INTRODUCTION

Nowadays, the major problems of the numerical computation of mathematical models for complex processes are solved by using different commercial and open source software packages. The representation of the models in these languages is often based on model equations. The bond graph representation allows a physical structural analysis which is based on the system energy structure. This facilitates the exchange of models and simulation specifications. Bond graph is a graphical representation methodology for modelling multidisciplinary physical systems (Jardin et al. 2005).

Heat exchange is an important unit operation that contributes to efficiency and safety of many processes (nuclear power plants, steam generators, automotive, heat pumps, etc.). A plate heat exchanger is a type of heat exchanger that uses metal plates to transfer heat between two fluids. The plate heat exchanger was invented by Dr Richard Seligman in 1923 and revolutionised methods of indirect heating and cooling of fluids (Crepaco 1987). Plate heat exchangers are widely used in many other applications (food, oil, chemical and paper industries, HVAC, heat recovery, refrigeration, etc.) because of their small size and weight, their cleaning as well as their superior thermal performance compared to other types of heat exchangers (Guo et al. 2012).

The plate heat exchanger model is one of over 200 0D/1D models of components belonging to the ThermosysPro library. This open source library, developed by EDF R&D, is used to model energy systems and different types of power plants (nuclear, conventional, solar, etc.) (El Hefni 2014, El Hefni and Bouskela 2006, El Hefni et al. 2011, 2012, Deneux et al. 2013). The Modelica model is developed in Dymola. Modelica representation leads to static analyses which are based on model equations, while BG representation permits a physical structural analysis which is based on the system’s energy structure. The bond graph representation in this paper is built using the graphical editor MS1. MS1, an acronym of Modelling System One, is an interactive environment for modelling, simulation and analysis of non-linear dynamic systems (Jardin et al. 2008).

In the literature, bond graph modelling of heat exchangers is widespread (Shoureshi and Kevin 1983, Hubbard and Brewer 1981, Delgado and Thomé 1999). Due to difficulties in handling entropy and heat transfer rate, many efforts have been made to develop pseudo-bond graph representations of thermo-fluid transport and heat exchange (Karnopp 1978, 1979, Ould Bouamama 2003). All these references mentioned above have different assumptions. For instance, in (Shoureshi and Kevin 1983) a temperature-entropy bond graph technique has been proposed based on three lump models to predict the reversal of flow. In this model, the authors have considered that the fluid domain is operated independently from the thermal domain. In (Karnopp 1978), pseudo bond graph strategies have been proposed with using the temperature and heat flow as effort and flow.

This paper uses pseudo bond graph method for heat/mass transfer modelling. Furthermore, multi-port C and multi-port R elements have been used. This method is based on finite volume approach considering the thermal and fluid bonds. First, fundamental theory of thermofluid is given. Then, the plate heat exchanger models are explained: the Modelica model and the BG model. In the section after that, simulation results are discussed. The last section contains conclusion and future research paths.

THERMOFLUID SYSTEM

Thermofluid or thermal fluid sciences involve the study of the thermodynamics, fluid mechanics, heat and mass transfer in complex engineering systems. In the open system case, the energy and mass equations for a thermodynamic system are formulated as (see nomenclature page 7)

$$\frac{dE_i}{dr} = \dot{Q}_{in} + \dot{W}_{in} + \dot{E}_i - \dot{E}_e$$

where \(\frac{dE_i}{dr}\) is the rate of increase in energy within the system, \(\dot{Q}_{in}\) is the rate at which heat enters the system, \(\dot{W}_{in}\) is the rate at which work enters the system, \(\dot{E}_i\) is the rate at which energy is brought in by the mass entering the system, and \(\dot{E}_e\) is the rate at which energy is removed by the mass leaving the system.

$$\frac{dm_i}{dr} = \dot{m}_i - m_e$$
where \((dm_i/dt)\) represents the rate of increase in mass within the system, and \(m_i\) and \(m_e\) represent the respective rates at which mass entering and leaving the system.

