DOMINANT SPECTRUM ASSIGNMENT FOR NEUTRAL TIME DELAY SYSTEMS: A STUDY CASE

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KEYWORDS

ABSTRACT
This paper is aimed at the presentation of a suboptimal numerical algorithm that has been designed for the assignment of a finite dominant part of the infinite eigenvalue spectrum of a strongly stable neutral time delay control system. Once a structure of a conventional finite-dimensional controller for a delayed controlled plant is suggested, desired loci of a sufficient number of feedback poles are selected, according to required dynamical properties. The goal is to bring the dominant spectrum to prescribed ones as close as possible. The first step of the procedure insists in the use of the Quasi-Continuous Shifting Algorithm (QCSA). Then, the results are enhanced by an optimization procedure for a suitable objective function. The presented methodology is demonstrated by a simulation example of control of an unstable time delay system (TDS) in the MATLAB®/Simulink® environment. Some possibilities how to adjust the algorithm are given to the reader as well.

INTRODUCTION
Although there have been derived and designed various unconventional ad-hoc control methods and strategies for linear time-invariant time delay systems (TDS) in the scientific literature during the early years of this millennium, see e.g. (Chiasson and Loiseau 2012; Richard 2003; Sipahi et al. 2012), the use of conventional proportional-integral-derivative (PID) controllers still plays a crucial role in modern control theory despite the made progress and advances (Åström and Hägglund 2006; Wang et al. 2009; Zítek et al. 2013).

If, however, a finite-dimensional controller is applied to a TDS plant, an infinite-dimensional control feedback system is obtained. This feature is characterized by the fact that the eventual characteristic quasipolynomial instead of a polynomial (the zeros of which usually agree with system poles) includes exponential terms. Simply, a PID law can not cancel delays in the feedback loop. In such cases, the task of controller tuning yields the problem of a suitable setting of a finite number of adjustable controller parameters faced with the infinite spectrum of system poles. There is a natural effort to develop tuning procedures for the aforementioned class of systems which are usable and understandable also for non-experts without an excessive mathematical formulation. A possible way is to shape the dominant feedback spectrum by means of pole placement controller parameters tuning principles.

A one-shot or direct pole assignment for controllable TDS has been presented e.g. in (Lee and Zak 1982; Zítek and Vyhlidal 2002). A more advanced idea is based on successive shifting the dominant poles to the left (stable) complex half-plane by using the Quasi-Continuous Shifting Algorithm (QCSA) (Michiels et al. 2002; Michiels and Vyhlidal 2005), or other methods (Michiels and Gumussoy 2014; Vyhlidal 2003). However, all these methods intend to minimize the spectral abscissa only. The pole-matching problem for retarded TDS operating in the state-space has been solved in (Michiels et al. 2010) where poles can not leave the prescribed positions and the unrestrained rest of the spectrum is attempted to be pushed to the left, which may results in a lengthy trial-and-error placing procedure.

In (Pekař and Navrátil 2014), we introduced an algorithm called the PPSA (Pole-Placement Shifting based controller tuning Algorithm) for retarded TDS where both poles and zeros are selected according to desired closed-loop dynamic properties represented by the finite-dimensional model. During the shifting procedure minimizing both spectral abscissas as a secondary objective function, poles and zeros can leave their prescribed positions but remain in their vicinities.

In this paper, ideas and methodology of the PPSA are applied to dominant low-frequency pole assignment in input-output neutral TDS model formulation. Neutral TDS spectral properties are more advanced, tricky and intricate compared to retarded ones, i.e. the so-called strongly stable system are to be reached. First, dominant poles are forced to move towards the prescribed positions. Then, the objective function reflecting the distance of prescribed poles from the actual ones and the abscissa of the rest of the spectrum is minimized by means of the Nelder-Mead technique (Nelder and Mead 1965). The whole procedure is simply implementable in standard program languages.
A detailed simulation example performed in the MATLAB®/Simulink® environment provides the reader with the procedure demonstration and performance verification.

