OPTIMIZATION OF A HEAT RADIATION INTENSITY AND TEMPERATURE FIELD ON THE MOULD SURFACE

Jaroslav Mlynek
Roman Knobloch
Institute of Mechatronics and Computer Engineering
Technical University of Liberec
Studentská 2, 461 17 Liberec, Czech Republic
E-mail: jaroslav.mlynek@tul.cz
E-mail: roman.knobloch@tul.cz

Radek Srb
Karel Mlýnek
Institute of Mechatronics and Computer Engineering
Technical University of Liberec
Studentská 2, 461 17 Liberec, Czech Republic
E-mail: radek.srb@tul.cz

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Intensity of heat radiation, optimization of temperature field, differential evolution algorithm, parallel programming, software implementation.

ABSTRACT
This article is focused on the infrared heating of shell metal moulds and optimization of temperature field on the surface of the mould. The upper part of the mould is heated by infrared heaters, and after the required temperature is attained the inner part of the mould is sprinkled with special PVC powder. The moulds are made of aluminium or nickel alloys. The described mathematical model allows us to optimize locations of heaters above the mould and thus we get an approximately uniform temperature field on the whole inner mould surface. In this way the whole surface of produced artificial leather has the same material structure and colour shade. A differential evolution algorithm is used to optimize the locations of heaters. A practical example of the optimization of the heaters locations and the calculation of the temperature field on the inner part of mould surface is included at the end of the article. The described process is one of the economical ways of artificial leathers production in the car industry.

INTRODUCTION
This article concerns economically beneficial technology for producing artificial leathers in the automotive industry. The artificial leathers are used as final parts for some components of car interior equipment (e.g. the inner surface of the door padding, the surface of the dashboard). In practice, a metal mould is at first preheated by infrared heaters located above the mould. Then the inner mould surface is sprinkled with a special PVC powder and the upper part of mould surface is continually heated for about 3 minutes to an approximate temperature of 250°C.

The infrared heaters have a tubular shape and their length is between 15 and 30 cm (see Figure 1). The heater is equipped with a mirror located above the radiating tube, which reflects heat radiation in the adjusted direction. Shell metal moulds of different proportions with variously complicated surfaces and with weights approximately up to 300 kg are used in the production of artificial leathers (see Figure 2). The shell mould thickness is constant. For various moulds this constant thickness is from 6 to 8 mm. Moulds are usually made of aluminium or nickel alloys.

An important requirement for manufacturing artificial leathers is approximately uniform temperature of the whole inner mould surface in the given time during heating of the mould. In this way we obtain the required uniform material structure and colour shade of the artificial leather surface. The above mentioned requirement of the uniform temperature can be fulfilled by finding suitable locations of heaters over the mould. The aim is to optimize the locations of the heaters, that is to find such locations that ensure the uniform radiation on the whole heated surface of the mould. Up to now, suitable locations of heaters were determined by manufacturing technicians based on their operational experience. Nevertheless, this approach did not provide a sufficiently uniform temperature and was time consuming.

The infrared heaters above the mould are usually located at a distance of between 5 and 30 cm over the mould surface. In our model we also do not know the heater...
distribution intensity function from the heater manufacturer. Therefore, experimental values for the heat radiation intensity by a sensor in the surroundings of the infra heater were measured. For radiation intensity calculations it was necessary to use transformations of Cartesian coordinate systems in Euclidean space \( E_3 \) and the linear interpolation at individual points of the mould surface.

During the optimization process of heaters locations we have to take into account the possible collisions of two heaters as well as collisions of a heater and the mould surface. Therefore, the optimization process is more complicated. The minimized function has many local extremes and for this reason it is not suitable to use gradient methods in the optimization. We used an evolutionary algorithm, specifically the differential evolution algorithm (using this algorithm, we obtain better results than when using a genetic algorithm, see (Mlynek and Srb 2012)). Evolutionary algorithms in general require a lot of operations and long computation time (especially when the mould volume is larger and we use a higher number of heaters). This is the main reason for us to use parallel programming techniques in the optimization process. The whole optimization process was programmed by the authors in the Matlab system and we used the Matlab Parallel Computing Toolbox.

For finding optimized locations of the heaters and the optimized heat radiation intensity on the whole irradiated surface of the mould we calculate temperature evolution (during the heating of the mould surface) on the inner part of the mould surface. We solve the partial differential parabolic equation of the heat conduction in the mould with initial and boundary conditions. We use the ANSYS software package to obtain the solution to this equation.

