MODELS AND ALGORITHMS FOR ABILITIES EVALUATION OF ACTIVE MOVING OBJECTS CONTROL SYSTEM

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ABSTRACT
One of the important problems in active moving objects control system (AMO CS) is the evaluation of goal abilities, i.e., potential of the system to perform its missions in different situations. Thus, the preliminary analysis of information and technological and goal abilities (GA and ITA) of AMO CS is very important in practice and can be used to obtain reasonable means of the AMO exploitation under different conditions. In the paper models and algorithms for abilities evaluation of AMO CS are proposed.

ABBREVIATIONS AND NOTATION
AMO — active moving objects  
AS — attainable sets  
CS — control system  
CTS — complex technical systems  
GA — goal abilities  
ITA — informational and technological abilities  
IZ — interaction zone  
NFDDS — nonstationary finite-dimensional differential dynamic systems  
OS — object-in service  
OPS — optimal program control  
SDC — structure-dynamics control  
\( D^- (t_f,t_0, x(t_0)) \) — approximation of AS  
\( x(t_0) \) — initial state vector of AMO CS  
\( x(t_f) \) — end state vector of AMO CS  
\( t_0 \) — initial point of time  
\( t_f \) — final time of the scheduling interval  
\( u(t) \) — control vector, represents AMO control program  
\( \varepsilon_{ij}(t) \) — element of the present time function of time-spatial constraints \( \varepsilon_{ij}(t) \in \{0,1\} \)

INTRODUCTION
The general objects of our investigation are active moving objects control system (AMO CS). The notion “Active Mobile Object” generalizes features of mobile elements dealing with different complex technical systems (CTS) types (Kalilin et al., 1985, Okhtilev et al., 2006, Ivanov et al., 2010). Depending on the type of CTS Active Mobile Objects can move and interact in space, in air, on the ground, in water, or on water surface. Active Mobile Object can be regarded as multi-agent system. We distinguish two classes of AMO. AMO-one, namely AMO of the first type. This type of AMO fulfills CTS principal tasks. AMO-two supports functioning of AMO-one. Objects-in-service (OS) can be regarded as external AMO. Analysis of the main trends of modern AMO CS indicates their peculiarities such as: multiple aspects and uncertainty of behavior, hierarchy, structure similarity and surplus for main elements and subsystems of AMO CS, interrelations,
The variety control functions relevant to each AMO CS level, territory distribution of AMO CS components. One of the main features of modern AMO CS is the variability of their parameters and structures as caused by objective and subjective factors at different phases of the AMO CS life cycle (Klir 2005, Okhtilev et al., 2006, Ivanov et al., 2010). In other words, we always come across the AMO CS structure dynamics in practice. Under the existing conditions the AMO CS potentialities increment (stabilization) or degradation reducing makes it necessary to perform the AMO CS structures control (including the control of structures reconfiguration). There are many possible variants of AMO CS structure dynamics control. For example, they are alteration of AMO CS functioning means and objectives; alteration of the order of observation tasks and control tasks solving; redistribution of functions, of problems and of control algorithms between AMO CS levels; reserve resources control; control of motion of AMO CS elements and subsystems; reconfiguration of AMO CS different structures.

According to the contents of the structure-dynamics control problems belong under the class of the AMO CS structure–functional synthesis problems and the problems of program construction, providing for the AMO CS development.

