OPTIMAL SCHEDULING OF TWO-STAGE REENTRANT HYBRID FLOW SHOP FOR HEAT TREATMENT PROCESS

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ABSTRACT

The reentrant hybrid flow shop for a heat treatment process is considered in this study. We consider job scheduling in a reentrant hybrid flow shop problem that consists of two statges in series. The first stage is washing, followed by heat treating in the second stage. Each job passes through the first and second stages, respectively, and then re-enter the first stage one more time. Since the first stage must process the jobs twice (with different processing times depending upon the type of the jobs), it becomes the bottleneck in this flow shop problem. To resolve this problem, the jobs needed to be better sequenced to balance the load among the first and the second stages. The objective is to minimize makespan of a set of jobs and increase the utilization of the both stages. This problem was formulated as a mixed integer program (MIP). The results from the data set show that the utilization of the second stage (heat treating) increased from 79.5% to their full capacity at 100%, exceeding the target set by the company at 95%.

INTRODUCTION

Heat treatment is a technique to enhance the properties of materials to a desired level. The heat treatment process is wildly used with automotive parts to increase their strength. The process consists of a heating and cooling cycle. In practice, the parts are heated in heat furnaces and cooled by passing through washing machines. However, prior to the heating parts need to be washed to clean the parts. This production process is equivalent to a two-stage flow shop, whereas the first stage is washing, then heating, and re-washing (reentrance). Each stage may consist of several machines working in parallel. This problem is called a reentrant hybrid flow show scheduling (RHFSS) (Choi et al. 2009).

The RHFSS problem was studied by Watanakich 2001. He studied a scheduling problem for a two-stage hybrid flow shop with machine setup time, and solved the problem using a heuristic. The heuristic is composed of two phases. The first phase constructs a schedule and the second phase assigns jobs with setup time consideration according to the constructed schedule. Vignier et al. 1996 applied a branch and bound based algorithm to schedule jobs in multi-stages flow shop to minimize the makespan. Pan and Chen 2005 formulated a multi-stage reentrant hybrid flow shop as a mixed integer program (MIP). Their study also aims to minimize the makespan of a set of jobs. Choi et al. 2008 studied a mixed binary integer program of two-stage reentrant hybrid flow shop in which a due-date constraint was added to prevent tardiness. Their application was a coating process in automotive manufacturing that jobs were first coated in the paint shop, passed through an oven, and then coated the second time in the paint shop. Yalaoui et al. 2009 proposed several algorithms to solve a scheduling problem of a re-entrant production line. Their objective was to minimize the total tardiness. The performance of their heuristic was better than those of the EDD algolithm.

This paper presents a two-stage reentrant hybrid flow shop problem with due date constraints. In this flow shop all new jobs pass through the first and second stages. Then jobs reenter the first stage again. After the reentrance, jobs exit the flow shop. There are several identical machines (and furnaces) working in parallel at these stages. Jobs require different heat treatment times, but the time needed to wash them are the same. This problem is formulated as a mathematical program, and solved by LINGO commercial software.

The organization of this paper is as the following. The next section describes the two-stage re-entrant hybrid flow shop of the heat treatment process. Then the problem is formulated as a mixed integer program. A numerical example is presented to illustrate an application of the formulation. Finally, the paper is concluded.

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PROBLEM DESCRIPTION

The heat treatment process is composed of three sequencial processes. The first process is washing. Then the parts are heat treated, and passed through the second washing. The washing machines and heat treating furnaces were arranged as a two-stage hybrid flow shop. The flow of the jobs starts from entering through one of the washing machines (working in parallel) to clean the parts. Then each of the jobs will pass through one of the heat treating furnaces. Then each of the jobs re-enters any of the washing machines one more time, as shown in Figure 1.