The used mathematical model, the process of calculation of the heat radiation intensity on the mould surface are described in more details in (Mlynek and Srb 2012) and the optimization process in (Mlynek and Srb 2014).

**MATHEMATICAL MODEL OF HEAT RADIATION ON THE MOULD**

We describe a heat radiation model in this chapter. The heaters and mould are represented in a 3-dimensional Euclidean space \( E_3 \) using a Cartesian coordinate system \((O, x_1, x_2, x_3)\), with basis vectors \( e_1 = (1, 0, 0) \), \( e_2 = (0, 1, 0) \) and \( e_3 = (0, 0, 1) \).

**Representation of a heater**

The heater is represented by a straight line segment with a given length (see Figure 3). The position of every heater \( H \) can be defined by the following 6 parameters

\[
H : (s_1, s_2, s_3, u_1, u_2, \phi) ,
\]

where the first three parameters are the coordinates of the heater centre \( S \), the following two parameters are the first two coordinates of the unit vector \( u \) of the heat radiation direction (the third coordinate is negative, i.e. the heater radiates “downward”). The last parameter is the angle \( \varphi \) between the vertical projection of vector \( r \) of the heater axis onto the \( x_1 x_2 \)-plane and the positive part of axis \( x_1 \) \((0 \leq \varphi < \pi)\), the vectors \( u \) and \( r \) are orthogonal.

**Figure 3: Schematic representation of the heater**

**Representation of a mould**

The upper mould surface \( P_{up} \) is described by elementary surfaces \( p_j \), where \( 1 \leq j \leq N \). It holds that \( P_{up} = \cup p_j \), where \( 1 \leq j \leq N \) and \( \text{int} p_i \cap \text{int} p_j = \emptyset \) for \( i \neq j \), \( 1 \leq i, j \leq N \). Each elementary surface \( p_j \) is described by its centre of gravity \( C_j = [c_{0j}^1, c_{0j}^2, c_{0j}^3] \), the unit outer normal vector \( v_j = (v_{1j}^2, v_{2j}^2, v_{3j}^2) \) at the point \( C_j \) (we suppose \( v_j \) faces “upwards” and therefore is defined through the first two components \( v_{1j}^2, v_{2j}^2 \)) and by the area of elementary surface \( w_j \). Therefore each elementary surface \( p_j \) can be defined by the following 6 parameters

\[
p_j : (c_{0j}^1, c_{0j}^2, c_{0j}^3, v_{1j}^2, v_{2j}^2, w_j).
\]

**Calculation of total heat radiation**

Now, we describe the numerical procedure for computation of total heat radiation intensity on the upper mould surface \( P_{up} \). We denote \( L_j \) as the set of all heaters radiating on the \( j \)-th elementary surface \( p_j (1 \leq j \leq N) \) for the fixed position of heaters, and \( I_{lj} \), the heat radiation intensity of the \( l \)-th heater on the \( p_j \) elementary surface (in our model we suppose \( I_{lj} \) is constant on the whole \( p_j \) and is calculated in its centre of gravity \( C_j \)). Then the total radiation intensity \( I_j \) on the elementary surface \( p_j \) is given by the following relation (more details in (Cengel 2007))

\[
I_j = \sum_{l \in L_j} I_{lj} .
\]

A detailed description of the procedure for calculating
the heat radiation intensity \( I_{j} \) of the \( i \)-th heater in a general position at the elementary surface \( p_{j} \) is shown in (Mlynk and Srb 2012).

**OPTIMIZATION OF HEAT RADIATION INTENSITY**

In this chapter we describe a procedure for optimizing the location of heaters. We need to determine such location of heaters that heat radiation intensity on the whole upper mould surface \( P_{up} \) will be approximately equal to the constant value of heat radiation intensity \( I_{w} \) recommended by the producer of artificial leathers.