One of the important problems in AMO CS structure-dynamic control is the evaluation of goal abilities, i.e., potential of the system to perform its missions in different situations. Thus, the preliminary analysis of information and technological and goal abilities (GA and ITA) of AMO CS can be used to obtain reasonable measures of GA exploitation under different conditions. The numerical estimations of AMO CS GA and ITA should be based on the system of measures. These measures can be regarded as characteristics of AMO CS potential effectiveness. The GA measures characterizing different levels of AMO CS are interrelated and have a hierarchical structure. The leading role of information and technological aspects of the goal-abilities (GA) evaluation is a result of the influence of the technology structure (the structure of AMO CS control technology) upon the other AMO CS structures (organizational structure, technical structure, etc.) So the information and technological abilities (ITA) of a system ought to be evaluated first of all. These abilities can be measured as AMO CS capacities. The following measures are to be evaluated: the total number of objects in a given macrostate over a fixed time period or at a fixed point of time; the total number of working operations performed over a fixed time period or by the time point . Parallel with the enumerated measures of ITA the following measures of GA can be used the total possible number of objects-in-service (OS) over the time period ; the total time that is necessary for the execution of all interaction operations with OS. If the uncertainty factors are considered (the stochastic, probabilistic, or fuzzy models can be applied) the measures of GA can be evaluated as the expectation (or the fuzzy expectation) of the number of serviced objects by a given time point; the probability (its statistical estimation) of successful service for the given objects. Similar measures can be proposed for ITA estimations, for example the expectation of the number of objects in a given macrostate, the probability of technological operations fulfillment.

The problem of AMO CS GA and ITA evaluation and analysis can be solved on the basis of structure dynamics control models (the model М and its components Mo, Mk, Mn, Me, Mg, Mv, Mc, Mp) (Okhtilev et al., 2006, Kalinin et al., 1985, 1987).

RESEARCH METHODOLOGY

The proposed approach is based on fundamental scientific results of optimal program control (OPC) theory regarding dynamic interpretation of scheduling problems and performance evaluation. The research methodology is based on the following basic principles.

The first feature of the research methodology is an original dynamic representation of AMO CS schedule as OPC control vector u(τ), represents AMO control programs (plans of AMO CS functioning) (Kalinin et al., 1985, 1987). The AMO CS scheduling is interpreted as dynamic process of operations control. From these points of view, the understanding of “dynamic scheduling” in this control theoretic study differs from the concept of dynamic scheduling in traditional rescheduling techniques (compare with Vieira et al. 2000). The advantages of the scheduling with the help of OPC have been extensively discussed in (Khmelnitsky et al., 1997, Ivanov and Sokolov, 2010).

For the stage of AMO CS scheduling, we formulate the OPC model as a linear non-stationary finite-dimensional controlled differential system with the convex area of admissible control. Such a model form is favourable because of (i) possibilities to calculate OPC and (ii) to approximate attainable sets (see further in this paper).

The calculation procedure for OPC is based on the Pontryagin’s maximum principle. With representing the AMO CS schedule as OPC, it becomes possible to perturb the parameters of AMO CS schedule at any point of time and of different severity (e.g., in the interval from zero to full resource breakdown) and to reflect non-stationary perturbations in the further calculation of robustness metric. Hence, the parameters and their variations in dynamics are explicitly expressed in the scheduling model and can be used for the robustness analysis in order to integrate the robustness objective as a non-stationary performance indicator in AMO CS scheduling.

The second feature of the research methodology is the dynamic representation of AMO CS schedule execution under different uncertainties based on attainable sets (AS). An AS of a controllable dynamic system in the state space is typically notated as , where
left \_0 \right \}$ is an initial point of time, $x(t_0)$ is an initial state vector, and $t_f$ is the final time of the interval.

The AS at current time $t_1 \in (t_0, t_f)$ includes all points of the system’s state trajectories (e.g., a set of all possible execution scenarios which may occur for the AMO CS schedule after the perturbations) at time $t_1$ under the following conditions: each trajectory begins at time $t = t_0$ in the state $x(t_0)$ and is formed through some allowable variations of control $u(t_0)$ within the time interval $(t_0, t_f)$ (Gubarev et al. 1988, Chernousko 1994, Clarke et al. 1995, Okhtiliev et al., 2006). It is important that the AS concept be applicable to multi-step procedures. It may be possible to derive the multiple decoupled attainable sets at each point of time that ensure that the overall schedule meets the performance requirements as long as the constituent steps are operated within the AS.

In less technical words, the AS approach is to determine a range of operating policies (the union of which is called as an AS) during the scheduling stage over which the system current performance can be guaranteed to meet certain targets, i.e., the output performance. Basically the AS is a fundamental characteristic of any dynamic system. If the AS is known its basic characteristics in essence replace with themselves all the information necessary about system dynamics, the stability of its functioning and output performance. The AS characterizes all possible states of the AMO CS schedule subject to different variations of AMO CS parameters in nodes and channels (e.g., resource capacity availability).

