

# MODELLING A PCT40 HEAT EXCHANGER FOR CONTROL PURPOSES

Frantisek Gazdos and Daniel Macek  
Faculty of Applied Informatics  
Tomas Bata University in Zlin  
Nam. T. G. Masaryka 5555, 760 01 Zlin, Czech Republic  
E-mail: gazdos@fai.utb.cz

## KEYWORDS

Modelling, Simulation, Heat exchanger, PCT40.

## ABSTRACT

This paper presents a simulation model of a PCT40 heat exchanger. It starts with a motivation, followed by description of the system and of the whole modelling process. The mathematical model includes simplified description of heating and cooling systems together with a more detailed description of a proportioning solenoid valve. It is formed using combination of both analytical and empirical approaches and the resultant simulation model is compared to real-time measurements in terms of both static and dynamic properties. The overall results indicate that although the model is relatively simple it can be used for control-oriented simulation experiments, which was the purpose of this work and saves time during experiments on this system.

## INTRODUCTION

Modelling and simulation courses are essential parts of education process in the field of control engineering. Students – prospective control experts have to be able to model a given system to be controlled in order to design a model-based controller. Prior to real-time implementation of the proposed controller it is also common nowadays to perform simulation testing to ensure the proposed control system is effective and safe at the same time. In order to prepare students for industrial practice properly the courses always include a form of laboratory exercises where students test their knowledge of control field on laboratory-scale industrial processes. These labs are usually included in the follow-up studies and students face real control problems here – from system identification, controller design to real-time implementation and tuning on different plants related to industrial practice. Similarly, during studies of Automatic Control & Informatics Master's degree course at the Faculty of Applied Informatics, Tomas Bata University in Zlin (FAI TBU in Zlin 2016), students have to complete the course “Real Process Control”. Here students work in the Department of Process Control laboratory with various laboratory-scale processes to prove their understanding of control engineering. In this lab there is a wide range of processes from different producers to ensure students test many different types of systems. For instance,

TecEquipment models (TecEquipment 2016) include CE107 Engine Speed Control Apparatus, CE108 Coupled Drives Apparatus, CE120 Controller, CE150 Helicopter Model, CE151 Ball and Plate Apparatus and CE152 Magnetic Levitation Model. AMIRA models (Amira 2016) consists of DTS200 Three-Tank-System, DR300 Speed Control with Variable Load and PS600 Inverted Pendulum. Then there are also other models from Feedback (Feedback 2016): 33-007-PCI Twin Rotor MIMO System, from Leybold (LD Didactic 2016): T 8.2.1.4 Gas Flow Control and from Armfield (Armfield 2016): PCT40 Multifunction Process Control Teaching System. While some of the processes are fast, e.g. the magnetic levitation model, coupled drives apparatus or speed control system – experiments with these models are relatively quick with responses in seconds, others can be really slow with responses in tens of minutes or even hours. These include e.g. the three-tank-system or a heat-exchanger from the multifunction process control teaching unit. In this case students have to wait even hours to complete their experiment. Therefore it is very useful for them to have simulation models of these systems that capture their main properties so that students or other faculty staff can test their control algorithms by simulation means, prior to real-time implementation. Simulation experiments with these models are much faster with responses in seconds which leads to more effective work. For instance, a simulation model of the three-tank-system was developed and presented in (Chalupa et al. 2012). Modelling of the PCT40 Heat Exchanger has been the subject of student's Master's thesis (Macek 2015) and it is described briefly in this contribution, including also main results. The modelling principles employed here follow the basic guidelines presented in the modelling classics, e.g. (Wellstead 1979; Ljung and Torkel 1994; Severance 2001). A simplified, first-principles mathematical model is derived analytically with parameters further tuned using real-time experiments.

The paper is structured as follows: after this introductory section it continues by a description of the PCT40 heat exchanger including also experimental conditions and main variables. The main body includes modelling of the basic process parts – heating/cooling systems and proportioning solenoid valve (PSV). Last section compares the developed model to real-time data and the paper concludes summarizing main results and suggesting future possible directions of development.

## PCT40 HEAT EXCHANGER

The heat exchanger modelled in this paper is a part of the PCT40 multifunction process control teaching system (Armfield 2016). This unit is designed for use in teaching a wide range of control methods and to demonstrate a variety of process control loops. Processes such as level, temperature, flow or pressure control can be easily implemented using the provided interface, multifunction I/O card and suitable software. More advanced aspects of control can be addressed by adding optional extras to the basic system, such as fluid property control (conductivity and pH probe), remote set points or dual loops (Armfield 2005). Illustrative overview of the unit is presented in Fig. 1. The heat exchanger is located on the right side and it is further depicted schematically in more detail in Fig. 2.