We can define the deviation function \( F \) (respectively \( \widetilde{F} \) ) of heat radiation intensity by the relations

\[
F = \frac{1}{W} \sum_{j=1}^{N} |I_{j} - I_{w}w_{j},
\]

\[
\widetilde{F} = \left( \frac{1}{W} \sum_{j=1}^{N} (I_{j} - I_{w})w_{j} \right)^{2},
\]

where \( W = \sum_{j=1}^{N} w_{j} \). We highlight that \( w_{j} \) denotes the area of the elementary surface \( p_{j} \). Our goal is to find the minimum of function \( F \) (respectively \( \widetilde{F} \) ). Function \( F \) (and analogously function \( \widetilde{F} \) ) has many local minima. Using the gradient method for minimizing function \( F \) (respectively \( \widetilde{F} \) ) is not appropriate because there is a high probability of failure of this method (finding only a local inconvenient minimum). Therefore, we use an evolutionary algorithm to find a minimum of function \( F \) (respectively \( \widetilde{F} \) ). In accordance with relation (1), the location of every heater \( H \) is defined by 6 parameters. Let us suppose we use \( M \) heaters for heating of the mould. Therefore, 6M parameters are necessary to define the positions of all \( M \) heaters. We successively construct populations of individuals \( y \) by the evolutionary algorithm (for more details see (Price et al. 2005)). Every individual \( y \) represents one possible location of heaters above the mould.

We seek the individual \( y_{\text{min}} \in C \) satisfying the condition

\[
F(y_{\text{min}}) = \min\{F(y); y \in C\},
\]

where \( C \subset E_{6M} \) is the examined set. Every element of \( C \) is formed by a set of \( 6M \) allowable parameters and this set defines just one location of heaters above the mould.

We keep track of the following three types of collisions during generation of individuals \( y \): a heater radiates more on another heater than the given limit, a heater has an insufficient distance from another heater, a heater has an insufficient distance from the mould surface. If a collision occurs then the corresponding individual \( y \) is penalized and excluded from the population. The finding of the individual \( y_{\text{min}} \) defined by relation (6) is not realistic in practice. However, we are able to determine an optimized solution \( y_{\text{opt}} \) that is satisfactory for the production requirements.

Every population of our evolutionary algorithm includes \( NP \) individuals. The generated individuals are saved in the matrix \( B_{NP \times (6M+1)} \). Every row of this matrix represents one individual \( y_{i} \) and its evaluation \( F(y_{i}) \).

Now, we briefly describe the used differential evolution algorithm.

**Differential evolution algorithm**

Our minimization problem is solved by the differential evolution algorithm named \( DE/rand/1/bin \) (for more details see (Price et al. 2005)) and was programmed in the Matlab code by the authors. We describe schematically the individual steps of the algorithm.

**Steps of the differential evolution algorithm**

**Input:** The initial individual \( y_{i,0} \), population size \( NP \), the number of used heaters \( M \) (dimension of the problem is \( 6M \)), crossover probability \( CR \), mutation factor \( f \), the number of calculated generations \( NG \).

**Internal computation:**

1. create an initial generation \((G=0)\) of \( NP \) individuals \( y_{i}^{G} \), \( 1 \leq i \leq NP \),
2. a) evaluate all the individuals \( y_{i}^{G} \) of the generation \( G \) (calculate \( F(y_{i}^{G}) \) for every individual \( y_{i}^{G} \)), b) store the individuals \( y_{i}^{G} \) and their evaluations \( F(y_{i}^{G}) \) into the matrix \( B \),
3. repeat until \( G \leq NG \)

a) for \( i=1 \) to \( NP \) do

(i) randomly select index \( k_{i} \in \{1, 2, \ldots, 6M\} \),

(ii) randomly select indexes \( r_{1}, r_{2}, r_{3} \in \{1, \ldots, NP\} \), where \( r_{i} \neq i \) for \( 1 \leq i \leq 3 \); \( r_{1} \neq r_{2}, r_{1} \neq r_{3}, r_{2} \neq r_{3} \);

(iii) for \( j=1 \) to \( 6M \) do

if \( (\text{rand}(0,1) \leq CR \text{ or } j=k_{i}) \) then

\( y_{i,j}^{\text{rad}} \leftarrow y_{r_{1},j}^{G} + f \left( y_{r_{2},j}^{G} - y_{r_{3},j}^{G} \right) \)

else

\( y_{i,j}^{\text{rad}} \leftarrow y_{i,j}^{G} \)

end for (j)

(iv) if \( F(y_{i}^{rad}) \leq F(y_{i}^{G}) \) then \( y_{i}^{G+1} \leftarrow y_{i}^{rad} \) else \( y_{i}^{G+1} \leftarrow y_{i}^{G} \)

end for (i)

b) store individuals \( y_{i}^{G+1} \) and their evolutions \( F(y_{i}^{G+1}) \) \((1 \leq i \leq NP)\) of the new generation \( G+1 \) into the matrix \( B \); \( G \leftarrow G+1 \)

end repeat.

