SIMULATION MODELS OF TWO DUOPOLY GAMES

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ABSTRACT

The paper presents simulation of two classic duopoly games, those of Cournot and Bertrand. The simulation is done with a very simple Discrete Event Simulation system, aGPSS, developed for economics students. With the Cournot model one can study under which conditions the Cournot solution will be obtained even if the parties lack knowledge about demand and costs. The Bertrand model, with stochastic demand, allows the study of cases when a firm can charge a higher price than the other firm and yet sell because the firm charging a lower price sells more than it has expected.

INTRODUCTION

In recent years there has been a strong trend towards modifying some original game theory ideas, completely based on mathematical deduction, by introducing behavioral aspects, often learnt from experiments with people playing games, modeled in accordance with the original game theory models. In contrast to usual game theory models, where one by deduction only obtains one single solution, the outcome in these experiments often vary greatly between each other, often depending on the types of participant, and one hence needs many experiments when studying a particular game.

Since these experiments are time consuming and costly, there has arisen an interest in running computer simulations, where one in the simulation model can introduce various behavioral assumptions. The behavioral assumptions are for both models of this paper based on evidence from experimental games with Swedish students. One can also allow for various types of uncertainty, e.g. about demand and costs.

We shall in this paper present two very simple simulation models, based on fundamental game theory. They both deal with duopoly, i.e. a market situation with two sellers, and homogeneous products, i.e. both firms produce identical products, like wheat or oil. The game theory behind these two models are taught in most micro-economic courses, often in the same class session, as the starting point of oligopoly theory. We shall after having presented our simulation models, discuss their relationship to earlier literature and how they can be expanded.

The first model deals with the oldest of all game theory models, the duopoly model of Cournot from 1838 (Cournot 1838). The distinguishing assumption is here that the two firms just decide on the quantity that they supply to the market and that price is dependent on the total supply. The second game example is the second oldest duopoly model, namely that of Bertrand from 1883 (Bertrand 1883). The main distinction from the Cournot game is that in the Bertrand game both firms set a price, and the quantity sold by each firm is dependent on the price it sets and the price set by the competitor. Furthermore, while there is no uncertainty in our Cournot model, the demand in our Bertrand model is characterized by stochastic variations. For both simulation models we assume that both firms have constant unit costs.

We shall simulate both models in a simple Discrete Event Simulation system, aGPSS. aGPSS is a stream-lined and very simplified version of GPSS (the General Purpose Simulation System), originally an IBM product. aGPSS is mainly used in business schools in shorter courses in Management Science (Born and Ståhl 2013, Ståhl 2007). In contrast to earlier GPSS systems, it has a GUI for building the model and built- in graphics. Both models, as well as the aGPSS system, can be downloaded from www.aGPSS.com. The two models have been used at the Norwegian School of Economics in Bergen in a simulation course focused on applications in the energy sector.

THE COURNOT GAME

Our first example deals, as mentioned, with a game of duopolies producing and selling homogenous goods, like wheat, oil or coal. Although the game is very small, our model contains the main characteristics of a game based on Discrete Event Simulation. To make the model very simple we assume that demand can be described by a linear demand function $p = a\text{-}bQ$, where $Q = \Sigma q$. Both

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producers have the same constant unit cost *c*. We shall first study the analytical solution. For this purpose, we can write firm *i*'s profit as

$$v_i = q_i(p-c) = q_i(a-bQ-c) = q_i b((a-c)/b-Q) = q_i b(A-Q), \text{ where } A=(a-c)/b$$

To simplify further we write this profit on a new scale as $V_i = v_i/b = q_i(A-Q)$

For this duopoly case, with $Q = q_1+q_2$ we hence have

$$V_1 = q_1(A - q_1 - q_2) \quad (1)$$

$= A\,q_1 - q_1^2 - q_1\,q_2$ and $V_2 = Aq_2 - q_2 q_1 - q_2^2$

The standard analytical solution of the game, the non-cooperative solution, found already by Cournot, is obtained when each firm regards the competitor's quantity as given. The optimal quantities are then determined by setting

$$V_1'(q_1) = A - 2\,q_1 - q_2 = 0 \quad \text{and}$$

$$V_2'(q_2) = A - q_1 - 2q_2 = 0$$

This implies in turn that $2\,q_1 + q_2 = A = q_1 + 2q_2$, i.e.

$q_1 = q_2$, i.e. $A-3\,q_1 = 0$, *i.e.*

$$q_1 = q_2 = A/3 \quad (2)$$

The optimization decisions behind this equilibrium presupposes that both parties know the parameters *a* and *b* of the demand function and also the unit cost *c*.

