same situation but in opposite meaning of the ball movement.

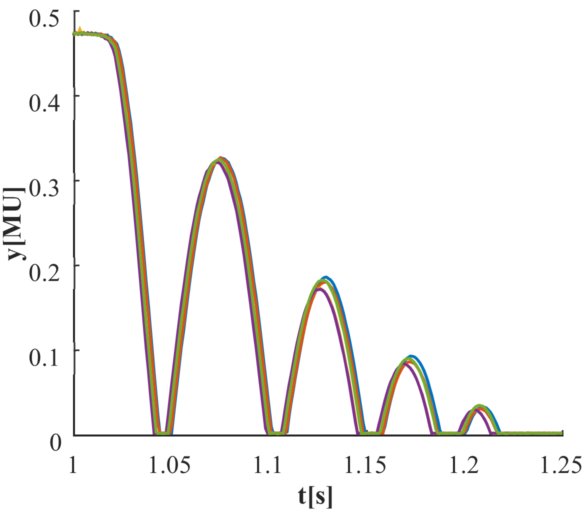

Figure 4: Reaction on input value change to zero

As it can be seen, there is some kind of delay, before the system starts to attract the ball. This is happening because coil first needs to attract the ball straight to the middle under its core and then the ball is lifted. This can be overcome by setting an optimal starting point of the ball. Even though, the small but non zero delay still remains and it is probably caused by internal processes in the system. Because of later comparison of the created model with the real system, counting with this delay should be taken into account. A small delay can be also observed when the ball rebound (see Figure 4). This behavior is caused by difference between zero position of the measurement system and position corresponding to the ball on the head of the position sensor (ball in the initial position).

MODELLING OF THE SYSTEM PARTS

Based on mentioned approaches and methodologies, an adequate model of CE152 magnetic levitation system is deduced in this section. According to above preview of the system specification and composition, model can be decomposed to following parts: D/A and A/D converter, position sensor, power amplifier and ball and coil subsystem. All parts can be identified separately. All parts are eventually interconnected and they form the final model of CE152 magnetic levitation system.

D/A converter

The D/A converter converts digital signal u_{MU} from PC into an analogue voltage signal u. The D/A converter can be described by a linear function (1) and represented by a Simulink block (Figure 5).

$$u = k_{DA} u_{MU} \tag{1}$$

where u is D/A converter output signal/coil input voltage [V], u_{MU} is D/A converter input signal [MU] and k_{DA} is D/A converter gain [V/MU]

The D/A converter maps the input signal range u_{MU} <-1 MU, 1 MU> to the range u <0 V, 5 V>. This leads to converter gain $k_{DA} = 10\ V/MU$. Real range of D/A input signal is $-10\ V$ to $10\ V$ but input to the CE152 magnetic levitation system is constructed for the range 0 V to 5 V, so signal input must be constrained.

A/D converter

The A/D converter converts analogue voltage signal y into a digital signal y_{MU}. The A/D converter can be described by a linear function (2) and represented by a Simulink block (Figure 6).

$$y_{MU} = k_{AD} y \tag{2}$$

where y represents A/D converter output signal/position sensor voltage [V], y_{MU} is A/D converter output signal [MU] and k_{AD} is A/D converter gain [MU/V]

The A/D converter maps the input signal range y <$-10\ V$, 10 V> to the range y_{MU} <$-1\ MU$, 1 MU>. This leads to converter gain $k_{AD} = 0.1\ MU/V$.