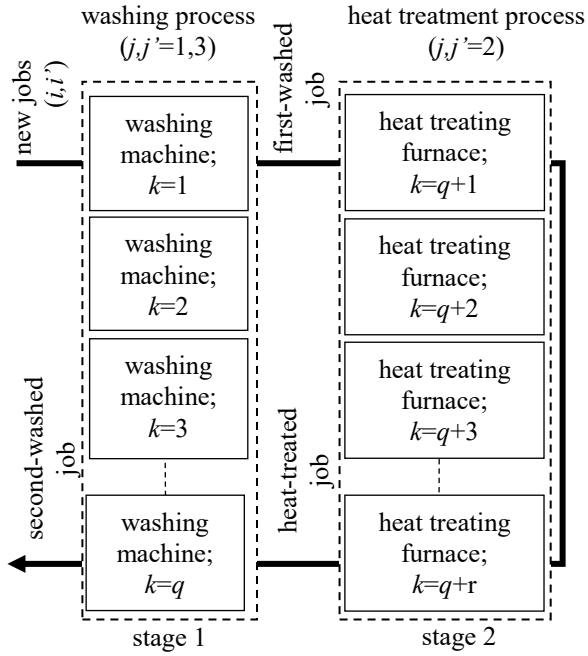

Figure 1: Two-stage Reentrant Hybrid Heat Treatment Flow Shop

The objective is to schedule jobs to these sets of the washing machines and the heat treatement furnaces so that the total makespan of all the jobs is minimized. This implies that the jobs must be sequenced to balance the utilization among these machines. To sequence the jobs, the starting times of each job to the washing machines and the furnaces, and their order must be determined. Since the jobs reenter the washing machines, each job needs two enter times to the washing machines. One for the first pass and the other for the second pass.

The next section describes this two-stage reentrant hybrid flow shop with various constraints as a mathematical program.

MATHEMATICAL FORMULATION

The mathematical model is a system of equations established to solve the problem of optimal scheduling of two-stage reentrant hybrid flow shop for heat treatment process. As earlier mentioned, the objective is to minimize the makespan.

Let X_{ijk} be the assignment of job i to machine (or furnace)k for processing operation j, and S_{ij} be the starting time of job i for processing operation j. The due date of job i is denoted by D_i. The followings are a complete list of all notations.

Indices

i, i' jobs : $i, i' = 1,2,3,\ldots,n$

j, j' operations : $j, j' = 1$: first washing
$j, j' = 2$: heat treatment
$j, j' = 3$: second washing

k machines at stage 1 : $k = 1,2,3,\ldots.q$
furnaces at stage 2 : $k = q+1, q+2, q+3,\ldots,q+r$

Parameters

D_i due-date of jobs i
M very large positive number
n number of jobs
P_{ij} processing time of job i processed at operation j
q number of machines at stage 1
r number of furnaces at stage 2

Decision Variables

S_{ij} starting time of job i processed at operation j

X_{ijk} $=1$ if job i processed at operation j on machine or furnace k, and 0 otherwise

$Y_{iji'j'}$ $=1$ if job i processed at operation j precedes job i' that processed at operation j', and 0 otherwise

Objective Function

The objective function is to minimize the makespan. The equation is :

Minimize C_{max} (1)

where C_{max} is described by Equations (10) and (11)

Constraints

1. For each job, the starting time of the next operation j' must follow the finish time of the current operation j :

$$\sum_{k=1}^{q} X_{ijk}(S_{ij}+P_{ij}) \le \sum_{k=q}^{q+r} X_{ij'k}S_{ij'},\ \forall i, j=1 \text{ and } j'=2 \quad (2)$$

$$\sum_{k=q}^{q+r} X_{ijk}(S_{ij}+P_{ij}) \le \sum_{k=1}^{q} X_{ij'k}S_{ij'},\ \forall i, j=2 \text{ and } j'=3 \quad (3)$$

2. Each job must be processed at operation j by only one machine (or furnace):

$$\sum_{k=1}^{q} X_{ijk} = 1, \forall i \text{ and } j = 1,3 \quad (4)$$

$$\sum_{k=q}^{q+r} X_{ijk} = 1, \forall i \text{ and } j = 2 \quad (5)$$

3. For the first stage, a job can be processed only on a machine at any time.

$$(2 - X_{ijk} - X_{i'j'k})M + (1 - Y_{iji'j'})M + S_{i'j'} - S_{ij} \le P_{ij}, \text{ where}$$
$$1 \le i < i' \le n, j, j' = 1,3 \text{ and } k = 1,2,3,...,q \quad (6)$$

$$(2 - X_{ijk} - X_{i'j'k})M + Y_{iji'j'}M + S_{ij} - S_{i'j'} \le P_{i'j'}, \text{ where}$$
$$1 \le i < i' \le n, j, j' = 1,3 \text{ and } k = 1,2,3,...,q \quad (7)$$