In many thermal applications, the reduced heat equation is used
\[
\dot{Q} = \frac{dQ}{dt} = mC_p\frac{dT}{dt} = KA(\Delta T),
\]
where \(\dot{Q}\) is the heat-flow-rate (named just heat rate), \(C_p\) is the specific heat capacity, \(m\) is the mass flow rate, the global heat transfer coefficient \(K\) (associated to a bounding area \(A\) and the average temperature jump \(\Delta T\) between the system and the surroundings).

More details about the above equations can be seen in reference (Martinez 1992).

The cooling water heat exchanger used in this study is an equipment for nuclear power plants. Two main approaches for the dynamic modelling of the heat exchanger are the moving boundaries (MB) and the discretized models, known as finite-volume models (FVM) (Bendapudi et al. 2004; Desideri et al. 2015).

The moving boundary method is useful for developing feedback controllers, in this approach the heat exchanger is divided into zones based on the fluid phase in each region and the location of the boundary between regions vary in time according to the current conditions. In finite-volume models the 1D flow is subdivided into several equal control volumes as shown in Figure [1].

The modelling technique used in this paper is based on finite volumes approach. A pictorial representation of the discretized counterflow heat exchanger is shown in Figure [2].

**PLATE HEAT EXCHANGER MODELS**

The plate heat exchanger is the component that transforms heat (thermal energy) from one fluid to another. Plate heat exchangers have a high heat transfer rate compared to other types of heat exchangers due to their large surface area.

**Modelling of Water/Water Heat Exchangers in Modelica**

The dynamic water/water heat exchanger component used belongs to the ThermoSysPro library. The core model of the heat exchanger was written in Modelica and simulated with the Dymola simulation environment. Figure [3] shows the schematic of the heat exchanger model in Dymola. This model has two parts: the upper part for hot water and the other part for cold water.

Figure 1: Staggered finite volume scheme

Figure 2: Schematic diagram of a typical discretized counterflow plate heat exchanger

Figure 3: Dymola layout of the heat exchanger

In this model, the rate of mass accumulation within the volume does not incorporate any dynamic effects. This means that the entering mass flow rate is exactly equal to the leaving one i.e. the steady balance for all volumes of the heat exchanger, yields
\[
\dot{m}_{b,i-1} - \dot{m}_{b,i} = 0
\]
where \(\dot{m}\) is the mass flow rate. Throughout the paper, the subscript \(b\) means the hot part when \((b \leftarrow h)\) or the cold part when \((b \leftarrow c)\).

To simplify the model, the mass flow rate is considered positive in both parts i.e. \(\dot{m}_{b,i} > 0\), and the pressure between each two volumes is defined as
\[
P_{b,i+1} = P_{b,i} - \Delta P_{b,i}/N
\]
where \(N\) is number of segments, \(\Delta P_{b,i}\) is the pressure drop.

The pressure drop \((\Delta P_{b,i})\), which has direct relationship to the size of the plate heat exchanger, is defined by
\[
\Delta P_{b,i} = k_{b,i} \cdot N u_{b,i} \cdot \frac{a}{\rho_{b,i}} \cdot \frac{M}{\rho_{b,i}} + 104.97 \cdot N u_{b,i}^{0.25}
\]
where \(a = 0.097\) and \(k_{b,i}\) correlation for the heat transfer \(N u\) (is called also Nusselt number), and pressure drop \(qu\) characteristics, are defined as
\[
\begin{align*}
N u_{b,i} & = \frac{\dot{m}_{b,i}/(M \cdot \rho_{b,i})}{qu_{b,i} / \rho_{b,i}} \\
k_{b,i} & = 14423.2 \left[ 1472.47 + \frac{1.54(M - 1)}{2} \right] c_{1,b} \rho_{b,i}^{0.25}
\end{align*}
\]
where \( c_{1,b} \) is a correction coefficient, and \( M = (n - 1)/2 \), 
\( n \) is the number of plates. The formulas above are
investigated experimentally, see [Cracow 2011].

The energy balance equation in each layer is given by:
\[
V_{b,i} \cdot \rho_{b,i} \cdot \frac{dh_{b,i}}{dt} = \pm (h_{b,i} \cdot \dot{m}_{b,i} - h_{b,i+1} \cdot \dot{m}_{b,i+1} - W_i)
\]  
(8)

where the plus-minus sign (\( \pm \)) indicates (+) for the hot part and (-) for cold part.