Figure 1: Basic Process Control Unit PCT40 with Process Vessel Accessory

### Heat Exchanger Description

It consists of a small clear acrylic vessel that incorporates an electrical element for heating water. A thermostat and level detector are incorporated in the vessel to prevent the heater from operating if the water is too hot ( $>80\text{ }^{\circ}\text{C}$ ) or the level in the vessel is too low. The vessel lid provides support for the stainless steel heating/cooling coil. Fittings at the inlet and outlet of the coil accommodate thermocouple-type temperature sensors and allow connection to the water supply. The lid also accommodates adjustable glands for a variable-height thermocouple sensor, a thermometer pocket and a temperature switch (thermostat) to be inserted into the vessel for the purpose of calibration. The main water inlet is connected to a pressure regulating valve with integral filter. The flow of water through the equipment can be varied by adjusting the setting of the regulator. A turbine type flow meter is fitted in series with the main water inlet to allow inlet flow rate measurement. It has a range from 0.2 to 1.5 litres/minute approximately. A proportioning solenoid valve (PSV) is also located near the main water inlet to enable continuous regulation of the inlet flow rate. The heater power can be controlled by SSR (Solid State Relay) drive in on/off sense only

with nominal power 2 kW. Three temperature sensors – k-type thermocouple probes ( $0\text{--}200\text{ }^{\circ}\text{C}$ ) are incorporated within the vessel. The first one is used to measure the fluid temperature inside the vessel while the two remaining located at the inlet/outlet of the heating/cooling coil are used to measure the fluid temperature as it enters and leaves the coil (Armfield 2005).

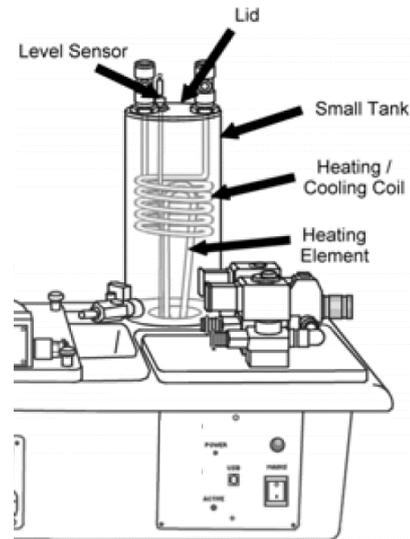


Figure 2: Heat Exchanger Schema

### Experimental Conditions

Within the “Real Process Control” course during experiments on this unit in the Department of Process Control laboratory, Faculty of Applied Informatics, Tomas Bata University in Zlin, students have a special set-up of this system. First the vessel is filled with water to a certain level to ensure that both heating element and heating/cooling coil are under water. The level stays the same during the experiments. The coil is used for cooling only and so it is connected to cold water inlet using the pressure regulating valve, proportioning solenoid valve and the flow meter. The inlet flow rate is set to 1500 ml/min approx. using the pressure regulating valve and then it can be controlled continuously using the PSV valve via the I/O card and MATLAB/Simulink software. Then students perform various experiments including e.g. measurements of the static properties of the PSV valve, pulse-width modulation (PWM) of the heater power for temperature control, identification of heating/temperature and cooling/temperature systems, temperature control using heating and cooling, etc.

### Main Variables for Modelling and Control

From the control theory point of view the main usual output variable to be controlled is the temperature inside the vessel  $T(t)$  [ $^{\circ}\text{C}$ ], measured via the k-type thermocouple sensor. Control inputs (manipulated variables) used for the temperature control are usually heater power  $P(t)$  [%] (regulated using PWM) and water flow rate through the cooling coil  $q_c(t)$  [l/min]

(measured by the turbine type flow meter and controlled using PSV). Therefore it can be generally seen as a MIMO system with 2 inputs and 1 output. For modelling purposes the main state-variables were selected as: water level in the vessel  $h(t)$ , temperature inside  $T(t)$  and (output) temperature of the cooling water  $T_C(t)$ .

### MATHEMATICAL MODEL

The modelling goal was to develop a relatively simple mathematical model for control purposes that captures main static and dynamic properties of the system rather than deriving a comprehensive mathematical model. Therefore the heat-exchanger is modelled as a lumped-parameters system using a set of ordinary differential equations (ODEs) only. The presented model consists of two main parts – model of the PSV valve and model of the heat exchanger that can be also divided into two interconnected systems – heating and cooling models.