PRELIMINARIES
Neutral TDS Spectral Properties
Let basic spectral and exponential stability properties of retarded and neutral TDS be introduced first. Consider a single-input single-output (SISO) TDS governed by the following transfer function

\[ G(s) = \frac{N(s)}{D(s)} \]  

where \( N(s), \) \( D(s) \) are quasipolynomials of the general form

\[ s^k \sum a_{ij}^k \prod_{l=1}^k (s^l - \lambda_l)^{\tau_l} \]  

in which \( \tau_l \) represent independent delays, \( \lambda_l \) and \( a_{ij} \) be the associated exponential polynomial related to \( s^X \). If \( s^D \), \( s^N \) are quasipolynomials of the general form,

Assumption 1. Assume that

\[ \{ s^D \} \cap \{ s^N \} = \emptyset \]

that is, there are no common roots of \( s^D \), \( s^N \).

Under Assumption 1, the roots of \( s^D \) coincide with system poles. For their spectrum, it holds the following properties (Hale and Verduyn Lunel 1993; Michiels and Niculescu 2007).

Property 1. For system (1) of neutral type it holds that:

1. If there exists a nonzero pair \( \{ t_{ij}, \lambda_{ij} \} \) for some \( t_{ij} > 0 \) and some \( i,j \), then \( \| \lambda \| = \infty \).
2. There exists a vertical chain of poles, \( s_k \), at \( \gamma = \sup \text{Re} \Sigma_a \) such that \( \lim_{k \to \infty} \text{Re} s_k = \gamma \), \( \lim_{k \to \infty} \text{Im} s_k = \infty \) where \( \Sigma_a \) is the zero set of \( X_a(s) \).
3. Isolated poles behave continuously and smoothly with respect to \( \tau \) on \( C \).

Neutral TDS Stability
Among many approaches to stability of neutral TDS, exponential and strong ones are matters of this contribution.

Proposition 1 (Michiels and Vyhlidal 2005). Neutral system (1) is exponentially stable if \( \alpha(p) < -\varepsilon, \varepsilon > 0 \).

Whereas the notion of exponential stability is well-known, the concept of strong stability is much more unfamiliar to researchers and engineers. It expresses the ability of \( \Sigma_a \) to persist in the left (stable) half-plane under small delay perturbations.

Definition 1 (Hale and Verduyn Lunel 1993; Michiels and Niculescu 2007; Michiels and Vyhlidal 2005). Neutral system (1) is said to be strongly stable if

\[ \tilde{\gamma} = \sup \{ \gamma(\tau + \delta \tau) | | \delta \| < \varepsilon \} < 0 \]

for any sufficiently small \( \varepsilon > 0 \).

Property 3 (Michiels and Vyhlidal 2005). Number of poles with \( \text{Re} s_k > \tilde{\gamma} + \varepsilon \) for a sufficiently small \( \varepsilon > 0 \) is always finite and they are isolated.

Proposition 2 (Vyhlidal 2003). The system is strongly stable if

\[ \sum_{j=1}^r | h_j | < 1 \]  

When achieving exponential stability it is desirable for practical reasons to satisfy strong stability as well.

DOMINANT SPECTRUM ASSIGNMENT ALGORITHM
Consider the closed-loop control feedback system governed by the transfer function (1) with the denominator including the number of \( r > 0 \) selectable parameters

\[ K = (K_1, K_2, \ldots, K_r) \neq 0 \in \mathbb{R}^r \]