Besides, if the AS is known, it becomes possible to analyse the dependence between the scheduling results subject to output performance (e.g., service level and delivery reliability) and the structure and properties (e.g., inventory quantity and location, lot-sizes, transportation channels and the intensity of their usage) of the start and end states $(X_0, X_f)$. In other words, it becomes possible to define the area in which permissible solutions (e.g., AMO CS schedules) are included. On the other hand, the AS analysis may show that, with the given resource and at the given time horizon, it is impossible to achieve the required output performance; hence, we should introduce additional resources or expand the supply cycle.

**MODELS AND ALGORITHMS FOR ABILITIES EVALUATION OF ACTIVE MOVING OBJECTS CONTROL SYSTEM**

The problem of AMO CS GA and ITA evaluation and analysis can be solved on the basis of structure-dynamics control models (the model $M$ and its components $M_0, M_k, M_n, M_c, M_g, M_v, M_e, M_p$). (Okhtiliev et al.,2006, Ivanov and Sokolov 2010). These models have a form of nonstationary finite-dimensional differential dynamic systems (NFDDS).

With reconfigurable structures. So the problem of GA and ITA evaluation can be regarded as a problem of NFDDS controllability analysis. The latter problem, in its turn, can be solved by the NFDDS attainability set $D(t_f, t_0, x(t_0))$ construction. If the attainability set (AS) is found, the solvability of the previously stated boundary problems for structure-dynamics control (SDC) can be checked in accordance with the sets of initial $X_0$ and final $X_f$ states $x(t_0) \in X_0, x(t_f) \in X_f$, with the considered period of time, with time-spatial, technical, and technological constraints.

Moreover, the problems of AMO CS GA and ITA evaluation and analysis can be formulated as follows:

$$J_{as}^*(x(t)) \rightarrow \min_{x(t) \in D(t_f, t_0, x(t_0))},$$

where $D(t_f, t_0, x(t_0))$ is the attainability set of the dynamic system (model) $M$; $J_{as}^*(x(t))$ – is the initial functional transformed to the form of Mayer’s functional. It is important that the alteration of objective functional does not imply the recalculation of the attainability set $D(t_f, t_0, x(t_0))$. If the dimensionality of AMO CS GA and ITA evaluation and analysis problems is high, then the construction of the attainability sets becomes a rather complicated problem. Therefore, the approximations of $D(t_f, t_0, x(t_0))$ ought to be used (Chernousko F.L. 1994). Now we introduce the algorithm of $D(t_f, t_0, x(t_0))$ construction. The boundary points of the set $D(t_f, t_0, x(t_0))$ are obtained as the solutions of the optimal control problems (Chernousko F.L. 1994, Okhtiliev et al.,2006, Ivanov et al., 2010):

$$J_{as}^*(x(t)) = c'x(t_f) \rightarrow \min_{x(t) \in D(t_f, t_0, x(t_0))},$$

where $c$ is a vector such that $|c| = 1$. For a given vector $c$ we obtain the optimal control $u^*(t)$, the appropriate state vector $x^*(T)$ that is equal to some boundary point of $D(t_f, t_0, x(t_0))$, and the hyperplane $c'x^*(t)$ to $D(t_f, t_0, x(t_0))$ at the point $x^*(t)$. Let $\overline{A}$ be the number of different vectors $c_{\overline{A}}$.

$\overline{A} = 1,...,\overline{A}$, then the external approximation $D^+(t_f, t_0, x(t_0))$ of the set $D(t_f, t_0, x(t_0))$ is a polyhedron whose faces lie on the corresponding hyperplanes, the internal approximation $D^-(t_f, t_0, x(t_0))$ of $D(t_f, t_0, x(t_0))$ is a polyhedron whose vertices are the points $x^*(t_f), i.e.,

$D^-(t_f, t_0, x(t_0)) = \text{Co}(x(t_f),...,x(\overline{A})(t_f))$. The bigger $\overline{A}$, the better approximation of the attainability set $D(t_f, t_0, x(t_0))$ can be obtained. It can be proved (Okhtiliev et al.,2006, Ivanov and Sokolov 2010, Ivanov et al., 2010) that the value $\overline{A}$ is defined by the total
number of possible interruptions for AMO CS interaction operations over a given time period \((t_0, t_f)\).