#### Model of Proportioning Solenoid Valve

This regulating valve has specific static and dynamic properties which has been measured and identified experimentally and then modelled mathematically. First, a static characteristics has been measured – during repeated experiments the valve was opening gradually and the resultant flow rate was recorded, and the same for the valve closing. Repeated experiments were averaged. The results are presented in Fig. 3 below.

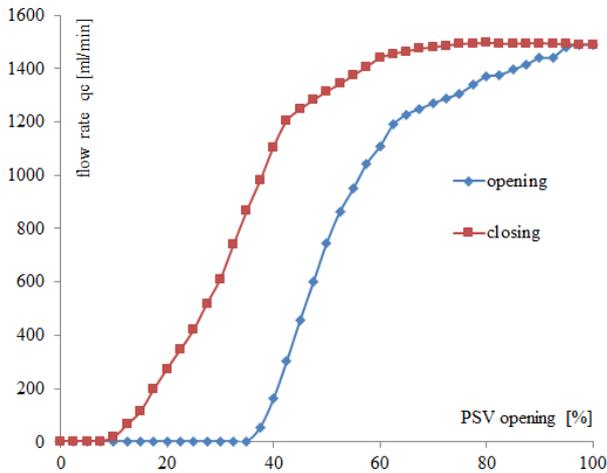


Figure 3: PSV – Static Properties

From the graph it is clear that the valve has a hysteresis and that suitable (nearly linear) area for control is between 40 – 60 [%] for opening and 20 – 40 [%] for closing approximately. The opening and closing curves were approximated using the polynomial regression of 5<sup>th</sup> degree and consequently the resultant equations are of the form (1) for opening and (2) for closing.

$$q_{c/o} = -1.754 \times 10^{-5} \cdot o^5 + 0.0061 \cdot o^4 - 0.83 \cdot o^3 + 53.23 \cdot o^2 - 1577.6 \cdot o + 17234 \quad (1)$$

$$q_{c/c} = -1.43 \times 10^{-6} \cdot o^5 + 5.21 \times 10^{-4} \cdot o^4 - 0.07 \cdot o^3 + 3.43 \cdot o^2 - 34.91 \cdot o + 62.31, \quad (2)$$

where the variable  $o$  [%] describes the opening/closing rate. The approximation is presented in Fig. 4 and shows relatively good fit of both curves to the measured data.

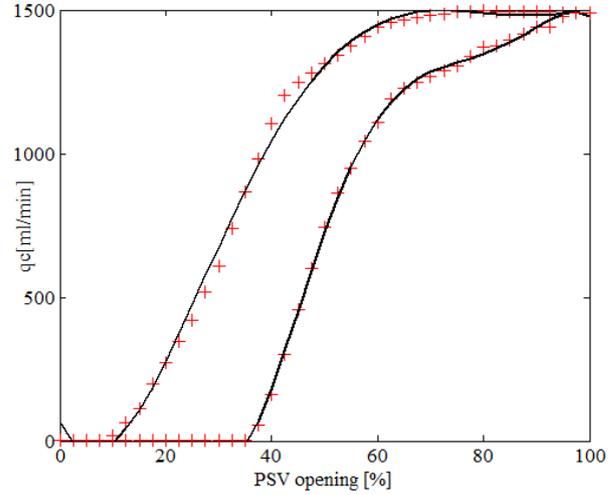


Figure 4: PSV – Approximation of Opening/Closing

Dynamical properties were measured as step-responses in different operating points for both opening and closing. As the responses were relatively similar the results were averaged, normed and identified as a first-order system with a time-constant  $T = 0.33$  [s] approximately, i.e. with a transfer function:

$$G(s) = \frac{1}{0.33s + 1}. \quad (3)$$

Comparison of this model for both opening and closing responses are given in the following graphs, Fig.5 and Fig.6. The results confirm good fit of this simple model.

