LIFETIME PROBABILITY OF DEFAULT MODELING FOR HUNGARIAN CORPORATE DEBT INSTRUMENTS

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KEYWORDS

IFRS 9, credit risk, probability of default, expected loss, Markov chain

ABSTRACT

The paper attempts to provide forecast methodological framework and concrete models to estimate long run probability of default term structure for Hungarian corporate debt instruments, in line with IFRS 9 requirements.

Long run probability of default and expected loss can be estimated by various methods and has fifty-five years of history in literature. After studying literature and empirical models, the Markov chain approach was selected to accomplish lifetime probability of default modeling for Hungarian corporate debt instruments.

Empirical results reveal that both discrete and continuous homogeneous Markov chain models systematically overestimate the long term corporate probability of default. However, the continuous non-homogeneous Markov chain gives both intuitively and empirically appropriate probability of default trajectories. The estimated term structure mathematically and professionally properly expresses the probability of default element of expected loss that can realistically occur in the long-run in Hungarian corporate lending. The elaborated models can be easily implemented at Hungarian corporate financial institutions.

INTRODUCTION

Credit risk analysis of corporate financial instruments is a central issue of corporate finances from theoretical and empirical points of view. One of the most important research fields, which is at the same time the fundamental credit risk parameter of the debtors, is the probability of default (PD) that can be quantified both from average PDs mapped to rating classes, or by using statistical PD estimation models.

As an industrial standard, PD models have traditionally been elaborated using cross sectional or some years of historical data, applying multivariate statistical classification methods, estimating PD for one year horizon. It has a rich literature and empirical results also in Hungary (see inter alia Kristóf 2008; Virág and Fiáth 2010; Kristóf and Virág 2012; Virág et al. 2013; Virág and Nyitrai 2014, Nyitrai 2015). Static, one-year PD estimation approach has met supervisory authority expectations and professional best practice for a long time.

However, as an aftermath of the recent financial crisis, substantial regulatory pressure has been made on the further development of credit risk models, laying emphasis on the timely recognition of credit losses, underpinning the establishment and implementation of IFRS 9 standards, coming into effect on 1st January 2018 (IASB 2014). The forward looking impairment model of IFRS 9 calls for the quantification of lifetime credit loss, if significant credit risk deterioration happens to the debtors, which requires lifetime PD modeling.

According to naïve approach, the constant annual PD might be extended to multiple periods. However, on the basis of practical experience, it is easy to see that the time behavior of PD is not constant and non-linear, thereby more complex modeling is necessary.

The aim of this paper is to publish forecast methodological framework and concrete models to estimate long run PD term structure for Hungarian corporate debt instruments, in line with IFRS 9 requirements.

METHODOLOGICAL APPROACHES

Lifetime PD modeling has fifty-five years of history in literature. According to our best knowledge, the first lifetime expected loss model was published by Cyert et al. (1962) for accounts receivables, applying the Markov chain method. Consideration behind the application of discrete Markov chain was the fact that accounts receivables month by month migrate among different delinquency states. Movements among delinquency states were described by migration matrices or transition matrices.

The structural approach of corporate default modeling appeared in the 1970s, the theoretical and methodological foundation of which was formulated by Black and Scholes (1973); Merton (1974); Black and Cox (1976) for corporate bonds. The pioneer publications assumed that the behavior of corporate receivables depend on the asset quality under certain conditions (interest rate, capital structure etc.). It was a difference, however, that Merton (ibid.) equated the time of default with the maturity of bonds, whereas according to Black and Cox (ibid.) a company might become defaulted any time before maturity. The default

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event and its probable time were approached by a perceived incident, when the asset quality of a company first time fell behind a predefined threshold.

Examination of relationships between term and default spreads lead to the appearance of the term structure models (Fama 1986). The three most important findings of these models were that default spreads stand in reverse ratio with term, they depend on economic cycles, and are not necessarily monotonous.