hence, \( D(s) = D(s, K) \), and let Assumption 1 hold hereinafter. The designed algorithm framework solving the pole placement matching problem for neutral TDS can be summarized as follows.
Algorithm 1.
1. It is given the feedback denominator \( D(s, K) \) as the input.
2. If \( D_n(s, K \neq 0) \) and condition (2) does not hold, abandon the algorithm; else, select the number of \( n < r \) poles and their loci according to desired feedback dynamics.
3. Place a subset of poles to prescribed positions by using a direct pole placement methodology. The initial setting \( K_0 \) is obtained.
4. If the placed spectrum is the rightmost (dominant) within the selected range of frequencies and (2) holds, terminate the algorithm (go to step 6); else, move the dominant roots to the desired loci by means of a shifting algorithm, and simultaneously, push the rest of the spectrum to the left as far as possible. Denote the eventual result as \( K_i \), where \( i \) expresses the achieved number of iterations.
5. If the shifting is successful (see step 4), terminate the algorithm; otherwise, minimize the cost function \( \Phi(K) \) reflecting the distance of dominant roots from prescribed ones and the spectral abscissa of the rest of the spectrum, by using an optimization iterative algorithm. The (sub)optimal solution \( K_{opt} \) is obtained.
6. Get \( K_0 \), \( K_i \) or \( K_{opt} \) as the output.

Algorithm Discussion
Going into details of Algorithm 1, step 2 means that if the feedback system is strongly unstable and the associated exponential polynomial can not be affected by selectable parameters, there is no sense to shape the spectrum anymore. The number of selected desired poles should be less the number of free parameters to remain some degrees of freedom to adapt \( \sigma(K) \).

In step 3, the reader is referred e.g. to (Vyhlidal 2003; Zitek and Vyhlidal 2002) for details. Once the initial spectrum is placed, its dominant part is checked. If it concurs with the desired loci, the placement is sufficient and there is no reason to made improvements (unless the user wants to enhance the spectral abscissa). Contrariwise, rightmost poles may be successively shifted towards the prescribed positions. Here in step 4, we have to highlight our observation: Although it has been stated in (Michiels and Vyhlidal 2005) that for a neutral TDS there is no reason to deal with poles \( \text{Re} s_i < \bar{\gamma} \) (see Property 1, and Definition 1), we have observed by simulations that it is desirable to control also poles left from this vertical line with a sufficiently small modulus in some cases. The idea can simply be explained as follows. Consider the vertical strip of poles introduced in Property 1, item 2, with some unperturbed \( \gamma \) and \( D_n(s, K \neq 0) \). If a finite number of isolated poles satisfies \( \text{Re} s < \gamma \) but they are right from the bunch of poles constituting the strip, the essential part of the system dynamics might be determined by this small low-frequency subset. Moreover, the eventual value of \( \bar{\gamma} \) can be adjusted. In other words, from the dynamical point of view, it is not reasonable to deal with the rightmost high-frequency poles. Nevertheless, their position must be checked with respect to exponential stability.

Step 5 of Algorithm 1 includes the optimization procedure that may be performed via several techniques. The crucial substep consists of the formulation of the objective function that must consider up to three factors: The distance of current dominant poles form the desired ones, the spectral abscissa of the rest of the spectrum, \( \sigma(K) \), and the condition (2). Thus, three subfunctions \( \phi(K), \pi(K), \varphi(K) \) are to be set, respectively, such that

\[
\Phi(K) = \varphi(K) + \pi(K) + \Phi(K)
\]

Simply, we have

\[
\phi(K) = \sum_{i=1}^{n} |\sigma_i - s_i|
\]

where \( \sigma_i \) stand for prescribed poles, whereas the current dominant ones are expressed as \( s_i \).

Function \( \pi(K) \) can be chosen as a penalty function (Fletcher 1987), for instance

\[
\pi(K) = \lambda_1 \sigma_i(K), \lambda_1 > 0
\]

\[
\pi(K) = \lambda_2 \max(0, \sigma_i - \theta(K)),
\]

\[
\sigma_i < 0.5 \max(\text{Re} \sigma_i) \lambda_i > 0, l \in \mathbb{N} 
\]

where \( \sigma_i \) represents the desired eventual abscissa margin for the rest of the spectrum, and \( \lambda_2 \) must be sufficiently high.