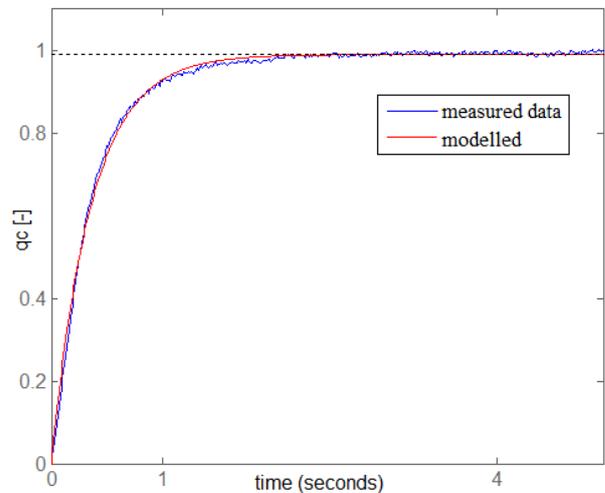


Figure 5: PSV – Dynamical Approximation of Opening

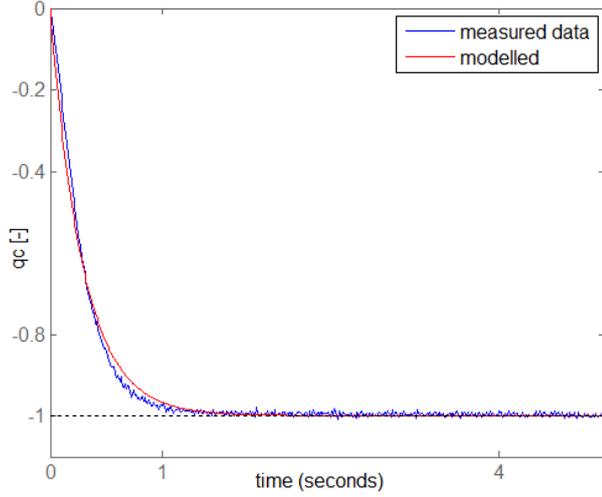


Figure 6: PSV – Dynamical Approximation of Closing

Both static and dynamic parts of this model were combined in the MATLAB/Simulink to form a simplified simulation model of the PSV. Input into the model is the value of valve opening in [%] and the model gives flow rate as the output variable in [ml/min]. There are also several auxiliary functions and blocks to detect opening/closing and for the hysteresis.

### Model of Heat Exchanger

As it has been explained before, the mathematical model of the heat exchanger can be divided into two main interconnected parts – the water filled vessel with a heating element and the coil with cooling water. Schematic picture of the heat exchanger is presented in the following figure (Fig. 7). In the picture,  $q_V$  describes the input flow rate into the vessel of temperature  $T_V$ , output flow rate is denoted as  $q$  with temperature  $T$ . Water volume  $V$  in the tank is function of water level  $h$  and the tank diameter  $D$ . Heater power is denoted as  $P$  and its efficiency as  $\eta$ . Heat transfer coefficients are described using symbols  $\alpha$  (heated water  $\leftrightarrow$  surroundings with temperature  $T_o$ ) and  $\alpha_c$  (heated water  $\leftrightarrow$  cooling water). Input/output flow rate of cooling water is denoted as  $q_c$  with input and output temperatures  $T_{CV}$ ,  $T_C$ , respectively. Cooling water volume describes variable  $V_C$ . As it has been explained earlier in the paper the main state-variables for modelling were selected as: water level in the tank  $h(t)$ , temperature inside  $T(t)$  and (output) temperature of the cooling water  $T_C(t)$ . Consequently the model is described by 3 basic equations derived using mass and energy balances of the whole system. The following simplified assumptions were taken into account for the modelling purposes: ideal mixing of both heated and cooling liquid, heat capacity of the vessel and cooling coil walls is neglected and all the following variables remain constant during the experiment: heat transfer coefficients  $\alpha$  and  $\alpha_c$ , densities of both fluids  $\rho$  and  $\rho_c$  and the same for their heat capacities  $c_P$ ,  $c_{PC}$ .

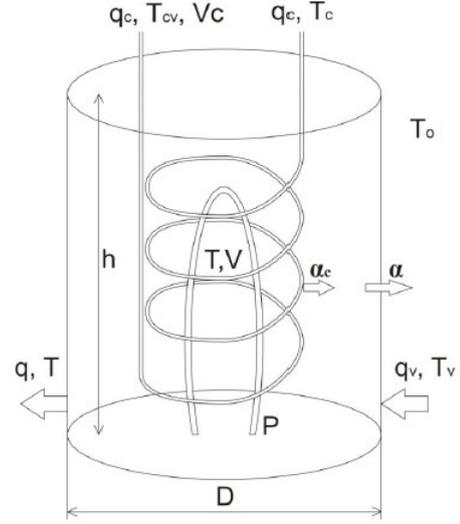


Figure 7: Heat Exchanger Main Variables

Material balance of heated water can be described by:

$$q_V(t) = q(t) + \frac{dV(t)}{dt}, \quad (4)$$

where the output flow rate can be modelled simply as:

$$q(t) = k\sqrt{h(t)} \quad (5)$$

for some constant  $k$ . The volume can be further described using the water level and cross-section area  $F$  as  $V = Fh(t)$  while ideally  $F = \pi D^2/4$ . In fact, the cross-section area  $D$  is smaller as there is a heating element and cooling coil inside the vessel. Therefore it was calculated experimentally for several volumes from the equation above and averaged. The result is  $F = 0.00705 \text{ m}^2$  approx. (while ideally it can be easily calculated using its diameter  $D = 0.1 \text{ m}$  as  $F_{IDEAL} = 0.00785 \text{ m}^2$ ). For the model, the averaged value of  $F$  was further used. Then the resultant ODE for heated water balance reads:

$$q_V(t) = k\sqrt{h(t)} + F \frac{dh(t)}{dt} \quad (6)$$

for some initial (steady-state) water level  $h(0) = h^S$ . Energy balance of the heated water provides:

$$q_V \rho_c c_P T_V + \eta P(t) = q \rho c_P T(t) + \alpha F_o [T(t) - T_o(t)] + \alpha_c F_c [T(t) - T_C(t)] + V \rho c_p \frac{dT(t)}{dt} \quad (7)$$

for some initial temperature  $T(0) = T^S$ , while the simplified energy balance of the cooling water gives:

$$q_c(t) \rho_c c_{PC} T_{CV} + \alpha_c F_c [T(t) - T_C(t)] = q_c(t) \rho_c c_{PC} T_C(t) + V_C \rho_c c_{PC} \frac{dT_C(t)}{dt} \quad (8)$$

for some initial cooling water temperature  $T_C(0) = T_C^S$ . In the equation above, the volume of cooling water was measured experimentally as  $V_C = 30 \text{ ml} = 0.00003 \text{ m}^3$  approx. It is obvious that heat transfer surface areas  $F_O$  (heated water  $\leftrightarrow$  surroundings) and  $F_C$  (cooling water  $\leftrightarrow$  heated water) are functions of the water level  $h(t)$ , i.e.  $F_O = \pi D h(t)$  and  $F_C = F_C(h)$  generally (this constant is approximated further using experiments). Then the basic set of ODEs describing the heat exchanger in a suitable form for numerical solution can be expressed as:

$$\begin{aligned} \frac{dh(t)}{dt} &= \frac{q_V}{F} - \frac{k\sqrt{h(t)}}{F}, \\ \frac{dT(t)}{dt} &= \frac{q_V T_V}{Fh(t)} + \frac{\eta P(t)}{F\rho c_p h(t)} - \frac{qT(t)}{Fh(t)} - \frac{\alpha\pi D}{F\rho c_p} T(t) \\ &+ \frac{\alpha\pi D T_O}{F\rho c_p} - \frac{\alpha_c F_C(h)T(t)}{F\rho c_p h(t)} + \frac{\alpha_c F_C(h)T_C(t)}{F\rho c_p h(t)}, \quad (9) \\ \frac{dT_C(t)}{dt} &= \frac{T_{CV}}{V_C} q_C(t) + \frac{\alpha_c F_C(h)}{V_C \rho_C c_{PC}} T(t) \\ &- \frac{\alpha_c F_C(h)}{V_C \rho_C c_{PC}} T_C(t) - \frac{1}{V_C} q_C(t) T_C(t), \end{aligned}$$

where the input, state and output variables are denoted as time-dependent. These equations represent a model of dynamics and consequently are described by a set of ordinary differential equations. Steady-state models are also useful as they can be used to assess static properties, e.g. present nonlinearities, suitable working areas, or they can be used to identify unknown parameters experimentally. The same idea was used here to estimate the parameters  $k$ ,  $F_C$ ,  $\alpha$  and  $\alpha_c$ . First, a steady-states model of (9) was obtained by putting all time-derivatives equal to zero and denoting the time-dependent variables as steady using the superscript “s”:

$$\begin{aligned} q_V^S &= q^S = k\sqrt{h^S} \\ \eta P^S &= \alpha F_O (T^S - T_O^S) + \alpha_c F_C(h^S) (T^S - T_C^S) \quad (10) \\ q_C^S \rho_C c_{PC} T_{CV} + \alpha_c F_C(h^S) (T^S - T_C^S) &= q_C^S \rho_C c_{PC} T_C^S \end{aligned}$$

The first equation can be used to identify the valve constant  $k$  experimentally. For several steady input flow rates  $q_V^S$  the steady-state water level in the vessel  $h^S$  was measured and averaged with the following approximate result:  $k = 0.128 \text{ m}^{5/2}/\text{s}$ . The second and third equations were used to estimate experimentally the heat transfer coefficient  $\alpha$ , and the term  $\alpha_c F_C$  which is a function of the water level  $h^S$ . Several repeated experiments were performed in usual operating points. The results were averaged and processed further to estimate the unknown parameters. The resultant approximation of  $\alpha$  was  $\alpha = 13.9 \text{ J.m}^{-2}.\text{K}^{-1}.\text{s}^{-1}$ . The other unknown term  $\alpha_c F_C$  was obtained as a function of the water level in the form of the following graph, Fig. 8, which has been approximated by a polynomial regression.