The study of Jarrow et al. (1997) represented a milestone in the literature that elaborated a continuous Markov chain model for corporate bonds, taking into account the credit rating. Changes of credit rating formulated the states of the Markov chain. The transition matrix expressed the probability of remaining in the existing rating class, and the migration to other rating classes.

Within the framework of a comparative analysis Lando and Skodeberg (2002) compared the performance of the continuous multistate Markov model to the traditional, cross sectional, discrete Markov model. The authors concluded that the continuous model outperformed the discrete model.

A problem of applying Markov chain in practice emerged from the observation that the behavior of data modeled by Markov chain is often non-homogeneous. Bluhm and Overbeck (2007) generated PD term structures using homogeneous and non-homogeneous, continuous Markov chains, and compared the results to the fifteen years of cumulated actual default rates published by Standard&Poors. Results with the non-homogeneous model were much better, from which it was concluded that the homogeneity assumption could be set aside.

Since the end of the 1990s – in parallel with the development of retail scoring models – survival analysis models have begun to spread, facilitating the estimation in the function of time, when a client is expected to default (Banasik et al. 1999). Survival time can be estimated with hazard function, which forecasts the magnitude of PD change for any future time.

A great part of PD term structure literature attempted to estimate the term structure from market data (Duffie and Singleton 1999; Jarrow 2001; Longstaff et al. 2005). PDs are often implied from default swap or bond data. Based on practical experience, however, since the majority of loan portfolios contain financial instruments not traded in secondary markets, there is no market data, particularly for loans, from which it would be possible to derive PDs.

A number of studies involved macroeconomic variables and economic cycles into PD modeling to ensure that the relationship between actual economic environment and credit risk is taken into account. Changes in credit risk state are usually explained by industry, location and changes in economic cycle (Gavalas and Syriopoulos 2014).

After studying various literature and empirical models, the Markov chain approach was selected to accomplish lifetime PD modeling for Hungarian corporate debt instruments. The formal description of the method is provided in the next chapter.

MARKOV CHAIN MODELING

A series of random variables formulate a Markov chain, if an observation is in any period in an initial i-th state, and the probability that it migrates to a j-th state in the next period, exclusively depends on the value of i. Let $(X_t)_{t\geq 0}$ denote the series of random variables with {1, 2, …, K} fixed number of classes, where K denotes the default state. The series is a finite first order Markov chain, if:

$$P\left(X_{t+1}=j \middle| X_0=x_0,\ldots,X_{t-1}=x_{t-1},X_t=i\right)=P(X_{t+1}=j|X_t=i) \quad (1)$$

for each t, and i, j∈{1, 2, …, K}

$P_t(i,j)=P(X_{t+1}=j|X_t=i)$ means the probability of transition in t-th period from i-th state to j-th state in (t+1)-th period, and represent the element of the K×K size P_t transition matrix.

The Markov chain is stationary, if $P_t=P$ for each $t\geq 0$. Then the transition matrices are identical in each time. In this case any multi-period transition matrix can be calculated by raising the annual transition matrix to power:

$$P(X_{t+k}=j|X_t=i)=P^k(i,j) \quad (2)$$

The continuous X_t Markov chain is timely homogeneous, if for each i, j state and t, s≥0 times:

$$P(X_{t+s}=j|X_t=i)=P(X_s=j|X_0=i) \quad (3)$$

In case of continuous Markov chain a transition matrix between 0-th and t-th period can be estimated by exponentiating the generator matrix. G generator matrix is such a K×K matrix, where $P(0,t)=\exp(Gt)$. Gt is scalar product, and the exponential function is:

$$\exp(Gt) = \sum_{n=0}^{\infty} \frac{t^n}{n!} G^n \quad (4)$$

The generator matrix has the following characteristics:

$G_{i,j}=0$ for each $i\neq j$

$G_{i,i}=-\sum_{j\neq i} G_{i,j}$.