To obtain \(D^+, D^-\) Krylov and Chernousko’s method was used (Chernousko F.L. 1994). Instead of the vector \(\varphi(t_0)\) of conjugate variables it is to be varied. Besides the general dynamic model of AMO CS functioning (the model M) its aggregated variants can be used for the attainability-set construction. Let us exemplify this approach via the models \(M_b, M_L\). Interaction operations of the object \(B_j\) will be regarded as one aggregated operation, the channels \(C^{(j)}_b\) will be replaced by one general channel \(C^{(j)}\). Besides, we prescribe \(\theta=\lambda=1 \forall i, a, j, \lambda\) and allow the interruptions of operations. So the aggregated models of object’s IO and channels can be stated as follows:

\[
\dot{x}_{ij} = \sum_{j=1}^{m} e_{ij}(t)x_{ij}^{(o)}, \tag{3}
\]

\[
\dot{\bar{x}}_{ij} = \sum_{j=1}^{m} \bar{u}_{ij}(t) \frac{h_{ij} - \bar{x}_{ij}}{s_{ij}} y_{ij} \gamma_{ij} \bar{x}_{ij}, \tag{4}
\]

where \(\bar{x}_{ij}^{(o)} = \sum_{a=1}^{s} x_{ia}^{(a)}, \bar{u}_{ij} = \sum_{a=1}^{s} u_{ia}^{(a)}\) are the aggregating functions. The classes \(\tilde{K}_{ij}^{(o)}, \tilde{K}_{ij}^{(k)}\) of allowable control inputs are defined as follows:

\[
\tilde{K}_{ij}^{(o)} = \left\{ \bar{U}_{ij}^{(o)} = \|\bar{u}_{ij}^{(o)}\| \sum_{j=1}^{m} \bar{u}_{ij}^{(o)} \leq 1, \right\} \tag{5}
\]

\[
\tilde{K}_{ij}^{(k)} = \left\{ \bar{U}_{ij}^{(k)} = \|\bar{u}_{ij}^{(k)}\| \sum_{j=1}^{m} \bar{u}_{ij}^{(k)} \leq 1, \right\} \tag{6}
\]

where \(\bar{x}_{ij}^{(o)}, \bar{u}_{ij}^{(k)}\) are function-theoretic constraints imposed on the classes of allowable controls.

We assume that the control inputs are piecewise continuous functions. We introduce vector \(\mathbf{x}^{(o)} = [x_{1}^{(o)}, ..., x_{m}^{(o)}]^{T}\) and vector \(\mathbf{\bar{x}}^{(k)} = [\bar{x}_{1}^{(k)}, ..., \bar{x}_{m}^{(k)}]^{T}\). Let \(\mathbf{x}^{(o)}(t_0) = 0\), \(\mathbf{\bar{x}}^{(k)}(t_0) = \mathbf{\bar{x}}^{(k)}\). Then the attainability set in the state space of the dynamic system (3–(4) can be obtained as follows:

\[
\tilde{D}_{(o,k)} = \left\{ \bar{x}_{ij} = \int_{t_0}^{t_f} \sum_{j=1}^{m} e_{ij}(t)\bar{u}_{ij}^{(o)}(t)dt, \right\}
\]

\[
\tilde{U}_{ij}^{(o)} \in \tilde{K}_{ij}^{(o)}, \tag{7}
\]

\[
\bar{x}_{ij}^{(k)} = \sum_{j=1}^{m} \bar{u}_{ij}(t)\bar{u}_{ij}^{(k)}(t)dt, \quad \tilde{U}_{ij}^{(k)} \in \tilde{K}_{ij}^{(k)},
\]

where \(x = [\mathbf{x}^{(o)}, \mathbf{\bar{x}}^{(k)}]^{T}\), \(\bar{u}_{ij} = \frac{h_{ij} - \bar{x}_{ij}}{s_{ij}} y_{ij} \gamma_{ij} \bar{x}_{ij}\).