**Output:** The row of matrix \( B \) that contains the corresponding value \( \min\{F(y_{i}^{G}); y_{i}^{G} \in B\} \) represents the best found individual \( y_{\text{opt}} \).

Comment: function \( \text{rand}(0,1) \) randomly chooses a number from the interval \( (0,1) \). The notation \( y_{i,j}^{G} \)
means the \( j \)-th component of an individual \( y^G \) in the \( G \)-th generation. The individual \( y_{opt} \) is the final solution and includes information about the location of each heater \( H \) in the form (1).

**CALCULATION OF TEMPERATURE FIELD IN THE MOULD**

By using the differential evolution algorithm described in chapter “Optimization of Heat Radiation Intensity” we obtain the optimized individual \( y_{opt} \). This means we receive suitable locations of heaters above the mould (each heater is defined in the form of relation (1)) and approximately uniform heat radiation intensity (close to the value \( I_{rev} \)) on the upper surface \( P_{up} \) of the mould (we know heat radiation intensity \( I_j \) for each elementary surface \( p_j \in P_{up} \)).

Now we describe the calculation of the temperature field in the mould for time of mould heating and for locations of heaters defined by optimized individual \( y_{opt} \).

**Mathematical model of the heat conduction**

In this section we solve the parabolic equation of the heat conduction

\[
c \rho \frac{\partial T(x,t)}{\partial t} = \lambda \Delta T(x,t) + Q(x,t)
\]

on the domain \( \Omega \subset E_3 \), where \( \Omega \) represents the shell mould. The function \( T(x,t) \) denotes a temperature field in relation (7), point \( x = (x_1, x_2, x_3) \in \Omega \), time \( t \in (0, \tau) \), where \( \tau \) is the duration of the heat radiation. The symbol \( \Delta \) stands for the Laplace operator with respect to space variables, i.e.

\[
\Delta T(x,t) = \sum_{i=1}^{3} \frac{\partial^2 T(x,t)}{\partial x_i^2}.
\]

The values \( c \) and \( \rho \) stand for the specific heat and mass density of the mould material. The value \( \lambda \) denotes the heat conductivity of the mould material. We suppose a homogeneous and isotropic material of the mould. The function \( Q(x,t) \) represents the volume density of the heat sources. Nevertheless, in our case there is no inner heat source in \( \Omega \) and so \( Q(x,t) = 0 \) in relation (7).

We consider the initial condition

\[
T(x,0) = T_0 \quad \forall x \in \Omega,
\]

where \( T_0 \) denotes the initial temperature of the mould.

We choose the Newton boundary condition which suits best the situation when the hot body is surrounded by an environment (air) and when the heat transfer between the body and environment is possible. The simple Newton boundary condition can be expressed in the form

\[
\lambda \frac{\partial T(x,t)}{\partial v} = -\alpha (T(x,t) - T_{air}),
\]

where \( \alpha \) is the coefficient of the heat transfer between the mould material and air, \( T_{air} \) is air temperature and \( v \) is the unit vector of the outer normal. Nevertheless, this linear formulation of the boundary condition is not sufficient for our problem. There are two main reasons: 1/ the temperature of the mould reaches up to 300°C and it is not possible to neglect the own heat radiation of the mould determined by Stefan-Boltzmann law; 2/ there are no volume heat sources \( Q(x,t) \) in the body of the mould and the heat is supplied exclusively by infrared heaters through the upper part \( P_{up} \) of the mould surface \( \partial \Omega \). When we consider these two facts, we obtain the following form of the boundary condition (see e.g. (Incropera 2007))

\[
\lambda \frac{\partial T(x,t)}{\partial v} = -\alpha (T(x,t) - T_{air}) - \varepsilon \sigma (T^4(x,t) - T_{air}^4) + I(x)
\]

on the upper part \( P_{up} \) of the mould surface \( \partial \Omega \) and for all other parts of the mould surface \( \partial \Omega - P_{up} \).

\[
\lambda \frac{\partial T(x,t)}{\partial v} = -\alpha (T(x,t) - T_{air}) - \varepsilon \sigma (T^4(x,t) - T_{air}^4).
\]

Here value \( \varepsilon \) denotes the emissivity of the mould and \( \sigma \) denotes the Stefan-Boltzmann constant, \( \sigma = 5.775 \times 10^{-8} \text{ Wm}^{-2} \text{K}^{-4} \). Value \( I(x) \) indicates the heat radiation intensity on the part of mould surface \( P_{up} \). In our case the value \( I(x) \) is defined by locations of the heaters given by optimized solution \( y_{opt} \) from chapter “Optimization of Heat Radiation Intensity”.