The global heat exchanged between the both fluids and the wall is
\[
W_i = K_i \cdot \Delta S \cdot (T_{b,i} - T_{c,i})
\]
(9)

The heat exchange surface \( \Delta S \), and the global heat transfer coefficient \( K_i \) are given by
\[
\begin{aligned}
\Delta S &= (n - 2) \cdot A/N_i, \\
K_i &= \frac{(h_b \cdot h_c)}{(h_b + h_c + \frac{h_b \cdot h_c}{\bar{h}_b})}, \\
\bar{h}_b &= 11.245 \cdot \left| N_t\rho_{b,i} \right|^{0.8} \cdot \rho_{b,i}^{-0.4} \cdot \lambda_{b,i},
\end{aligned}
\]
(10)

where \( h_b \) is the convection heat transfer coefficient between the fluid and the wall. The Prandtl number \( Pr_{b,i} \) is defined as,
\[
Pr_{b,i} = \mu_{b,i} \cdot C_p_{b,i} / \lambda_{b,i}
\]
(11)

In Equations (7) to (11), \( \rho_{b,i}, T_{b,i}, h_{b,i}, \mu_{b,i}, \) and \( C_p_{b,i} \) are the density, temperature, thermal conductivity, dynamic viscosity, and heat capacity respectively. In Modelica model, water properties are expressed as
\[
\begin{aligned}
\rho_{b,i} &= \rho_{b,i} (P_{b,i}, T_{b,i}) \\
\lambda_{b,i} &= \lambda_{b,i} (\rho_{b,i}, T_{b,i}, P_{b,i}) \\
\mu_{b,i} &= \mu_{b,i} (T_{b,i}, P_{b,i}) \\
C_p_{b,i} &= C_p_{b,i} (P_{b,i}, T_{b,i})
\end{aligned}
\]
(12)

The pressure \( P_{b,i} \) and the specific enthalpy \( h_{b,i} \) at the volume are given by
\[
\begin{aligned}
P_{b,i} &= \frac{P_{b,i} + P_{b,i+1}}{2}, \\
h_{b,i} &= \frac{h_{b,i} + h_{b,i+1}}{2}.
\end{aligned}
\]
(13)

Then the temperature at the volume is calculated from the tables of the water thermodynamic properties, and it is defined as
\[
T_{b,i} = T_{b,i} (P_{b,i}, h_{b,i})
\]
(14)

In Modelica model, water temperature \( T \) and density \( \rho \) are calculated from tables of water thermodynamic properties, depending on water pressure \( P \) and specific enthalpy \( h \). However, for most of the elements presented above, the calculation of pressure depends on water density, which introduces an algebraic loop in the calculation scheme.

**Modelling of Water/Water Heat Exchangers using Bond Graph Approach**

The bond graph model of this complex dynamic behavior and nonlinear system in thermofluids engineering allows to non-expert to get a better understanding of the model and to analyse the properties of this physical system [Karnopp et al. 1990, 2012]. In this approach, the specific enthalpy and some other properties of water are calculated from tables depending on pressure and temperature. The bond graph developed here has more degree of freedom compared with the Modelica model due to the consideration of the rate of mass accumulation within the volume.

The same finite volume method used in previous section is applied, where the heat exchanger has to be divided into \( n \) small successive heat exchangers, where each has uniform hot and cold temperature. Figure 4 shows the causal bond graph representation for one small heat exchanger. For modelling of each lump, the following elements were used

- **Multi-port C-element**, 

The multi-port C-element is used here for a thermodynamic accumulator, and is in its pseudo-bond graph form since the variables at its ports are not strictly power variables (see Figure 5). The thermodynamic accumulator has two degrees of freedom (2-DOF) since its volume is fixed here. The two state variables chosen are the water mass \( (m) \) for the fluid domain, and the specific enthalpy \( (h) \) for the thermal domain. Therefore, several modifications must be done compared to the Modelica model. The outputs from the multi-port C-element are the pressure and temperature, which are given by following relations:

\[
\text{Fluid Bond} \quad P_{b,i} = P_{b,i} (\rho_{b,i}, h_{b,i})
\]
(15)
The function of the water, as well detailed in reference [Wagner and Kretzschmar 2008] and the ThermoSysPro model. The function is calculated from the following one-dimensional energy equation

\[
\frac{dm_{b,i}}{dt} = \dot{m}_{b,i} - \dot{m}_{b,i+1}
\]

where \( m_{b,i} \) is the mass, \( \dot{m}_{b,i} \) and \( \dot{m}_{b,i+1} \) are respectively the control volume entering and leaving mass flow rates, as shown in Figure 5 by the dashed bonds.