The elements of the generator matrix relate to the time spent in each rating class. The remaining time in i-th class can be characterized by exponential distribution having $-G_{i,i}$ parameter. Timely homogeneous probabilities of transitions in any horizon can be expressed in the function of the same generator matrix. However, in case of non-homogeneous transitions, the generator matrix depends on time, and can be formulated as follows:

$$P(0,t)=\exp\left(\int_0^T G(t)dt\right) \quad (5)$$

EMPIRICAL RESEARCH

In Markov chain modeling the first research task is to construct a transition matrix based on observed changes of states. In case of corporate credit risk modeling it generally means an annual transition matrix, reflecting the change in rating. The transition matrix can be assembled from internal or external data. It is important to note, however, that only financial institutions possess appropriate internal data for this modeling purpose. Since it is not possible to publish models using internal banking data, we have considered the long run global corporate annual probabilities of transitions of Fitch and Standard&Poors rating agencies.

In line with the objective of the paper, Hungarian idiosyncrasies should be considered in modeling. Since the credit rating history of the best Hungarian corporations strongly correlate with the sovereign rating of Hungary, and in recent years it was in the BBB and BB classes at both agencies, it is assumed that neither Hungarian company can be better than the credit risk characteristics of companies in the BBB classes. Accordingly all 'A' category rating classes were excluded from both transition matrices, thereby the ten remaining classes plus the default class represented the object of analysis. Furthermore it was necessary to handle the problem of withdrawn rating. Assuming that withdrawn rating does not mean upgrading or downgrading, the matrices were normalized by simple scaling. The so constricted and normalized transition matrices showed very corresponding tendencies and results. For further calculations the average transition matrix of Fitch and Standard&Poors was used (Table 1). The PD of each class is reflected by the probability of transition to the D class. If the classification of debtors were already default in the initial period of transition, both annual and lifetime PD of such debtors are 100%. The default class is absorbing state, regardless the fact where the migration is from.

On the basis of the transition matrix a discrete homogeneous, a continuous homogeneous and a continuous non-homogeneous Markov chain model have been elaborated.

Discrete Homogeneous Model

In line with the assumption system of the discrete Markov chain, probabilities of transitions for future terms can be estimated, by raising the transition matrix into power. PD term structure was estimated for twenty years. Matrix multiplication results in cumulated PDs.

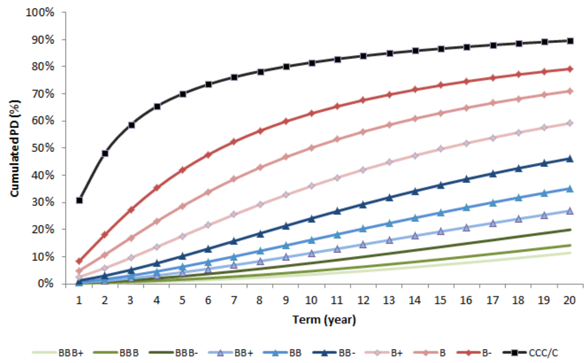

Figure 1: PD Term Structure Estimation for the ten Classes with the Discrete Homogeneous Model

Analyzing the results from practical corporate lending viewpoints, it can be argued that intuitively the PD term structure of the worse classes (B+ and down) might seem to be acceptable, since the trajectories follow a shape with decreasing progress, nevertheless, in particular in the second ten years the rate of growth appears to be exaggerated. However, in case of the better classes (BB- and up) the requirement of decreasing growth in time is not at all met, accordingly the results of discrete Markov chain model must be handled with doubts, since the estimated lifetime PD, as a consequence the expected loss, could be unduly high.