The following theorem [20] expresses characteristics of the attainability set.

**Theorem 1.** Let the functions \(e_{ij}(t)\) be nonnegative bounded functions having at most denumerable points of discontinuity, let the classes of allowable controls be defined by (5), (6), then the attainability set \(\tilde{D}_{(o,k)}\) meets the following conditions:

a) It is bounded, closed, and convex. It lies in the nonnegative orthant of the space \(\tilde{X} = \mathbb{R}^{(m+n+m)}\);

b) \(D_{(o,k)}^{-} \subseteq \tilde{D}_{(o,k)} \subseteq D_{(o,k)}^{+}\). \tag{8}

Here

\[
\tilde{D}_{(o,k)}^{-} = \left\{ \bar{x}_{ij} \leq 0, \bar{u}_{ij} \leq 0, \bar{u}_{ij} \in [0,1]; \tilde{K}_{ij}^{(o)} \right\},
\]

\[
\tilde{D}_{(o,k)}^{+} = \left\{ \bar{x}_{ij} \leq \bar{x}_{ij}^{(o)}, \bar{u}_{ij} \leq \bar{u}_{ij}^{(o)}, \bar{u}_{ij} \in [0,1] \right\},
\]

where \(\bar{x}_{ij}^{(o)} = \int_{t_0}^{t_f} \max_{j=1,...,m} e_{ij}(t)dt\) under the conditions

\[
x_{ij}^{(k)} = 0 \forall t, \forall i, \varphi_{ij}^{(k)} = 0. \tag{9}
\]

\[
\tilde{x}_{ij}^{(o)} \leq 0 \forall i, \varphi_{ij}^{(o)} = \max \{h_{ij}^{(o)}\} \forall j.
\]

The theorem is of high importance for the preliminary analysis of AMO CS control processes, as the calculation of the values \(\bar{x}_{ij}^{(o)}\) is rather simple, while the sets \(D_{(o,k)}^{-}, D_{(o,k)}^{+}\) let, in many cases, verify the end conditions and calculate the range of variation for the measures of AMO CS ITA.

The sets \(D, D^{+}, D^{-}\) and their images in the criteria space can be represented in a graphic form by Cartesian display.

Fig.1-3 illustrate different representation variants for attainability sets. So they show three service situations for the object \(B_3\) interaction with the objects \(B_1\) and \(B_2\) (see the expressions (3), (5)).

The first service situation (see Figure 1) demonstrates the absence of conflicts (interaction zones (IZ) do not intersect).
The second service situation (see Figure 2) shows the whole intersection of IZ and maximal conflicts. The third service situation (see Figure 3) is intermediate and shows the partial intersection of IZ. If the number of interacting pairs grows, then several Cartesian systems or polar diagrams may be used.

Figures 1: The first service situation demonstrates the absence of conflicts in AMO CS.

Figures 2: The second service situation shows maximal conflicts in AMO CS.
CONCLUSION

An attainability set (AS) is a fundamental characteristic of any dynamic system (in our case – AMO CS). The AS approach determines the range of execution policies in the presence of disturbances over which the system can be guaranteed to meet certain goals. An AS in the state space depicts the possible states of AMO CS schedule to variations of the model parameters (e.g., different capacities and processing times (Ivanov et al. 2010b). In order to interconnect the schedule execution and the performance analysis to the AS in the state space, an AS in the performance space has to be constructed.

Besides, if an AS we know it becomes possible to analyse the dependence between the AMO CS scheduling results subject to the schedule performance and the start and end states. In other words, it becomes possible to define the area in which permissible solutions (e.g., schedules) are included. On the other hand, an AS analysis may show that, with the given resources and at the given time horizon, it is impossible to achieve the required output performance; hence, additional resources should be introduced or the supply cycle shall be expanded (here, the AS approach is similar to goal programming). Limitations of using AS are their dimensionality. However, in most cases, it is possible to approximate AS, e.g., to a rectangular form while estimating the outcomes at four points of an AS.

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