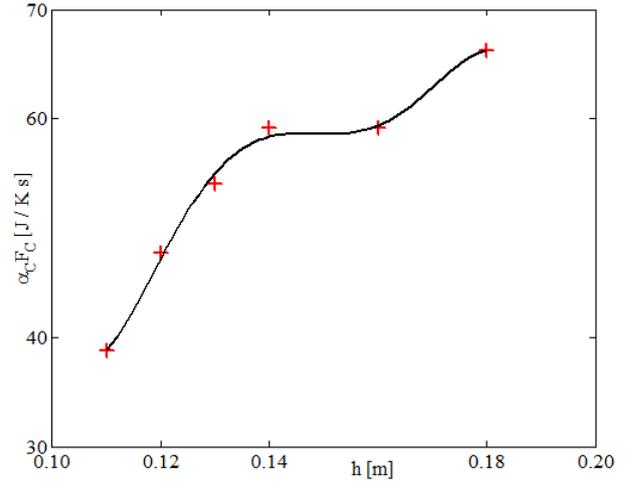


Figure 8: Approximation of the Term  $\alpha_c F_C$

The resultant regression equation reads:

$$\begin{aligned} \alpha_c F_C &= -292269698h^5 + 213321855h^4 \\ &- 61608458h^3 + 8792620h^2 - 619260h + 17242. \quad (11) \end{aligned}$$

This equation was further used in the resultant simulation model which was implemented in the MATLAB/Simulink environment and it is formed using the PSV model, described earlier in the paper, together with the heat exchanger model explained in this section.

## SIMULATION RESULTS

The developed model was tested in various experimental conditions usual in the course. The tests comprised both open and closed-loop experiments for which suitable controllers were proposed. All the main experiments, linearization, control-oriented analysis and design together with corresponding results are included in (Macek 2015). Here, due to the limited space, only some of them are presented. Comparison of open-loop responses of the model and real-time measurements on the PSV are presented in Fig. 9 and Fig. 10 where the latter one is zoomed in to enable closer inspection.

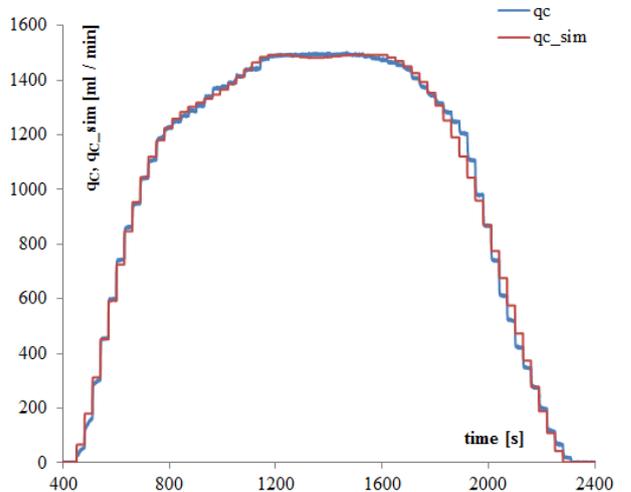


Figure 9: PSV – Open-Loop Comparison of Responses

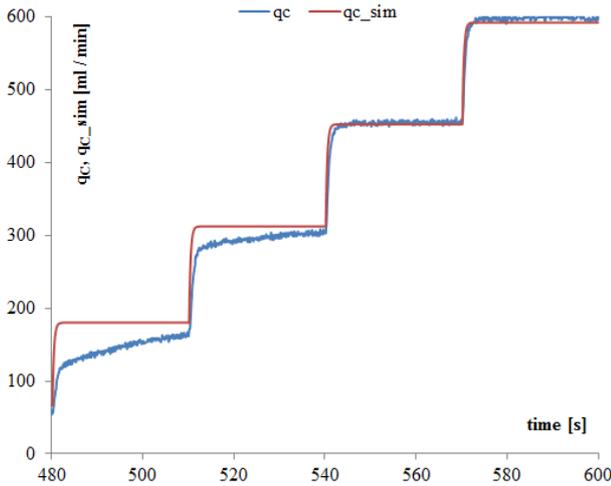


Figure 10: PSV – Detailed Open-Loop Comparison

Overall behaviour during the measurements of the static characteristics presented in Fig. 9 shows relatively good static approximation of the valve. Detailed inspection of the responses in Fig 10 reveals lower accuracy in the dynamic part of the model, especially in the lower flow-rates, which is given by the only one (averaged) simple approximation of the dynamics (3) in the whole working range of the valve. Closed-loop responses obtained using an auxiliary PI-controller are presented in Fig. 11.