Continuous Homogeneous Model

For continuous Markov chain modeling it is essential to construct a generator matrix. It is easy to see that neither the simple root nor the logarithm of the annual transition matrix is appropriate, because the

Table 1: The applied Annual Transition Matrix

From rating class		To rating class 1 BBB+	2 BBB	3 BBB-	4 BB+	5 BB	6 BB-	7 B+	8 B	9 B-	10 CCC/C	Default D
1	BBB+	87.35%	9.14%	2.02%	0.43%	0.43%	0.11%	0.21%	0.11%	0.00%	0.11%	0.11%
2	BBB	8.04%	81.80%	6.77%	1.59%	0.74%	0.32%	0.32%	0.11%	0.00%	0.11%	0.21%
3	BBB-	1.38%	10.00%	77.86%	5.75%	2.45%	0.96%	0.43%	0.32%	0.21%	0.32%	0.32%
4	BB+	0.55%	2.18%	13.08%	70.56%	7.20%	3.27%	1.20%	0.76%	0.22%	0.55%	0.44%
5	BB	0.22%	0.67%	2.67%	10.91%	71.05%	8.69%	2.56%	1.34%	0.45%	0.78%	0.67%
6	BB-	0.11%	0.34%	0.46%	2.28%	10.81%	69.49%	9.68%	3.64%	1.02%	0.91%	1.25%
7	B+	0.12%	0.12%	0.12%	0.35%	1.85%	9.12%	70.91%	9.81%	3.00%	2.08%	2.54%
8	B	0.00%	0.11%	0.00%	0.23%	0.34%	1.58%	9.73%	68.87%	9.39%	4.87%	4.87%
9	B-	0.11%	0.11%	0.11%	0.11%	0.22%	0.56%	2.89%	12.24%	62.61%	12.69%	8.35%
10	CCC/C	0.12%	0.12%	0.12%	0.00%	0.23%	0.47%	1.40%	3.38%	10.62%	52.74%	30.81%

Source: calculations based on Fitch (2015) and S&P (2016) characteristics of generator matrix are not necessarily realized and negative results might arise. The empirical transition matrix might in itself possess such properties that exclude the existence of a generator matrix, and the same transition matrix might be resulted starting from more generator matrices (Israel et al. 2001).

Within the framework of this empirical research an approximated generator matrix was elaborated applying the regularization procedure published by Kreinin and Sidelnikova (2001) guaranteeing very good fit to the transition matrix considering Euclidean distance.

The first step of regularization is to take the natural logarithm of the annual transition matrix. Where negative values are resulted apart from the diagonal, they must be substituted with zero, so an initial G matrix is received. To achieve that the generator matrix contains zero sums of rows, non-positive diagonal values and non-negative non-diagonal values, the rows of the matrix must be modified considering the relative contribution of each element (Kreinin and Sidelnikova ibid.), formulating a $\tilde{G}$ matrix, the elements of which are calculated as follows:

$$\tilde{g}_{ij}=\left|g_{ij}\right|\frac{\sum_{j=1}^{N} g_{ij}}{\sum_{j=1}^{N}\left|g_{ij}\right|} \qquad (6)$$

The difference of the two matrices gives $\hat{G}$ generator matrix (Table 2), in which the sums of rows are zero:

$$\hat{G}=G-\tilde{G} \qquad (7)$$

In line with the assumption system of the continuous Markov chain, probabilities of transitions for – even fractional – terms can be estimated, by exponentiating the generator matrix to the desired power. Figure 2 summarizes the estimated PD term structure for twenty years.

It is visible from the PD trajectories that results are very similar to the discrete model, accordingly the drawn critical observations are also valid for the continuous homogeneous model. Hence, despite the fact that the exponentiation of the generator matrix almost perfectly estimates the annual transition matrix, the forward looking results are disappointing, since the model systematically overestimates the realistically expected default.

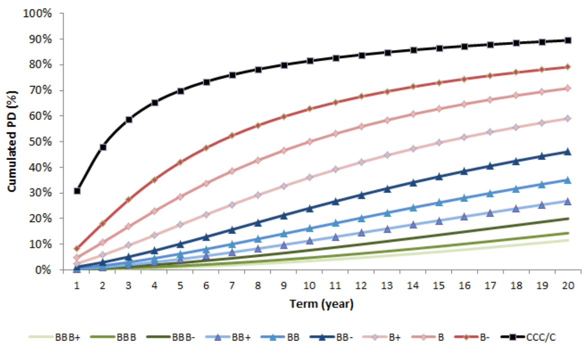

Figure 2: PD Term Structure Estimation for the ten Classes with the Continuous Homogeneous Model