Finally, for \( \varphi(K) \), a barrier function is better to be taken instead of penalty one, due to the constrain (inequality) in (2), e.g.

\[
\varphi(K) = -\log(1 - e - \sum_{i=1}^{n} |\theta_i(K)|)0 < e << 1
\]

see (Michiels and Gummussoy 2014), for which the initial setting must be made such that \( 1 - e > \sum_{i=1}^{n} |\theta_i(K)| \).

SIMULATION EXAMPLE
Let us demonstrate the procedure described above on a simulation example in the MATLAB®/Simulink® environment. Controller structure design is omitted.
Assume a non-minimum phase unstable TDS plant modeled by the transfer function

\[
P(s) = \frac{(s-4)e^{-s}}{s+1-2e^{-4s}} \quad (6)
\]

Although model (6) is of a retarded type, any attempt to design a feasible feedback controller results in the TDS control system of a neutral type. For instance, the use of the stabilizing general finite-dimension controller governed by the transfer function

\[
R(s) = \frac{q_0s^2 + q_1s + q_2}{s^2 + p_1s + p_0} \quad (7)
\]

within the well-known habitual simple negative feedback loop yields the following characteristic quasipolynomial

\[
D(s, K) = \left( s + 1 - 2e^{-4s} \right)^2 + \sum_{i=0}^{1} [p_i s^i] + \left( (s-4)e^{-s} \sum_{i=0}^{1} q_i s^i \right)
\]

\[
= \left( 1 + q_1 e^{-s} \right)^2 + \left( 1 - 2e^{-4s} + p_1 + (q_1 - 4q_2) e^{-s} \right)s^2 + \left( p_0 \left( 1 - 2e^{-4s} \right) + p_1 + (q_1 - 4q_2) e^{-s} \right)s + p_0 \left( 1 - 2e^{-4s} - 4q_1 e^{-s} \right) \quad (8)
\]

the roots of which constitute the system spectrum. Strong stability condition (2) apparently reads

\[
|q_1| < 1 \quad (9)
\]

which must hold during the evolution of the selectable parameters vector \( \mathbf{K} = (p_1, p_2, q_1, q_2) , r = 5 \).

Following step 2 of Algorithm 1, let the prescribed (desired) poles be represented by a conjugate pair \( \sigma_{\pm} = -0.1 \pm 0.2j \), i.e. \( n = 2 \). Note that it can be calculated that the vertical line introduced in Property 1, item 2, is located in \( \gamma = -\log(1/|q_2|) = \log|q_2| \), see e.g. (Bonnet et al. 2011). Hence, in order to get the vertical strip of neutral poles left from the desired pair, the inequality \( \log|q_2| < -0.1 \) must hold, the solution of which reads

\[
|q_2| < 0.9048 \quad (10)
\]

By placing roots of (9) directly to the desired loci, the initial feedback spectrum found within the selected range \( R = [-6.15, 0.40] \) and parameters setting read

\[
\Sigma_0 = \begin{cases} 
0.1458 \pm 3.94j, -0.1 \pm 0.2j, -0.4731 \pm 22.114j, \\
-0.4737 \pm 16.0155j, -0.5067 \pm 9.709j, \ldots
\end{cases}
\]

\[
\mathbf{K}_0 = (1.0317, 1.2722, 0.71174, 0.2978, 0.5927)\text{T}
\]

respectively. This setting gives exponentially unstable yet strongly stable feedback system with \( \sigma(\mathbf{K}_0) = 0.1458 \) and \( \gamma = \theta = -0.5231 \). It indicates that the spectrum must be enhanced according to step 4 of Algorithm 1.