In line with behavioral game theory (Camerer 2003), we now want to simulate a market, where the two firms know neither demand nor costs, but only their own profits and own quantities offered in each period. In experiments, done e.g. by Grubbström (1972) and Edman (2005), one has found that, under certain conditions, the Cournot solution will be obtained after some time also under these assumptions of no knowledge about demand and costs. The question is under what general conditions this can occur. In order to investigate this, a simulation model is necessary, since one in contrast to costly experiments can afford to test a great number of conditions by inexpensive simulations.

The aGPSS model used here for these simulations is quite small and we shall present it in four parts.

Table 1: Part 1 of Cournot model

```
      INPUT   x$oneOld
      INPUT   x$oneNew
      INPUT   x$twoOld
      INPUT   x$twoNew
      INPUT   x$aval
      INPUT   x$chg
      LET     x$size=1
prof1 VALUEOF x$one*(x$aval-x$one-x$two)
prof2 VALUEOF x$two*(x$aval-x$one-x$two)
```

The first part deals with the starting values, before the game begins. For each of the two firms, we first input two starting quantities, *oneOld* and *oneNew* for firm 1, and *twoOld* and *twoNew* for firm 2.

We also input the intercept of the linear demand function *aval* ($A = (a-c)/b$) and a change factor *chg*. We set the initial value of *size* to 1. *chg* and *size* are both, as discussed below, used to try to ensure that we get some convergence of the quantities. We note that variables are preceded by x$ in aGPSS.

At the bottom of table 1 we define two expressions. First, *prof1=one*(aval-one-two),* which corresponds to equation (1) $V_1 = q_1(A - q_1 - q_2)$. Likewise we define the profit function *prof2* for firm 2.

We next proceed to part 2, where the simulation starts.

Table 2: Part 2 of Cournot model

```
      GENERATE  ,,0,1 !  Start conditions
      LET       x$one=x$oneOld! 1's start q
      LET       x$two=x$twoOld! 2's start q
      GRAPH     cl,x$one,x$two
      LET       x$qch1=x$oneNew-x$oneOld
      LET       x$qch2=x$twoNew-x$twoOld
      TERMINATE
*
      GENERATE  ,,1,1      !  Time 1
      LET       x$one=x$oneNew ! 1's first move
      GRAPH     cl,x$one,x$two
      TERMINATE
```

We here have two segments. A first single event is generated at time 0. This puts the starting quantities of the two firms into a graph, after first having given them to the variables *one* and *two*. Next we calculate the initial changes in quantity, *qch1* and *qch2.* Another single event is then generated, at time 1, which puts *oneNew,* by the way of *one*, into the graph at time 1.

Part 3 contains the most important actions of firm 1.

Table 3: Part 3 of Cournot model

```
     GENERATE  2,,2 !  1 moves in 2,4,6...
     LET       x$one=x$oneOld! 1's earlier q
     LET       x$two=x$twoOld! 2's earlier q
     LET       x$oldP1=v$prof1 ! 1's old profit
     LET       x$oneNew=x$oneOld+x$qch1
     LET       x$one=x$oneNew ! 1's new q
     LET       x$newP1=v$prof1!  1's new profit
     IF        x$newp1>x$oldp1,inc
     IF        x$newp1<x$oldp1,dec!
     GOTO      dec,0.5 ! If equal,50 % to DEC
inc  LET       x$qch1=(x$oneNew-x$oneOld)*x$size
     GOTO      join
dec  LET       x$qch1=(x$oneOld-x$oneNew)*x$size
join LET       x$size=x$size*x$chg! Change size
     GRAPH     cl,x$one,x$two ! q1+q2 to graph
     LET       x$oneOld=x$oneNew! Oldq now newq
     TERMINATE 1
```

Firm 1 makes its moves in periods 2, 4, 6, etc., while firm 2, as seen in Table 4 below, makes its moves in periods 3, 5, 7, etc. aGPSS will with the block GENERATE 2,,2 in Part 3 produce a loop where the segment of firm 1's moves will be gone through every other period. The parties will hence take turns making their moves. This sequential quantity setting is fundamental for the model, since it ensures that each party can see the effect of one's own latest move.