4. For the second stage, a job can be processed only on a machine at any time.

$$(2 - X_{ijk} - X_{i'jk})M + (1 - Y_{iji'j})M + S_{i'j} - S_{ij} \le P_{ij}, \text{ where}$$
$$1 \le i < i' \le n, j = 2 \text{ and } k = q+1, q+2, q+3,...,q+r \quad (8)$$

$$(2 - X_{ijk} - X_{i'jk})M + Y_{iji'j}M + S_{ij} - S_{i'j} \le P_{i'j}, \text{ where}$$
$$1 \le i < i' \le n, j = 2 \text{ and } k = q+1, q+2, q+3,..., q+r \quad (9)$$

5. For each job, the completion time must be less than the makespan.

$$\sum_{k=1}^{q} X_{ijk}(S_{ij} + P_{ij}) \le C_{max}, \forall i \text{ and } j = 3 \quad (10)$$

$$C_{max} \ge 0 \quad (11)$$

6. For each job, the completion time must not exceed the due date.

$$\sum_{k=1}^{q} X_{ijk}(S_{ij} + P_{ij}) \le D_i, \quad \forall i \text{ and } j = 3 \quad (12)$$

7. Constraints describing the decision variables.

$$S_{ij} \ge 0, \forall i, j \quad (13)$$

$$X_{ijk} \in \{0,1\}, \forall i, j = 1,3 \text{ and } k = 1,2,3,...,q \quad (14)$$

$$X_{ijk} \in \{0,1\}, \forall i, j = 2 \text{ and } k = q+1, q+2, q+3,...,q+r \quad (15)$$

$$Y_{iji'j'} \in \{0,1\}, 1 \le i < i' \le n \text{ and } j, j' = 1,3 \quad (16)$$

$$Y_{iji'j} \in \{0,1\}, 1 \le i < i' \le n \text{ and } j = 2 \quad (17)$$

NUMERICAL EXAMPLE

In this example, 15 jobs were processed on two washing machines and two heat treating furnaces. We denote k = 1 and 2 to represent the two washing machines, and k = 3 and 4 to represent the two heat treating furnaces. There are four different types of jobs depend on the heat treatment time; 180, 300, 420 and 600 minutes. Each job was washed for 45 minutes during the first washing and then another 45 minutes for the second washing. All jobs must be completed in four days (or 5,760 minutes) as shown in the Table 1. We assume that the first job starts at time zero.

Table 1: Job Processing Times and Due Dates

Job (i)	Processing time (P_{ij}), min.			Due date (D_i), min.
	First washing j=1	Heat treatment j=2	Second washing j=3	
1	45	180	45	5,760
2	45	300	45	5,760
3	45	300	45	5,760
4	45	420	45	5,760
5	45	600	45	5,760
6	45	180	45	5,760
7	45	300	45	5,760
8	45	300	45	5,760
9	45	420	45	5,760
10	45	600	45	5,760
11	45	180	45	5,760
12	45	300	45	5,760
13	45	300	45	5,760
14	45	420	45	5,760
15	45	600	45	5,760

Results

LINGO software is used to solved this problem. The results are shown in Table 2. The table shows the starting time and completion time of the jobs, or their schedule. The makespan is then calculated by taking the difference of the starting time of the first job that enters the flow shop and the completion time of the last job that leaves the flow shop.