The position sensor

An inductive position sensor is used to measure the ball position x. Maximum declared height l is calculated as a difference between physical distance of the coil and the sensor l_0 and the ball diameter d_k. The position of the ball is obtained by reading the voltage from A/D converter's output. Sensor voltage varies with ball position. The relation between ball position and voltage is approximately linear.

$$y = k_x x + y_0 \tag{3}$$

where x is ball position [m], y_0 position sensor offset [V], y position sensor voltage [V], k_x - position sensor gain [V/m]

Calibration experiment must be done. First of all, a travelling distance of the ball l has to be calculated. It is a difference between physical distance of the coil and the sensor $l_0 = 18.4 \cdot 10^{-3}\ m$ and the ball diameter $d_k = 12.7 \cdot 10^{-3}\ m$.

$$l = l_0 - d_k = 5.7 \cdot 10^{-3}\ m$$

Final results of the boundary values were noted (Table 1), and based on calculation with these values, position sensor gain is obtained.

Table 1: Calibration data of the position sensor

i	$x_i[m]$	$y_{MUi}[MU]$	$y_i[V]$
1	0	0.00254	0.02537
2	0.0057	0.47384	4.73840

Position sensor offset y_0 is taken as an initial value from the inductive sensor, when input action signal is zero.

$$y_0 = y_1 = 0.02537\ V \tag{4}$$
$$k_x = \frac{y_2 - y_1}{x_2 - x_1} = 826.8525\ V/m \tag{5}$$

The power amplifier

The power amplifier works as transconductance amplifier whose differential voltage between input voltage u from the D/A converter and u_i produces an output current supplied to the coil. The power amplifier essentially represents a source of constant current with the current stabilisation. Internal structure of the amplifier is presented in Figure 5 where k_{am} stands for amplifier gain [-] k_s stands for current sensor gain [-], L represents coil inductance [H], R_c is coil resistance [Ω] and R_s is resistance of feedback resistor [Ω]

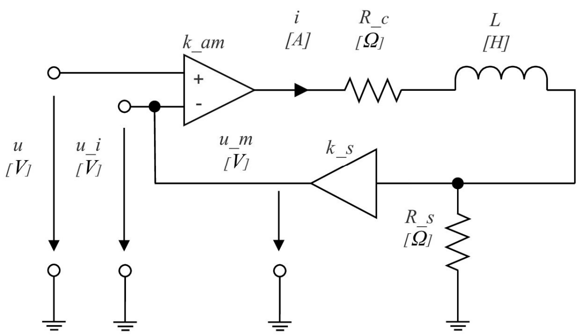

Figure 5: Internal structure of the power amplifier

The power amplifier can be described by the following set of equations:

$$u_m = \frac{di}{dt}L + i(R_c + R_s) \tag{6}$$
$$u_m = k_{am}(u - R_s k_s i) \tag{7}$$

Using direct Laplace transform with zero initial conditions leads to:

$$\frac{I(s)}{U(s)} = \frac{\frac{k_{am}}{R_c + R_s + k_{am} R_s k_s}}{\frac{L}{R_c + R_s + k_{am} R_s k_s} s + 1} \tag{8}$$

Simulation diagram is then shown in Figure 9.

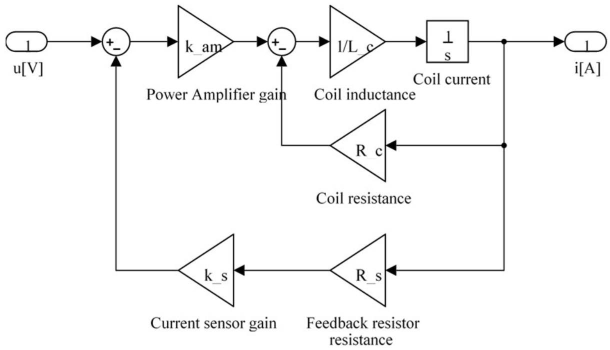

Figure 6: Power amplifier detailed simulation diagram

We can also do simplification of equation (8) and define it by transfer function of 1st order:

$$G_{PA}(s) = \frac{I(s)}{U(s)} = \frac{k_i}{T_a s + 1} \tag{9}$$

Constant k_i is an amplifier gain and T_a is an amplifier time constant. Manual (Humusoft 1996) states, that typical parameters of each component of the coil and power amplifier are:

$$k_{am} = 100$$
$$k_s = 13.33$$
$$L = 0.03\ H$$
$$R_c = 3.5\ \Omega$$
$$R_s = 0.25\ \Omega$$