The equation (7) together with initial condition (8) and boundary conditions (9) - (10) describes the heat conduction in the mould \( \Omega \). It is a nonstationary heat conduction problem, time \( \tau \) of duration of heat radiation is usually 3 minutes. For needs of the production it is most important to know the temperature on the inner part of the mould surface \( P_{in} \) (the artificial leather is produced on this part of the mould surface).

**Use of software package ANSYS**

We use the ANSYS 15.0.7. software package to determine the solution of the parabolic equation of the heat conduction (7) with conditions (8) - (10). The input parameters for the ANSYS system are the mould surface described by triangle elementary surfaces in Euclidean space \( E_3 \), heat radiation intensity on each elementary surface and thickness of the shell mould. The finite element method is used to solve our thermal problem by the ANSYS system. The corresponding base functions are quadratic.
PRACTICAL EXAMPLE

We find the optimized locations of the infrared heaters over the mould, and we calculate the temperature of the mould (especially the temperature on the inner surface $P_{\text{in}}$ of the mould).

I. Input parameters
a/ heaters
Type of infrared heater: Ushio, length 15 cm, width 4 cm, power 1000 W, all the heaters are of the same power and shape, number of heaters: 16,
b/ mould
Mould size: 0.9x0.4x0.15 m$^3$, mould thickness: 8 mm, material of the mould: aluminium alloy, number of the elementary surfaces of the mould $N = 2178$, specific heat $c = 875$ J/kgK, mass density $\rho = 2770$ kg/m$^3$, heat conductivity $\lambda = 160$ W/mK, coefficient of the heat transfer between the mould material and air $\alpha = 20$ W/mK$^2$, emissivity of the mould $\varepsilon = 1$,
c/ heat radiation
Recommended heat radiation intensity $I_{\text{rec}}$ by producer of the artificial leather: $I_{\text{rec}} = 47$ kW/m$^2$, duration of the heat radiation $t = 180$ s,
d/ temperature
Temperature of air $T_{\text{air}} = \text{initial temperature } T_0 = 22$ °C.

II. Starting locations of the heaters in optimization process
The heaters in the initial locations lie in the plane given by axes $x_1$ and $x_2$ and at a distance of 10 cm over the mould top and with heater axis $r$ parallel to axis $x_1$. Initial locations of heaters over the mould are represented by individual $y_1$ and corresponding heat radiation intensity on the upper part $P_{\text{out}}$ of mould surface is displayed in Figure 4. The value of the function $F$ (defined by relation (4)) for the initial individual $y_1$ is $F(y_1) = 20.87$.

We receive the optimized individual $y_{\text{opt}}$ by the differential evolution algorithm described in section “Steps of the differential evolution algorithm” with the following input parameters: crossover factor $CR = 0.98$; mutation factor $f = 0.60$; population size $NP = 200$; number of calculated generations $NG = 10000$. The locations of heaters over the mould corresponding to individual $y_{\text{opt}}$ and appropriate heat radiation intensity are shown in Figure 5. Deviation function $F$ (defined by relation (4)) has for individual $y_{\text{opt}}$ value $F(y_{\text{opt}}) = 2.30$.

b/ optimized locations of heaters and heat conduction
We solve the parabolic equation of the heat conduction (7) with conditions (8) – (10) for heat radiation intensity on the upper part of the mould surface $P_{\text{opt}}$ corresponding to initial individual $y_1$, $y_{\text{opt}}$ and for the case when the heat radiation intensity has the constant value $I = I_{\text{rec}}$ (recommended heat radiation intensity by the producer of artificial leathers).

For producers of leathers the most important knowledge is the temperature field on the inner part of the mould surface $P_{\text{in}}$ (here the artificial leather is gradually formed and it is necessary that the deviations of temperatures at specific time of warming are less than 15°C). The temperature field at the heating time $t = 180$ s on the inner part $P_{\text{in}}$ of the mould surface for above three mentioned cases are successively displayed in Figures 6, 7 and 8.

III. Results
a/ optimized locations of heaters and heat radiation intensity

Figure 4: The initial locations $y_1$ of the heaters and heat radiation intensity $I$ (kW/m$^2$) on the mould surface.