Continuous Non-homogeneous Model

Perceived problems of the homogeneous models are expected to be resolved by giving up the homogeneity assumption. It ensures the flexibility that estimated PD term structure better reflects realistic default trajectories. Again the $\hat{G}$ generator matrix (Table 2) is the starting point, however, it is no more assumed that the transitions are identical, and a timely dependent generator is applied:

$$\hat{G}_t=\phi(t)\times\hat{G} \qquad (8)$$

where $\times$ is matrix multiplication and $\phi(t)=(\varphi_{ij}(t))_{1\leq i,j\leq R}$ is such an R×R diagonal matrix, where:

$$\varphi_{ij}(t)=\begin{cases}0 & \text{if } i\neq j\\ \varphi_{\alpha,\beta}(t) & \text{if } i=j\end{cases} \qquad (9)$$

$\varphi_{\alpha,\beta}(t)$ can be formulated in the function of non-negative α and β parameters per rating class as follows (Bluhm and Overbeck 2007):

$$\varphi_{\alpha,\beta}(t)=\frac{(1-e^{-\alpha t})t^{\beta-1}}{1-e^{-\alpha}} \qquad (10)$$

Table 2: The applied Generator Matrix

	BBB+	BBB	BBB-	BB+	BB	BB-	B+	B	B-	CCC/C	D
BBB+	-14.05%	10.72%	1.97%	0.33%	0.44%	0.06%	0.22%	0.10%	0.00%	0.13%	0.07%
BBB	9.50%	-21.17%	8.27%	1.70%	0.72%	0.27%	0.34%	0.08%	0.00%	0.12%	0.19%
BBB-	1.07%	12.47%	-26.25%	7.46%	2.82%	0.93%	0.36%	0.30%	0.22%	0.39%	0.23%
BB+	0.47%	1.71%	17.57%	-36.38%	9.66%	3.93%	1.18%	0.78%	0.11%	0.71%	0.27%
BB	0.19%	0.52%	2.25%	15.31%	-35.93%	11.97%	2.61%	1.32%	0.34%	1.00%	0.43%
BB-	0.09%	0.35%	0.18%	2.06%	15.31%	-38.30%	13.38%	4.15%	0.87%	0.94%	0.98%
B+	0.13%	0.09%	0.08%	0.16%	1.60%	12.90%	-36.29%	13.55%	3.30%	2.44%	2.04%
B	0.00%	0.13%	0.00%	0.26%	0.19%	1.34%	13.71%	-39.71%	13.68%	6.42%	3.98%
B-	0.13%	0.11%	0.12%	0.11%	0.20%	0.40%	2.89%	18.28%	-50.12%	21.73%	6.15%
CCC/C	0.15%	0.14%	0.16%	0.00%	0.29%	0.54%	1.63%	3.81%	18.43%	-66.36%	41.20%
D	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%

In case of t=1 the diagonal matrix purely consists of $\varphi_{\alpha,\beta}(1)=1$. In the numerator $(1-e^{-\alpha t})$ denotes the exponential distribution of the random variable, while $t^{\beta-1}$ serves for convexity or concavity adjustment. Hence both the flexibility of free parameter selection and the application of well known functions from probability theory are met. By proper selection of α and β parameters, the generator matrix can interpolated to empirically given cumulated PD rates, achieving satisfactory estimation accuracy.
To optimize α and β parameters the long-term actual global corporate cumulated default rates of Fitch and Standard&Poors have been considered, for horizons where data was available at both agencies. The first-year rates equal to the probabilities of transitions to the default class in the annual transition matrix.