When these values are substituted into equations (8) and (9) then power amplifier gain and time constant have following values:

$$T_a = 8.90210^{-5}\ s$$
$$k_i = 0.297\ A/V$$

Time constant is very small and can be neglected, then only power amplifier gain can remain. Difference between response of the detailed model and the simplified model with only power amplifier gain was assessed as minimal. Because of this minimal difference and faster computation of a simulation, simplified model was used.

The ball and coil subsystem

Lagrange's method can be used for modelling ball and coil subsystem. The motion equation is based on the balance of all acting forces. Final equation (11) is a nonlinear second order differential equation where input variable is electric current and output variable is ball position. Gravitational force F_g depends on ball mass and it is constant. This force acts against electromagnetic force F_m created by coil, when electric current pass through it. It is clear that to lift the ball up, electromagnetic force must be greater than gravitational force (accelerating force greater than zero). From the equation (11) we can figure out that electromagnetic force relies on two parameters, the amount of electric current i and actual ball position x. Damping force is also considered in this model:

$$F_a = F_m - F_g \tag{10}$$

$$m_k\ddot{x} + k_{fv}\dot{x} = \frac{i^2 k_c}{(x-x_0)^2} - m_k g \tag{11}$$

where:
F_m - electromagnetic force [N]
F_a - accelerating force [N]
F_g - gravitational force [N]
g - gravitational acceleration [m.s–^{2}]
x - ball position [m]
m_k - ball mass [kg]
x_0 - coil offset [m]
kc - coil constant [-]
i - coil current [A]
k_{fv} - dumping constant [N.s.m–^{1}]
Parameter of the ball mass m_k can be calculated from its diameter $d_k = 12.7\cdot 10^{-3} m$ and density $\rho = 7800\ kg.m^{-3}$:

$$m_k = \rho V_k = \rho \frac{4}{3}\pi\left(\frac{d_k}{2}\right)^3 = 8.37\cdot 10^{-3} kg \tag{12}$$

Parameters of the coil constant are obtained through the measurement of the steel ball stable positions, which means that the system have to be controlled somehow. Required data was obtained from closed loop control. Design of the control was taken over from Humusoft real time toolbox control simulation model for CE152 magnetic levitation system. Calculation of desired parameters has been done by two point calibration method with data given by (Table 2).

Table 2 Two point calibration data

i	y^s_{MUi} [MU]	x^s_i [m]	i^s_i [A]	u^s_i [V]
1	0.1500	0.018	0.6579	2.2152
2	0.3500	0.0042	0.4144	1.3951

Two stable points were measured and based on the following equation desired parameters were calculated.

$$x_0 = 0.0083[m];$$
$$k_c = 8.0915\cdot 10^{-6}[N.m^2A.^{-2}]$$

Parameter of the ball damping constant k_{fv} cannot be measured directly or by a dedicated experiment. It was identified by the trial-and-error method using real experimental data for comparison:

$$k_{fv} = 0.0195[N.s.m^{-1}]$$

The ball and coil simulation diagram is presented in Figure 7.

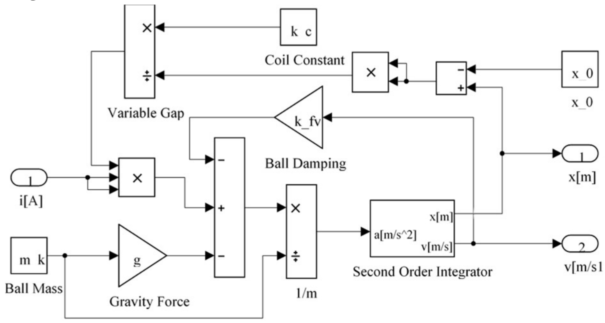

Figure 7: The ball and coil simulation diagram

Bouncing

A coefficient of restitution is used for bouncing ball behaviour. This coefficient accounts for the loss of kinetic energy during each bounce. Bouncing behaviour of the model isn't very close to the behaviour of the real system, because it doesn't take into account friction and other impacts of physics. Comparing can be found in section dedicated to validation and verification of the model. Bouncing model in placed inside the Second order integrator block presented in Figure 7. Bouncing subsystem itself is presented in Figure 8.