Figure 6: Temperature field corresponding to initial individual $y_1$. 
Furthermore, we determine the temperature deviation $D$ on the inner part of the mould surface $P_m$. The value of $D$ is defined by the following relation

$$D(y, t_D) = \frac{\int_{P_m} (T(x, t_D, I_y) - \bar{T}(t_D, I_y))^2 \, dx}{\sum_{I_y} (T(C_y, t_D, I_y) - \bar{T}(t_D, I_y))^2 w_j}.$$ (11)

Here $T(x, t_D, I_y)$ denotes the temperature at point $x \in P_m$ at heating time $t_D$ for heat radiation intensity $I_y$ corresponding to individual $y$. The term $\bar{T}(t_D, I_y)$ represents the average temperature on the surface part $P_m$ for the given time $t_D$ and heat radiation intensity $I_y$. We remind that $C_y$ denotes the centre of gravity of elementary surface $p_j$ and $w_j$ is the area of this elementary surface.

Figure 7: Temperature field corresponding to optimized individual $y^{opt}$.  

Figure 8: Temperature field corresponding to the recommended constant heat radiation intensity $I_{rec}$ on the upper part of the mould surface.

We calculated deviations $D(y_1, t_D)$, $D(y^{opt}, t_D)$ and $D(y_{rec}, t_D)$, where $y_{rec}$ denotes a virtual individual corresponding to the recommended fully homogeneous heat radiation intensity $I_{rec}$ on the mould surface part $P_{up}$. We computed these values for $t_D = 180s$:

$$D(y_1, 180) = 19,332$$  

$$D(y^{opt}, 180) = 1,522$$  

$$D(y_{rec}, 180) = 1,135.$$  

The last two values demonstrate that the optimized solution $y^{opt}$ differs only very little from the ideal solution $y_{rec}$ and is therefore fully acceptable for the practical production needs.

**CONCLUSIONS**

The mentioned technology of artificial leather manufacturing in the automotive industry is economically feasible (low energy consumption during production). The suitable locations of the heaters above the mould and their connection to an auxiliary constructions has been up to now conducted by experienced technicians. However, this approach was very laborious and time consuming (approximately 2 weeks). Furthermore, the optimized solution obtained by using our mathematical model is much more accurate and the calculation time when using parallel programming tool is on average approximately 4 hours.

We are now able to determine the temperature of the mould during its heating for the optimized locations of the heaters. This information is important for the manufacturer of artificial leathers, but it was not available before. Only temperature sensors were up to now positioned at some points of the mould. Knowledge of the temperature is important in particular on the inside surface part of the mould (where the artificial leather is produced). The temperature differences on this part of the mould surface have to be maintained at less than 15°C at the given moment of the mould heating. Such a temperature distribution ensures a uniform material structure and colour shade on the whole leather surface.

We obtained better results using the differential evolution algorithm than using a genetic algorithm (Mlynek and Srb 2012) in numerical experiments. The described mathematical model and algorithm allows us to find the optimized locations of the heaters over the mould and in this way to determine the corresponding heat radiation intensity on the irradiated mould surface and to calculate the temperature field on the inner surface of the mould. In addition, using our mathematical model and calculation does not add any additional economic costs for the manufacturer.

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**REFERENCES**


AUTHOR BIOGRAPHIES

**JAROSLAV MLYNEK** was born in Trnava, Czechoslovakia and went to the Charles University in Prague, where he studied numerical mathematics at the Faculty of Mathematics and Physics and he graduated in 1981. In his work he focuses on the computational problems of heating and thermal losses in components of electrical machines and on mathematical models of thermal convection in electric machines. Currently he works as an associate professor at the Technical University of Liberec, the Czech Republic. His e-mail address is: jaroslav.mlynek@tul.cz.

**ROMAN KNOBLOCH** was born in Turnov, the Czech Republic. He completed his studies at the Charles University in Prague, the Faculty of Mathematics and Physics where he studied scientific physics and teaching of mathematics and physics. His main areas of interest are: modelling of physical phenomena, stationary and non stationary heat processes and heat and transport phenomena in continuum mechanics. He works as an assistant professor at the Technical University of Liberec where he also participates in the PhD study programme. His e-mail address is roman.knobloch@tul.cz.

**RADEK SRB** was born in Mladá Boleslav, the Czech Republic and went to the Technical University of Liberec, the Czech Republic, where he studied computer science and programming at the Faculty of Mechatronics. He graduated in 2005. He focuses on problems concerning the automated control of production. He works as a teacher and he is a PhD student. His e-mail address is: radek.srb@tul.cz.