The results show the optimal schedule of the jobs in this two-stage reentrant hybrid flow shop by minimizing the makespan under a specific due date. Each job is assigned to a specific machine and a furnace every time it passes through these stages, and only allowed to access the machine or the furnace at a specific starting time. For example Job 1 enters the washing machine No.2 (k=2) for 45 minutes, starting at the time 1,500 minute, then passes through the heat treating furnace No.2 (k=4) for 180 minutes immediately after leaving the washing machine. The job leaves the furnace at time 1,725 minute. Then the job waits to reenter the washing process through the first washing machine (k=1) at time 2,700 minute. The job is completed at time 2,745 minute. Other jobs can be described similarly.

From the table, the first jobs that enter the flow shop are Jobs 10 and 12, and the last jobs that leave are Jobs 2 and 4. The makespan of all the jobs is 2,790 minutes. With the job schedule details in Table 2, the utilization of heat treating furnaces is at 100% and all the jobs are completed prior to the due date. Details of the schedule of all the jobs by washing machines and heat treating furnaces can be found in Figure 2.

Table 2: Job Schedule

Job (i)	Process (j)	Processing time (P_{ij}), min.	Job assignment, X_{ijk}				Starting time (S_{ij}), min	Completion time, min
			Washing machine		Heat treating furnace			
			k=1	k=2	k=3	k=4		
1	1	45	0	1	0	0	1,500	1,545
	2	180	0	0	0	1	1,545	1,725
	3	45	1	0	0	0	2,700	2,745
2	1	45	0	1	0	0	2325	2,370
	2	300	0	0	1	0	2,445	2,745
	3	45	1	0	0	0	2,745	2,790
3	1	45	1	0	0	0	2,100	2,145
	2	300	0	0	1	0	2,145	2,445
	3	45	0	1	0	0	2,445	2,490
4	1	45	0	1	0	0	2,280	2,325
	2	420	0	0	0	1	2,325	2,745
	3	45	0	1	0	0	2,745	2,790
5	1	45	1	0	0	0	480	525
	2	600	0	0	1	0	525	1,125
	3	45	0	1	0	0	2,370	2,415
6	1	45	0	1	0	0	300	345
	2	180	0	0	1	0	345	525
	3	45	0	1	0	0	2,235	2,280
7	1	45	1	0	0	0	1,170	1,215
	2	300	0	0	1	0	1,845	2,145
	3	45	0	1	0	0	2,145	2,190
8	1	45	0	1	0	0	1,080	1,125
	2	300	0	0	1	0	1,125	1,425
	3	45	0	1	0	0	1,425	1,470
9	1	45	0	1	0	0	90	135
	2	420	0	0	1	0	1,425	1,845
	3	45	1	0	0	0	1,845	1,890
10	1	45	1	0	0	0	0	45
	2	600	0	0	0	1	45	645
	3	45	0	1	0	0	645	690
11	1	45	1	0	0	0	90	135
	2	180	0	0	0	1	645	825
	3	45	1	0	0	0	2,655	2,700
12	1	45	0	1	0	0	0	45
	2	300	0	0	1	0	45	345
	3	45	1	0	0	0	2,610	2,655
13	1	45	1	0	0	0	135	180
	2	300	0	0	0	1	825	1,125
	3	45	1	0	0	0	1,125	1,170
14	1	45	1	0	0	0	45	90
	2	420	0	0	0	1	1,125	1,545
	3	45	1	0	0	0	1,545	1,590
15	1	45	0	1	0	0	45	90
	2	600	0	0	0	1	1,725	2,325
	3	45	1	0	0	0	2,325	2,370

Although there are merely 15 jobs in this example, the problem requires approximately two and a half hours to obtain the final optimal solution by LINGO. First, the software found an upper bound of the optimal solution, then this bound was entered to the software to find the final solution. This shows that scheduling jobs in this type of flow shop is not an easy task.

CONCLUSION

A two-stage reentrant hybrid flow shop with washing and heat treatment processes as the two stages is addressed in this paper. The reentrance occurs only at the first stage, after the jobs leave the second stage. The problem is formulated as a mixed integer program. A numerical example is used to illustrate the application of the formulation, and solved by a commercial software, LINGO. In the example, 15 jobs are scheduled to minimize the makespan under a specific due date. It was found that the schedule of the jobs to the first washing affects the utilization of the heat treating furnaces, whereas the schedule of the second washing is critical to the final makespan. The schedule obtained balances the workload among the machines and furnaces. As a consequence, the utilization of the furnaces increased from 79.5% to 100% (theoretically), exceeding the 95% target set by the company. This model can also be extended to a flow shop with more than two stages, and the jobs may reenter to any of the stages. For large-scale problems, a heuristic may be developed to quickly solve the problem because a multi-stage reentrant hybrid flow shop scheduling is considered as a hard problem.