The evolution of \( |\sigma_{\pm} - s_j| , \sigma(\mathbf{K}) \) and \( \sigma_j(\mathbf{K}) \) via the QCSA for the number of 16000 iterations can be seen in Figure 1, and that of \( \mathbf{K} \) is displayed in Figure 2. As can be seen, the value of \( q_2 \) is being improved during the shifting and system has been stabilized. However, the dominant pair is still quite far from the desired one. A slump in the plot of \( |\sigma_{\pm} - s_j| \) is caused by closeness of real parts of the dominant pair and a close real pole that appears right from the pair within some iterations.

Figure 1: The Evolution of \( |\sigma_{\pm} - s_j| , \sigma(\mathbf{K}) \) and \( \sigma_j(\mathbf{K}) \) by Using the QCSA

Figure 2: The Evolution of \( \mathbf{K} \) by Using the QCSA
The eventual dominant spectrum and the values of $K$ are the following

$$
\Sigma_{16000} = \left\{ -0.0988 \pm 2.0394j, -4.5531, -4.6077, \\
-4.6077, -4.7493, -4.7533 \pm 18.5866j, \ldots \right\}
$$

$$
K_{16000} = \{ 0.316, 4.59, -0.0568, -0.0984, -0.0054 \}^T
$$

These values are taken as initial conditions for the NM optimization procedure (Nelder and Mead 1965), the results of which are provided to the reader in Figures 3 and 4 where $\Delta$ stands for the simplex edge length. The objective function has been brought together from (4), (5a) with $\lambda = \lambda_i = 0.2$ and $\phi(K) = 0$.

Our simple test has revealed the setting $\lambda = 0.2, \Delta = 2$ as a suitable one for the optimization here since it gives the minimal $\Phi(K)$ from three possibilities. In Figure 5, we further try to compare this setting with the setting pair $\lambda = 0.05, \Delta = 10$. Although results for both pairs in Figure 5 are almost comparable, only the selected setting yields the dominant pair of poles in the close vicinity of desired positions (see the solid thin line approaching the zero).
Then the optimal low-frequency spectrum and the final controller parameters’ values read

\[
\Sigma_{opt,250} = \begin{cases} 
0.1 \pm 0.2j, -3.59 \pm 4.2255j, \\
-3.5903 \pm 18.9356j, -3.7083 \pm 37.6507j, \\
-3.7579 \pm 31.6205j, -3.9273 \pm 25.0812j, \\
\end{cases} \\
K_{opt,250} = \begin{pmatrix} 
13.4082, -6.3073, -0.3543, -1.7256, \\
-0.0238 
\end{pmatrix}^T
\]

Apparently, the dominant poles agree with desired ones and the feedback system can be judged as strongly exponentially stable (Michiels and Gumussoy 2014) since \(\alpha(K_{opt,250}) = 0.1458\) and condition (9) holds.

The result is also demonstrated by displaying a part of the dominant system spectrum, see Figure 6. Since the rest of the spectrum is quite far from the prescribed pair, it has only a minor effect on the system dynamics.

![Figure 6: The Eventual Obtained Spectrum](image)

**CONCLUSIONS**

We have introduced the basic concept of a suboptimal numerical dominant pole assignment procedure for neutral TDS. The goal is to shape the dominant part of the spectrum such that it matches a finite number of prescribed desired poles loci and to push the rest of the spectrum as left as possible. The algorithm consists of three steps: the direct pole placement, successive quasi-continuous shifting and the optimization procedure. The process may stop whenever the desired spectrum is reached. The novelty but also the main drama consists in that neutral TDS are considered. These systems have quite complex spectral and stability issues including the sensitivity to infinitesimally small delays. The method has been verified and demonstrated in the MATLAB®/Simulink® environment via a simulation example of control of an unstable TDS time delay system (TDS). Some possibilities how to adjust the algorithm are given to the reader as well.

The algorithm can be improved mainly by a more sophisticated optimization; namely, the selection of the objective function and optimization methods (and its parameters), the use of faster software and hardware tools, or by the development of a better poles loci computation.

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