We start by calculating firm 1's old profit *oldP1*. For this, we call on the *prof1* value expression by *V$prof1* at the bottom of table 1. To use this expression we first have to set `one=oneOld` and `two=twoOld`. We next calculate the new q_1 for firm 1, *oneNew*, by adding *qch1*, determined in a preceding period, to the old q_1. On the basis of this new q_1 we calculate the profit *newP1* of firm 1 to be obtained in this period. If this new profit is higher than the previous profit, we calculate the next change of q_1, *qch1* (at the line called *inc*), as an increase, i.e. as *(oneNew-oneOld)*size*, where *size* is a factor going over time from 1 to eventually be close to 0. If the new profit is lower than the previous profit, we calculate instead the next change of q_1 (at the line *dec*) as a decrease *(oneOld-oneNew)*size*. In case of equal, i.e. unchanged profits, we go with 50 percent probability to *inc* and with 50 percent probability to *dec*.

Finally in each period 2, 4, 6, etc., we first update the value of *size*. The variable *size* is used when calculating the change at *inc* and *dec*, as seen in the previous paragraph. *size*, which is initially set at 1 (see Table 1), is here multiplied by *chg*. If *chg* is input e.g. as 0.9, then *size* will take the values 0.81, 0.729, etc. This is meant to make convergence possible. We also put the new value of q_1 into a graph and give the value of *oneNew* to the variable *oneOld* to update this for the next period.

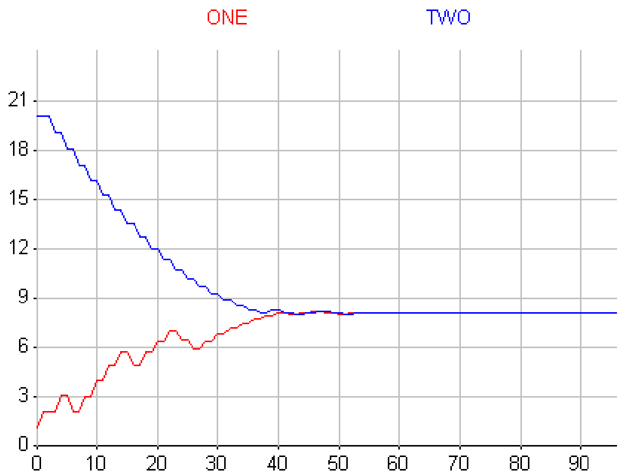
There is a similar segment for the actions of firm 2 in periods 3, 5, 7, etc. This is mainly obtained by letting the variables of firms 1 and 2 in Table 3 switch names. This segment is presented in table 4 below.

Table 4: Part 4 of Cournot model

```
        GENERATE 2,,3! 2 moves in 3, 5, 7..
        LET    x$one=x$oneOld
        LET    x$two=x$twoOld
        LET    x$oldP2=v$prof2
        LET    x$twoNew=x$twoOld+x$qch2
        LET    x$two=x$twoNew
        LET    x$newP2=v$prof2
        IF     x$newp2>x$oldp2,inc2
        IF     x$newp2<x$oldp2,dec2
        GOTO   dec2,0.5
inc2    LET    x$qch2=(x$twoNew-x$twoOld)*x$size
        GOTO   join2
dec2    LET    x$qch2=(x$twoOld-x$twoNew)*x$size
join2   LET    x$size=x$size*x$chg
        GRAPH  cl,x$one,x$two
        LET    x$twoOld=x$twoNew
        TERMINATE 1
        START 100
        END
```

Besides the blocks similar to those of Part 3, Part 4 also contains a statement START 100, allowing for a total of 100 periods, and a finishing END statement.

When a user runs this model, aGPSS will first ask for the values of *oneOld, oneNew, twoOld, twoNew, aval* and *chg*. Let us exemplify with the values 1, 2, 20, 19, 24 and 0.995. In this case, with the initial values of q_1 and q_2 fairly far apart, we get the following graph.



Figures 1: Example of Cournot Game Graph

We see that we after around 40 periods get a convergence on the quantity 8. In this case with *aval* = *A*=24, this is equivalent to the theoretical Cournot equilibrium of 24/3 = 8 of equation (2).

If we input other values, we can see that convergence might not be so fast and in several cases there will not be any convergence. We first keep the value of *aval* as 24 and of *chg* as 0.995, but change the starting quantities. With e.g. 5, 6, 15, 14 or 6, 7, 10, 9, we get convergence a little later and somewhat more oscillations than in Figure 1.