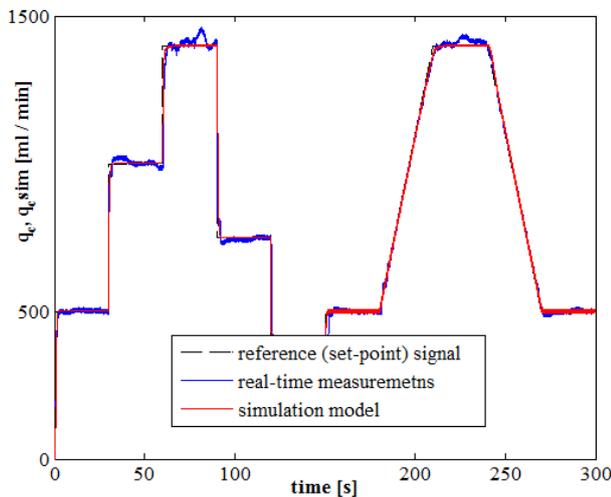


Figure 11: PSV – Closed-Loop Comparison

Here, it is clear that the model approximates the real valve in the closed loop system well and can be further used for simulation experiments. The heat exchanger model presented in the previous section has been also tested properly in both open and closed-loop settings. Several open-loop responses are presented further, other (especially closed-loop ones) are beyond the space limit of this contribution and can be found in (Macek 2015). Next graphs present open-loop dynamic responses for the following usual experimental conditions:  $h = 20 \text{ cm}$ ,  $q_C = 1200 \text{ ml/min}$  and  $P = 5 \%$ , in different regimes. The first one (Fig. 12) presents behaviour without any heating and cooling, i.e. it is a process of heating the

water inside the vessel to ambient temperature  $T_O$ . From the graph it is clear how the process can be slow – with responses in hours. The second graph, Fig. 13, present the case of heating with prescribed power ( $P = 5 \%$ ).

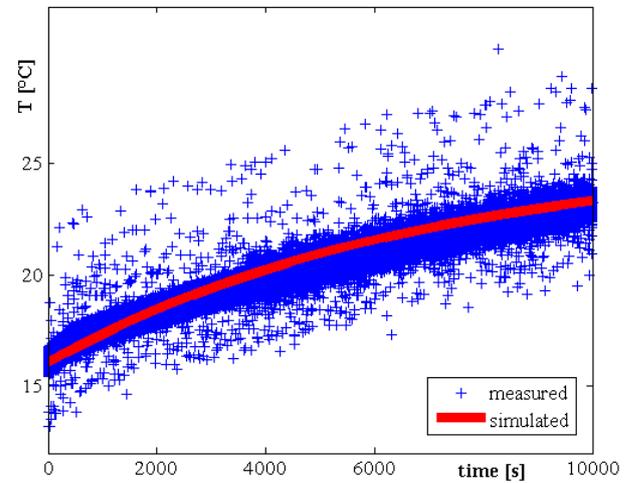


Figure 12: Heat Exchanger – Open-Loop Comparison of Responses (Heating & Cooling OFF)

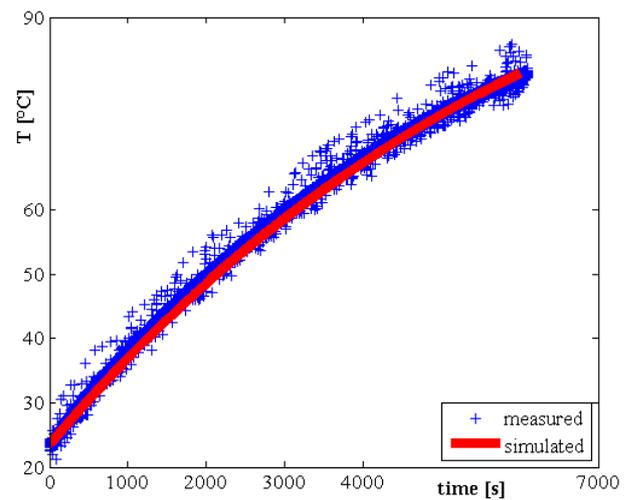


Figure 13: Heat Exchanger – Open-Loop Comparison of Responses (Heating ON)

Both responses reveal good model tracking of the real-time data. Next responses present behaviour in the cooling case – Fig. 14 for cooling only and Fig. 15 for simultaneous heating and cooling. The cooling comparison (Fig. 14) reveals a small acceptable mismatch in the simulation model for cooling. The last graph, Fig. 15, shows high noise ratio during the measurements which has led to the need of data filtration which has been implemented simply using a first order low-pass filter.