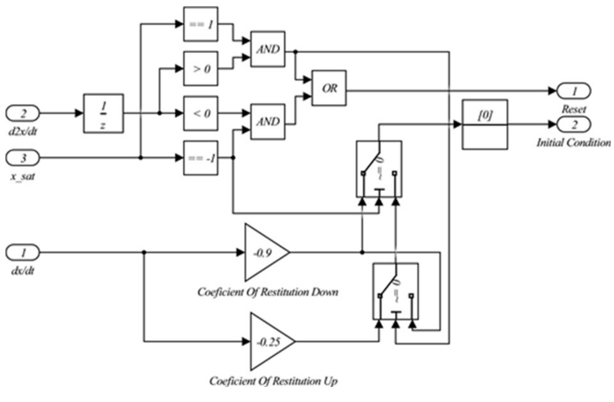

Figure 8: Hard stop and bouncing block diagram

THE WHOLE SYSTEM MODEL

The whole system consists of created subsystem, which were included into subsystem blocks and interconnected. Interconnection of these blocks is shown in Figure 9.

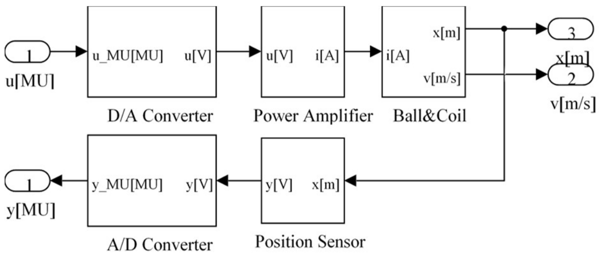

Figure 9: Interconnection of subsystem blocks

Finally, subsystem blocks were turn into a parent subsystem block as presented in Figure 10.

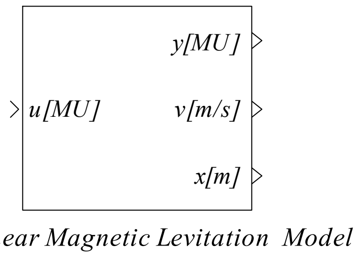

Figure 10: CE152 magnetic levitation system

VERIFICATION AND VALIDATION OF THE MODEL

From the perspective of a verification, created model is satisfactory. Behaviour of the model is just like the behaviour of the real system template. When we take a look on results of a validation, they are surprisingly good. The control input values have the same course and same is it with output values. The resulting characteristics depends strictly on estimated parameters. Figure 11 presents the response of the created model of the system to the input value $u = 0.45$ *MU* and its confrontation with response of the real system.

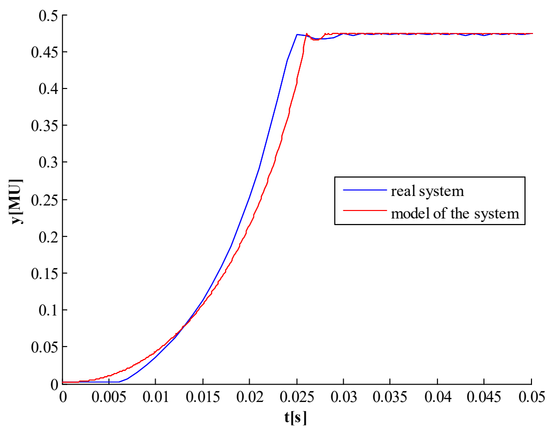

Figure 11: Comparison of step responses of the model and the real system

Response of the real system is then moved a little backwards, because of the delay discussed in a chapter dedicated to the system behaviour. From the same reason also response on the change of input value to zero needs to be moved a little backwards. This corresponds to an immediate response of the system. Now if responses are compared it can be said, that created model have the same behaviour. Waveforms have the same direction and tendency.