If we, however, next keep the originally input values of 1, 2, 20, 19 and of *chg*=0.995, but set *aval* to 30 or 42, we still get convergence, but with *aval* = 45 or 48 there is no convergence. We next keep all the originally input values except for *chg,* which we set to 0.9 or 0.99. In both cases we do not get convergence. If we set *chg* to 1, we get repeated oscillations between 7 and 9, even if we run for e.g. 300 periods.

By downloading this Cournot model from aGPSS.com one can test out any combination of values. One can then possibly get ideas for how to construct experiments, e.g. as regards the number of periods to be played, but one can also change the parameter *aval* of the demand function. It would also be very simple to change the demand function, e.g. to the constant elasticity function of the Bertrand model below.

THE BERTRAND GAME

Also in the Bertrand game we have two firms that sell identical products. If, as in original model from 1883, demand is deterministic, marginal costs are constant, production is made to order, there are no limits to production capacities and no inventories, then the firm with the lower unit cost will according to theory drive the price down to just below the unit cost of the competitor, who will then not be able to sell anything.

If one, however, assumes that demand is stochastic and that the firms produce to inventory, this conclusion does not hold. This was seen when playing a small DES game, partly similar to the one presented below, in a set of experiments, run with Swedish business students (Ståhl 1993, Ståhl 2010).

The firm with the higher unit cost could survive by being able to sell at a higher price, since the other producer would run out of inventories from time to time, due to the stochastic demand variations, and the buyers would then purchase from the firm with inventories on hand, even if it charged a higher price.

The model presented here is different to the model used for these experiments, by the fact that values are only input once at the beginning and the prices are changed by a simple “robot” algorithm in the program, where prices are changed automatically depending on the size of the inventory. Another assumption is that prices are never set below cost. These behavioral assumptions were influenced by experience from the experimental game runs with the Swedish business students.

The aGPSS Bertrand model is also very simple. We shall present it in four parts.

The first part of the Bertrand program is shown in Table 5 below. We here first input the initial prices and the unit costs of the two firms. We next input the two constants of the demand function, the size factor *a*, here called *aSize*, and the absolute value of the price elasticity *b*, here called *elast*.

Table 5: Part 1 of Bertrand model

```
        SIMULATE 1
        INPUT x$price1
        INPUT x$price2
        INPUT x$cost1
        INPUT x$cost2
        INPUT x$asize
        INPUT x$elast
        INPUT x$priLow
        INPUT x$priAdd
        INPUT x$protim
        LET   x$lowpr=x$price1
stock1  CAPACITY ! 2,000,000,000
stock2  CAPACITY ! 2,000,000,000
demand  VALUEOF asize*x$lowPr^(-x$elast)
```

We also input the two constants *priLow* and *priAdd*, used for changing the prices based on stocks. We finally input *proTim*, the production time.

Since we name the firms 1 and 2 so that firm 1 is the firm with the initially lower price, we also set the temporary lower price, *lowrPr*, equal to *price1*.

Next the program contains the definition of the capacities of the inventories of the firms. Since we do not want to limit these, we give them a very high capacity.

Finally in part 1, we define the demand function, *demand*, that a firm faces, if it charges a lower price, *lowrPr,* than the competitor. This is equivalent to the formula $q = ap^{-b}$. We believe that this demand function in general is more realistic than the linear demand function of the Cournot model above.

The Bertrand model next contains a small segment on the initial values of sales and production quantities shown in Table 2 below.

Table 6: Part 2 of Bertrand model

```
        GENERATE  ,,0,1,1  !  Initial  decisions
        LET x$salLa1=v$demand/52!Sold last week
        LET x$salLa2=v$demand/52
        GRAPH cl,x$price1
        GRAPH cl,x$price2
        TERMINATE
```

The events are generated once, at time 0, with priority to assure that they precede those of the first week in part 3. In order to determine production in period 1, done in part 3, we set for each firm the sales of the preceding week (before the start), *salLa1* and *salLa2*, as possible annual sales divided by 52. We then put the initially read-in prices into graphs.

The next part of the Bertrand program is shown in Table 7 on the next page. We here generate a report and decision event each week of 7 days, starting at time 0. We here first print the number of the week, CL/7, where the clock time CL is the number of days since simulation start. Then the profits as well as then number of units in stock of each firm are printed. All printing is done with 0 decimals, set at the first PRINT block.

Next we check if firm 1 has no stocks. If so, the price is too low and *price1* is increased by *priAdd.* If not, i.e. stocks are not empty (NE), price has been too high and we decrease *price1,* at *lower1,* by *priLow* times the number of units in stock, so that price is lowered more if the stocks are large. The price may, however, not be lower than unit cost. In that case *price1* is set to *cost1*.