The overall results indicate that the developed simulation model, although simple, can be successfully used for simulation experiments comparable to real-time measurements when there is no need for high accuracy in terms of absolute values, which is the common case of control-oriented simulation.

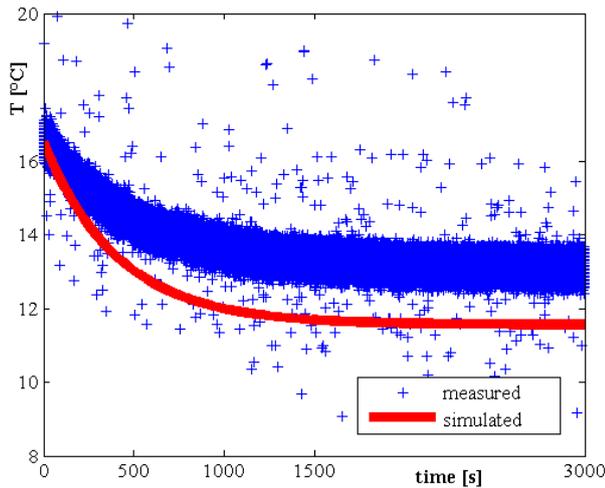


Figure 14: Heat Exchanger – Open-Loop Comparison of Responses (Cooling ON)

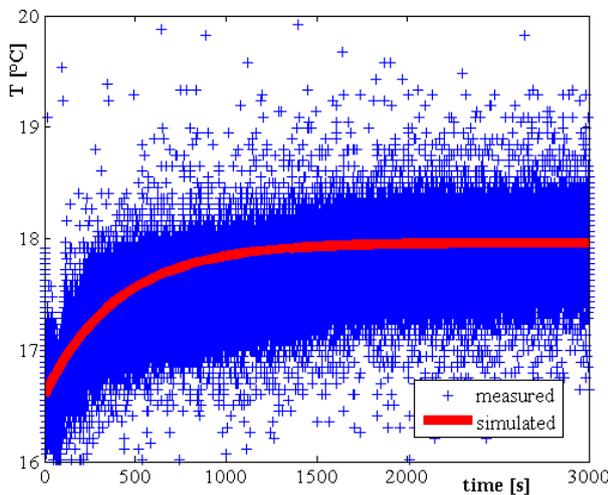


Figure 15: Heat Exchanger – Open-Loop Comparison of Responses (Heating & Cooling ON)

## CONCLUSIONS

This paper has presented a simulation model of the PCT40 heat exchanger. It was developed using both analytical and experimental methods – the basic model structure was obtained analytically and its unknown parameters were identified using repeated real-time measurements and corresponding data processing. The resultant model, though relatively simple, exhibits good fit in terms of main dynamic properties, which is sufficient for the outlined purposes, i.e. control-oriented simulation. As a result, both students and teachers working in the laboratory can save time during their experiments – they can easily and quickly test their control algorithms prior to the real-time implementation. The whole system could be, of course, modelled more precisely, especially the cooling part, resulting in a distributed-parameters system described by partial differential equations (PDEs). However, this was not the goal of the presented modelling & simulation study and

can be a theme for further work, comparing the presented simplified model with the more precise, distributed one.

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## AUTHOR BIOGRAPHIES



**FRANTIŠEK GAZDOŠ** was born in Zlín, Czech Republic in 1976, and graduated from the Brno University of Technology in 1999 with MSc. degree in Automation. He then followed studies of Technical Cybernetics at Tomas Bata

University in Zlín, obtaining Ph.D. degree in 2004. He became Associate Professor for Machine and Process Control in 2012 and now works in the Department of Process Control, Faculty of Applied Informatics of Tomas Bata University in Zlín, Czech Republic.

He is author or co-author of more than 80 journal contributions and conference papers giving lectures at foreign universities, such as Politecnico di Milano, University of Strathclyde Glasgow, Universidade Técnica de Lisboa and others. His research cover the area of process modelling, simulation and control. His e-mail address is: [gazdos@fai.utb.cz](mailto:gazdos@fai.utb.cz).