Bouncing behaviour of the model is presented in Figure 12 and differs from the behaviour of the real plant. But this is obvious, because this behaviour was modelled in the simplest form.

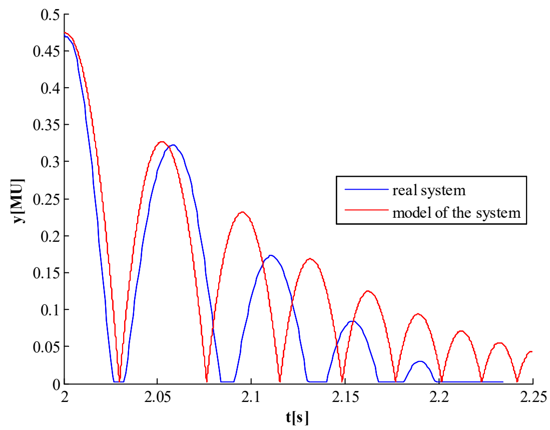

Figure 12: Comparison of bouncing of the model and the real system

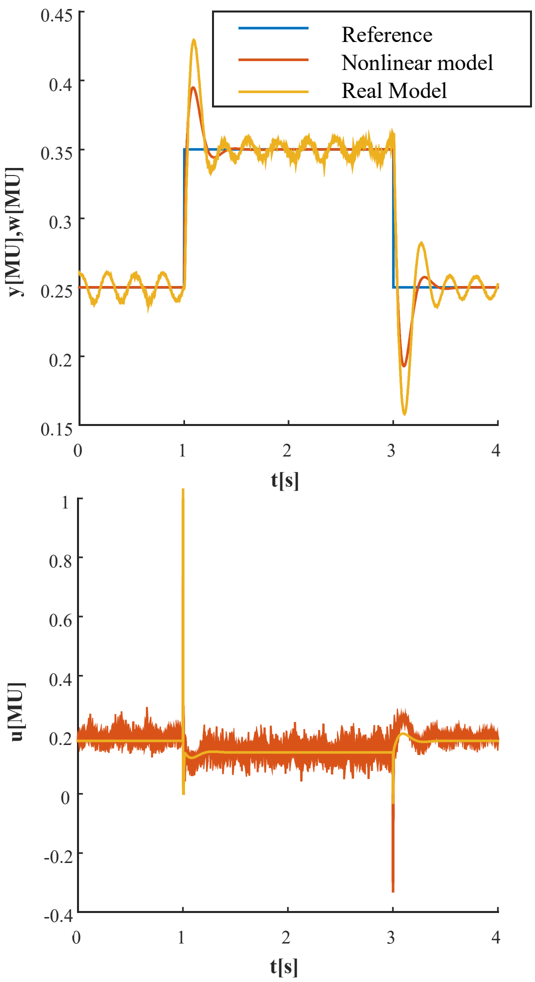

Figure.13: Control experiment

Validation was also performed by control experiments on the system and then the same experiments are repeated on the model. Compared values are output values and input action values as presented in Figure 13. It can be concluded that even from the perspective of validation is created model sufficiently adequate, because both the output and even the control input signal have the same behaviour as the output and control input signal obtained from the real system.

CONCLUSION

The CE152 Magnetic levitation system was investigated and its first principle model was derived. This model was created in the MATLAB/Simulink environment. Obtained model was made more precise by incorporating data from several real-time experiments.
Validation and verification proved a good correspondence of the nonlinear Simulink model and the real time plant.
The model will be further improved and used or control design of a model predictive controller for the magnetic levitation system.

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