To convert Equation (17) into a more useful form to obtain the density in the control volume, the following relationship is used

\[
\rho_{b,i} = \frac{m_{b,i}}{V_{b,i}}
\]

where \( V_{b,i} \) is the volume of the control volume.

Assuming a C-element with a constant volume, which leads to

\[
\frac{dP_{b,i}}{dt} = \frac{\dot{m}_{b,i} - \dot{m}_{b,i+1}}{V}
\]

In Equations (16) and (15), the specific enthalpy \( h_{b,i} \) is calculated from the following one-dimensional energy equation

\[
V_{b,i} \cdot \dot{P}_{b,i} \cdot \frac{dh_{b,i}}{dt} = \pm (h_{b,i} \cdot \dot{m}_{b,i} - h_{b,i+1} \cdot \dot{m}_{b,i+1} - W_t)
\]

- **Multi-port R-element**, the structure of the multiport pseudo-bond graph model R-element is defined in Figure 6.

\[
\dot{m}_{b,i} = \dot{m}_{b,i}(T_{b,i-1}, T_{b,i}, P_{b,i-1}, P_{b,i}), \quad q_{h_{b,i}} = q_{h_{b,i}}(T_{b,i-1}, T_{b,i}, P_{b,i-1}, P_{b,i})
\]

where the specific enthalpy flow rate \( q_{h_{b,i}} \) is represents the quantity \( m_{b,i}h_{b,i} \) shown in the Figure 6.

Figure 7: Comparison between the pressure difference

To calculate the mass flow rate \( \dot{m}_{b,i} \) as a function of \( \Delta P_{b,i} \) using the Equation 6 is quite difficult. Hence, to obtain an approximate solution, we ignore the second term \( 104.97 \cdot (Na_{b,i})^{-0.25} \) in the Equation 6 because its effect on \( \Delta P_{b,i} \) is negligible (see Figure 6). The pressure drop can be written as

\[
\Delta P_{b,i} = k_{b,i} \cdot Na_{b,i}^{a-2} \cdot \left( \frac{\dot{m}_{b,i}}{M} \right)^{2} - a
\]

where \( k_{b,i} = k_{b,i} \cdot Na_{b,i}^{a-2} \cdot M^{a-2} \). Thus, the mass flow rate \( \dot{m}_{b,i} \) can be calculated using the following formula

\[
\dot{m}_{b,i} = \exp \left( \frac{\ln(\Delta P_{b,i}) - \ln(k_{b,i})}{2 - a} \right)
\]

Figure 8 shows the comparison between both mass flow rates of the hot water in Dymola and MS1, in which calculated by Equation (23). The blue curve represents the mass flow rate obtained by the ThermoSysPro model,
In Figure 9, the 1-junction means that the rate of heat
is calculated in R-Element and materialized by the blue
color of water are given in terms of the pressure and the
thermodynamic properties of water is different compared
to Equation (12). Here, the thermodynamic properties
of water are given in terms of the pressure and the
temperature as following

\[
\begin{align*}
 b \dot{h}_b &= \rho_b, i \left( P_{b,i}, T_{b,i} \right) \\
 b \dot{h}_b &= \lambda_b, i \left( \rho_b, i, T_{b,i} \right) \\
 b \dot{\mu}_b &= \mu_b, i \left( P_{b,i}, T_{b,i} \right) \\
 b \dot{C}_p &= C_p b, i \left( P_{b,i}, T_{b,i} \right)
\end{align*}
\]  

(25)

where, the pressure \( P_{b,i} \) and the temperature \( T_{b,i} \) at the
multi-port R-element are given by

\[
\begin{align*}
 P_{b,i} &= \frac{P_{b,i-1} + P_{b,i}}{2} \\
 T_{b,i} &= \frac{T_{b,i-1} + T_{b,i}}{2}
\end{align*}
\]  

(26)

The functions in Equation (25) of the water properties
are based on the Industrial Formulation IAPWS-IF97 which consists of a set of equations for different water regions (more details are given in Wagner and Kretzschmar [2008]).