Table 3: Cumulated PD Calibration Targets

	Year 1	Year 2	Year 3	Year 4	Year 5	Year 10
BBB+	0.11%	0.34%	0.51%	0.80%	1.11%	2.31%
BBB	0.21%	0.58%	0.83%	1.31%	1.77%	3.78%
BBB-	0.32%	1.03%	1.63%	2.34%	3.11%	6.59%
BB+	0.44%	1.69%	3.07%	4.33%	5.45%	8.93%
BB	0.67%	2.42%	3.82%	5.49%	7.00%	12.30%
BB-	1.25%	3.01%	4.82%	6.45%	7.86%	13.18%
B+	2.54%	5.59%	7.83%	10.21%	11.89%	16.22%
B	4.87%	8.26%	10.41%	13.51%	16.12%	19.90%
B-	8.35%	12.55%	15.09%	18.91%	22.31%	25.40%
CCC/C	30.81%	35.64%	37.78%	40.35%	42.70%	45.29%

Source: calculations based on Fitch (ibid.); S&P (ibid.)

During optimization the monotonically increasing cumulated PDs, the accurate estimation of empirical default rates, and the realistic reflection of practical corporate lending experience also played important role. Table 4 summarizes the so optimized parameters.

Table 4: Optimal α and β Parameters for the Classes

	α	β
BBB+	0.40	1.85
BBB	0.08	0.32
BBB-	0.78	0.21
BB+	0.47	0.96
BB	0.35	0.92
BB-	0.09	0.87
B+	0.11	0.59
B	0.05	0.14
B-	0.93	1.43
CCC/C	0.80	0.01

In line with the assumption system of the continuous non-homogeneous Markov chain, probabilities of transitions for – even fractional – terms can be estimated, by exponentiating the timely changing generator matrix to the desired power. Figure 3 summarizes the estimated PD term structure for twenty years.

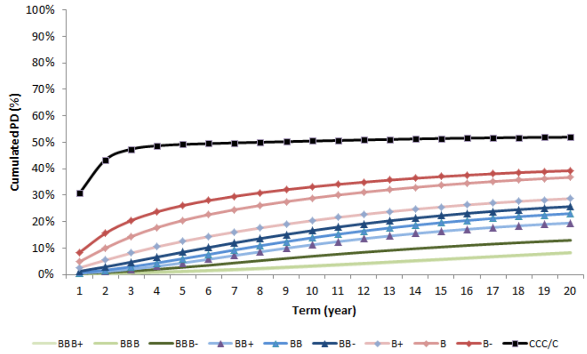

Figure 3: PD Term Structure Estimation for the ten Classes with the Continuous Non-homogeneous Model

The application of continuous non-homogeneus model results in the reduction of marginal default rates in the forecasting horizon, which is most observable in the CCC/C class. It is, however, not surprising, considering that initially this class has the highest PD, and the probability of remaining in the same class is the lowest in the transition matrix.
The non-homogeneous Markov chain intuitively and empirically gives suitable PD term structure in the light of actual default rates published by rating agencies, and also from the viewpoint of practical corporate lending experience. The estimated PD term structure mathematically and professionally properly expresses the PD element of expected loss that can realistically occur in the long-run in Hungarian corporate lending.

CONCLUSIONS

The paper attempted to provide forecast methodological framework and concrete models to estimate long-run PD term structure for Hungarian corporate debt instruments, in line with IFRS 9 requirements.
Lifetime PD modeling has fifty-five years of history in literature. PD term structure can be estimated by various methods. It was concluded that the most viable method to estimate long term PDs for Hungarian corporate debt instruments is the Markov chain approach.
In Markov chain modeling the first task is to construct an annual transition matrix, for which the normalized average of long run global corporate annual probabilities of transitions of Fitch and Standard&Poors rating agencies were considered, diminished to 10+1 rating classes, reflecting the credit risk characteristics of Hungarian corporate debtors.
On the basis of the transition matrix a discrete homogeneous, a continuous homogeneous and a continuous non-homogeneous Markov chain model were elaborated. PD term structures were estimated for twenty years.
Empirical results revealed that both discrete and continuous homogeneous models systematically overestimated the long term corporate PDs. However, the continuous, non-homogeneous Markov chain gave both intuitively and empirically appropriate PD term structure.

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