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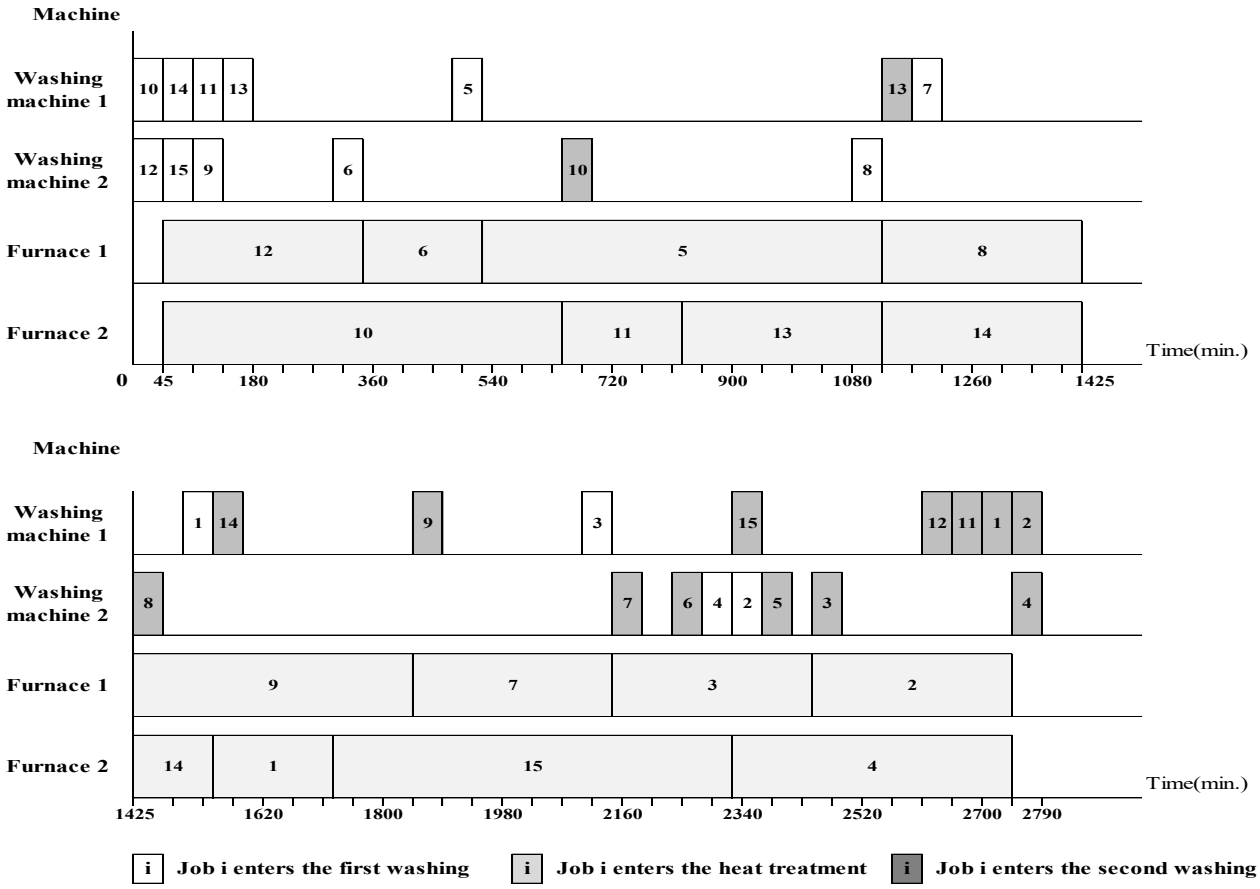

Figure 2: Job Schedule by Machines and Furnaces

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