RESULTS AND DISCUSSION

The bond graph model of the exchanger has been vali-
dated by simulations. Figure 10 shows the bond graph
model of the PHE. The PHE model consists of total
(N = 5) numbers of plates, each layer being represented
as a small heat exchanger (shown in Figure 4). The
inputs of bond graph model are the pressure and the
temperature, while the outputs are the mass flow rate
and enthalpy flow rate for hot and cold parts.

The Modelica and the BG models were run for 400
seconds of simulation time. Figure 11 shows the pres-
sures, where the black curves represent the pressure at
the boundaries, the blue curves represent the pressure
at each volume in the ThermoSysPro model, and the
dashed green curves represent the pressure at each
volume (C-element) in the BG model. Figure 12 shows
the error between obtained pressures (Dymola and MS1)
at first volume. From the comparison of the simulation
results, the conclusion that can be made is that the both
models have similar dynamic behavior.

CONCLUSION

In this paper, a multi-port pseudo BG model of a plate
heat exchanger system has been presented. The model
can be used in transient system simulations and can be
extended to cover other heat exchanger types. The
comparison of the simulation results of bond graph
model with the Modelica model indicates that the model
predicts the dynamic behavior of the heat exchanger
well.

Future developments will include developing bond graph
models of centrifugal pump, regulating valve, feeding
on-off valve, and pipes for nuclear power plants. The
main objective is to study the observability, initial
conditions, structural inversibility by physical structural
analysis to improve systems diagnosis and operation.
Figure 10: Pseudo bond graph of the plate heat exchanger (counterflow)

Figure 11: Comparison of bond graph model with Dymola model in hot flow

Figure 12: Error between obtained pressures (Dymola and MS1) at first volume

APPENDIX

The following algorithm has been developed for pressure calculation in the C-accumulator of the heat exchanger. Based on the density function it was developed to determine water pressure in terms of specific enthalpy and density. The structure of the algorithm is shown in Algorithm 1. This is a very fast and accurate method in all regions except solid region of the water. This algorithm is used because, as far as we know, there are no tables to calculate the water pressure directly in terms of specific enthalpy and density. In the solid region, the calculation of the water pressure is based on incompressibility consideration.

Data: $\rho, h$

- $p_{\text{min}} = 0.00611657$;
- $p_{\text{max}} = 1000$;
- $p_s = 100$;
- $\rho_s = \rho_s(p_s, h)$

while $|\rho - \rho_s| > 1E^{-7}$ do
  $\rho_s = \rho_s(p_s, h)$
  if $\rho_s \geq \rho$ then
    $p_{\text{max}} = p_s$
  else
    $p_{\text{min}} = p_s$
  end
  $p_s = (p_{\text{min}} + p_{\text{max}})/2$
end

$p = p_s$;

Algorithm 1: Calculation of the water pressure using the density function

Figure 13 shows how the thermodynamic property of the water pressure is very sensitive to small changes in density of the water.
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NOMENCLATURE

\[
\begin{align*}
\lambda_m & : \text{Metal thermal conductivity} \\
\dot{m} & : \text{Mass flow rate} \\
\Delta S & : \text{Heat transfer surface} \\
L & : \text{Heat exchanger length} \\
\eta & : \text{Number of plates} \\
\mu & : \text{Dynamic viscosity} \\
es & : \text{Metal wall thickness} \\
K_i & : \text{Heat transfer coefficient} \\
N & : \text{Number of segments} \\
\end{align*}
\]

Chosen abbreviations

- PHE: Plate Heat Exchanger
- BG: Bond Graph
- FVM: Finite-Volume Method
- MB: Moving Boundaries

Subscripts

- \( I \) in the volume \( I \) & \( b = c \): Cold side
- \( i \) entering the volume \( I \) & \( b = h \): Hot side

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