In the blocks started with the address *price2* we execute the changes in price done by firm 2, which are similar.

Table 7: Part 3 of Bertrand model

```
month   GENERATE 7,,0 ! At start of every week
        PRINT 'Week',cl/7,0
        PRINT 'Profits firm 1',x$sales1-x$tcos1
        PRINT 'Profits firm 2',x$sales2-x$tcos2
        PRINT 'Stocks firm 1',s$stock1
        PRINT 'Stocks firm 2',s$stock2
        IF stock1=NE,lower1 ! Stocks empty?
        LET+ x$price1,priAdd
! Price 1 up if everything sold
        GOTO price2
lower1  LET- x$price1,s$stock1*x$priLow!
! Price 1 reduced due to remaing stocks
        LET x$price1=fn$max(x$price1,x$cost1)
! Set price to cost if reduced price < cost
price2  IF stock2=NE,lower2 !Same as for firm 1
        LET+ x$price2,priAdd
        GOTO prod1
Lower2  LET- x$price2,s$stock2*x$priLow
        LET x$price2=fn$max(x$price2,x$cost2)
Prod1   ADVANCE x$proTim      ! Production time
        LET x$ordq1=x$salLa1   ! q =last sales
        IF s$stock1>=x$ordq1,prod2! Stocks large?
        ENTER stock1,x$ordq1   ! Into stocks
prod2   LET x$ordq2=x$salLa2   ! q =last sales
        IF s$stock2>=x$ordq2,finish
        ENTER stock2,x$ordq2
finish  GRAPH cl,x$price1
        GRAPH cl,x$price2
        LET x$salLa1=0 ! Set last sales = 0
        LET x$salLa2=0
        x$lowrPr=fn$min(x$price1,x$price2)
! New lowest price set
next    TERMINATE 1
```

Then at the address *prod1* we deal with production. Firm 1 here decides on a production quantity. We assume that it is ignorant about the price set by firm 2, so firm 1 just sets its production *ordq1* equal to the amount it sold in the preceding period, i.e. *salLa1*, determined in part 4 (see Table 8). Firm 2 production at *prod2* is similar.

After a potential production period of *proTim* days, the products are ready to be put into inventory to be available for sales. However, if firm 1's stock, *stock1*, is already larger than its planned production, then the blocks dealing with putting production into stocks are skipped, implying that no production takes place this month. Similar conditions apply to firm 2.

We next put the possibly new prices into graphs and next we set the accumulated sales of the week, *saLa1* and *salLa2*, to 0, so that they can be updated correctly in part 4 (see Table 8).

We also determine what is now the lower price of *price1* and *price2*, and set this as *lowrP*r. Finally, at the end of each week we will with TERMINATE 1 decrease the termination counter, initially set to 52 in Part 4, by 1. In week 52 the counter becomes 0, which stops the simulation.

The rest of the Bertrand model is shown in Table 8.

Table 8: Part 4 of Bertrand model

```
        GENERATE fn$xpdis*52*7/v$demand
* Mean IAT = 364/annual sales
        IF x$price1<x$price2,sltes1 ! p1<p2
        IF x$price1>x$price2,sltes2 ! p2<p1
        GOTO sltes2,0.5! If p1=p2 50/50
sltes1  IF stock1=NE,sell1 ! 1 sells if stocks
        IF stock2=NE,sell2 ! Else 2 if stocks
        GOTO sell1
* If 2 also 0 stocks, go to firm 1 to wait
sltes2  IF stock2=NE,sell2 ! 2 sells if stocks
        IF stock1=NE,sell1 ! Else 1 if stocks
        GOTO sell2
* If 1 also 0 stocks, go to firm 2 to wait
sell1   WAITIF stock1=E! Wait if 1 has 0 stocks
        LEAVE stock1 ! Take 1 unit from stocks
        LET+ x$salLa1,1 ! 1 more sold for 1
        LET+ x$sal1,x$price1 ! Increase revenue
        LET+ x$tcos1,x$cost1 !Incr. total costs
        TERMINATE
sell2   WAITIF stock2=E
        LEAVE stock2
        LET+ x$salLa2,1
        LET+ x$sal2,x$price2
        LET+ x$tcos2,x$cost2
        TERMINATE
        START 52
        END
```

In the GENERATE block we generate every single order from customers wanting to buy at the lower price. The average time between two such orders, measured in days (with 52*7 days a year), is 364 divided by the number units demanded annually, *V$demand*, obtained from the *VALUEOF* in Table 1. The actual time between two orders varies, however, stochastically, since we multiply this average time by *fn$xpdis*, representing the negative exponential distribution. This usually gives a value between 0 and 8, with values below 1 in 67 percent of the times.

If *price1<price2*, i.e. firm1 has the lower price, we go to the address *sltes1*, where we test if firm 1 has any stocks. If so, firm 1 can sell and we go to *sell1*. If *price1>price2*, i.e. firm 2 has the lower price, we go to the address *sltes2*, where we test if firm 2 has any stocks. If so, firm 2 can sell and we go to *sell2*. If the firms have equal prices, we proceed by random with 50 percent chance to *sltes2* and with 50 percent chance to the next block *sltes1*

At the address *sell1* we might first have to wait until there is some unit in firm 1's stocks. We then take one unit out of firm 1's stocks and increase the number of sold units this week by 1 as well as firm 1's revenues by the price of the product. We also add up firm 1's costs of goods sold by the unit cost of the product and then terminate this sales event. At the address *sell2* the corresponding happens to firm 2.

This completes the blocks of the Bertrand model. The model is then, as seen at the bottom of Table 8, finished by START 52 and END.

When this model is run, it produces a table and two graphs. We exemplify with the following input values: *price1*=24, *price2*=26, *cost1*=12, *cost2*=13, *asize*=30000, *elast*=1.5, *priLow*=0.1, *priadd*=0.5, and *proTim*=5.

In table 9 we see an excerpt of the table on weekly data on profits and stocks that is then obtained. .

Table 9: Example of Profits and Stocks

```
Week        1
Profits firm 1      50
Profits firm 2       0
Stocks firm 1        0
Stocks firm 2        4
Week        2
Profits firm 1     102
Profits firm 2      13
Stocks firm 1        0
Stocks firm 2        3
Week        3
Profits firm 1     115
Profits firm 2      51
Stocks firm 1        3
Stocks firm 2        3
```

We see that also firm 2 with higher unit costs can make profits, which is in contrast to the original Bertrand solution, according to which firm 2 would get 0 profits

In Figure 2 we present one of the graphs, namely that of the prices of firm 2, i.e. the firm with the higher costs. The graph of the prices of firm 1 is not very different.

Figure 2: Example of Bertrand Price 2 Graph

We here see that development is quite different from that of the original theory, where the price of the firm with lower cost would go down to be just below the unit cost of the competitor, i.e. in this case 13. Here the price also goes up and, when going down, never to 13.

SIMULATION, EXPERIMENTS AND THEORY

We shall finally comment briefly on the relationship between the two simulation models presented above and the experiments and original theories inspiring the models to try to bring out what is special with the two models above.

We have as a background for our thoughts in this area glanced at some 60 entries on Google, under the headings Cournot and Bertrand, with the subheadings theory, experiments and simulations.

It should first be noted that both regarding the Cournot and the Bertrand game there are lot of issues and complications, like more than two players, that have not at all been touched upon in this paper.

As regards the literature items on Cournot, it should first be noted that many of them deal with the issue of whether there will a cooperative solution, i.e. collusion, or a non-cooperative solution, i.e. a Nash equilibrium, which in this case is the original Cournot solution. In particular many of the experiments deal with this issue (e.g. Thorlund-Petersen 1990). It seems that in many experiments with two and also three players the results are closer to the cooperative solution, but with four and more firms the solution is closer to the non-cooperative one (Huck *et al.*, 2004).

As regards the Bertrand game, there is both much theory and also some experiments dealing outright with capacity restraints, which can lead to the result that also the firm with higher costs an sell (e.g. Brown Kruse *et al.* 1994). It should be stressed that in our Bertrand model there are no capacity restraints. Our result with also the higher cost firm selling is due to our assumption of stochastic demand, an assumption that is rare in the literature.

As regards duopoly simulations in the literature, they mainly refer to the Cournot game and most of them are done in Excel, sometimes with the spreadsheet made available on the web. A few other simulations are stated to have been made in Java and JavaScript, but we have not been able to find the programs. We have not either been able find any simulation of duopoly games done in a discrete event simulation package.

By providing both the simulation system, aGPSS, the two models as well as introductory lessons, free of charge, on the web at www.aGPSS.com, we hope that it will be possible for other oligopoly researchers to extend the two models to cover important aspects left out in our simple models. Extensions to three firms and to different unit costs for the firms in the Cournot model seem like suitable first steps.

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