INTERMEDIARY ACTIVITIES ON DECENTRALIZED FINANCIAL MARKETS

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ABSTRACT
Financial intermediary institutions often compete and cooperate with each other at the same time. These financial actors provide services to their investors, and enter into transactions with them. Moreover, these players very often trade with each other to mitigate their market risks related to their exposures against their clients. Decentralized inter-dealer markets differ from the Walrasian textbook markets in three characteristics: transactions are bilateral, market players form a network, market players possess diverse bargaining power. We develop and simulate a single-period model to describe the benefits of those dealers on the network, who not only mitigate the risk, but connect other dealers on these markets.

1 INTRODUCTION
A significant part of the turnover on financial markets is transacted by financial intermediaries. These players often act as market makers on the market formed by their own investors. The investment banks and brokerages, however, are also in contact with each other. Trading between intermediaries has been drawing increasing attention since the bankruptcy of Lehman Brothers, and rightfully so, since the bank was a major player on the market, and its collapse endangered the entire financial system. The counterparty risk induced systemic risk has been significantly diminished since trading was forced through the clearing houses. Trade among actors of the financial market, however, still remains an important issue.

In this paper we examine the behaviour of intermediaries in a situation where they trade not only with their clients, but also with each other. Their goal with the latter is to decrease the risk of their positions that arise through client trading. Such structure can be observed on the interbank loan-deposit and IRS market, as well as on the market of US treasury bonds, municipal bonds, or credit derivatives (especially CDS).

The most important attribute of the inter-dealer market is that finding a counterparty is costly, therefore different players meet each other with possibly different probabilities. It is important to understand how market makers find an appropriate exchange partner under such friction and how they determine the price of the exchanged assets in the bilateral trades.

We investigate the bargaining and trade mechanisms of the decentralized financial markets. Our simulation based analysis extends the theoretical model of Havran and Szűcs (2016). We use simulations to explain the connection between the profits gained from risk mitigation and intermediation, and the costs of accessing and bargaining with other dealers. We set up several core-periphery networks to show that gains from these intermediary activities are highly influenced by the network centrality of the players.

2 LITERATURE
The first theoretical paper to focus on the trading that is driven by a risk mitigation motive was Borch (1962), investigating the mechanisms of the reinsurance market. In his model, risk averse players enter the market to trade away their risky positions. Borch deduces the market equilibrium as well as the market price, but ignores the network aspect. Transaction prices may differ within the network. The question is how the players determine the transaction prices. The mechanism proposed by Borch is one way to allocate risk, but there are alternatives to it, such as in Csóka et al. (2009).

The past few years have seen numerous theoretical papers that explain trading on financial networks. Atkeson et al. (2013) model CDS markets in such a framework. In their model, intermediary banks that enter the market must face a fixed cost, and all banks have counterparty limits. Banks diminish their exposure on the inter-dealer
market. When two players trade they must agree on the price and quantity. Zawadowski (2013) relates to this, examining the systemic risk of over-the-counter markets. Malamud and Rostek (2012) builds a more general model where risk averse players trade on a fixed network, but, instead of searching and bargaining, their players use the network to trade simultaneously. These players have different price impact and liquidity, which they consider during the trading process. The price impact of a player is, however, independent of their endowment. In the equilibrium players maximize their utilities by trading on the network. In the market model of Babus and Kondor (2013) differently informed risk neutral players are bargaining on a fixed network. The authors present the dispersion of information in the network, and also deduce the market equilibrium.

In this paper we apply the search and bargain approach with risk averse players, similarly to Atkeson et al. (2013), Viswanathan and Wang (2004) and Zhong and Kawakami (2016). The logic of our inter-dealer market model, however, also relates to the theory of Malamud and Rostek (2012). Building on our formerly developed one-shot model (Havran and Szúcés, 2016), we extend this approach by adding new structures to the trading network and re-defining the bargaining process.

3 MODEL

Following the foundations of Havran and Szúcés (2016), we employ a certain set of assumptions to describe the daily routines of an over-the-counter market. All trades are made within a single time period. It means that players must decide in advance what other players to approach and how much to trade with them. However, before the trade there is a tatonnement process. The price is determined during the procedure through bilateral bargaining, and it is a function of the transacted quantity. To sum up, in a bilateral relation, each traded asset is subject to the same price, but the price may vary between different counterparties.

Theory

Let us assume there is a single risky asset on the market, with unknown end of day value. The distribution of this value is common knowledge among all the players: $v \sim N(\mu, \sigma^2)$. Players can also hold a risk free asset that serves as money in the trading process. We define the current wealth of a player as $w = v(\alpha + y) + c$, where $\alpha \in \mathbb{R}$ is the amount of risky assets obtained from customers, $y \in \mathbb{R}$ is the quantity traded with fellow intermediaries, while $c \in \mathbb{R}$ is the amount of money payed or received during the trading. There are $K$ rational, risk averse players on the markets. Players have a single period mean-variance utility function:

$$U_i(y_i) = E(w(y_i)) - \frac{1}{2}\lambda \cdot \text{var}(w(y_i))$$

The $\lambda$ risk aversion coefficient may differ among players.

Each $i$ player has a $\gamma_i \in \mathbb{R}^K$ searching preference vector, that shows the ratios in which the player intends to contact the other players (the reciprocal of the searching friction). Contacting another player does not necessarily result in trading. The sum of the weights is unity: $\sum_{k=1}^{K} \gamma_{ij} = 1$. Players do not trade with themselves, therefore: $\gamma_{ii} = 0$. The searching preference matrix consists of the vectors of all the players as follows:

$$\Gamma = \begin{bmatrix} \gamma'_{11} & \cdots & \gamma'_{1K} \\ \vdots & \ddots & \vdots \\ \gamma'_{K1} & \cdots & \gamma'_{KK} \end{bmatrix}$$

The equation describing the trading process is the following. Let $\tau \in \mathbb{R}^K$ denote the vector of transactions initiated by a certain market maker. The balance of initiated and incoming transactions must equal the final transaction goal, namely $\tau - \Gamma' \tau = y$. The $\tau$ quantity of transactions initiated on the searching network must satisfy the equation below:

$$\begin{pmatrix} I - \Gamma' \tau \end{pmatrix} \tau = \begin{pmatrix} y \\ 0 \end{pmatrix}$$

Where $I$ in the matrix on the left side is the identity matrix of order $K$, while $\downarrow$ is a column vector with all $K$ elements being ones, and $0$ denotes the scalar zero. Using the $\tau$ vector, the quantity of the actual transactions between players $i$ and $j$ can be expressed as follows $t_{ij} = y_{ij} - y_{ji} - y_{ij}$. Players both $i$ and $j$ may gain extra profit via any transaction. The sum of the traded volume conducted by player $i$ is $\sum_{j} t_{ij} = y_i$. The extra profit they achieve combined is allocated between them through Nash bargaining. Depending on their bargaining power, player $j$ gives $d_{ij}$ amount of money to player $i$ based on the following sharing rule:

$$d_{ij} = \theta_{ij} \left( \phi_{ij} [U_j(y_j) - U_j(0)] \right) - \left( 1 - \theta_{ij} \right) \left( \phi_{ij} [U_i(y_i) - U_i(0)] \right)$$

Where we define $\phi_{ij} \in [0, 1]$ as the individual utility increment allocation rule, that shows the ratio of the total utility increment of player $i$ is from the trade with player $j$. The total contribution of the players to dealer $i$ is $\sum \phi_{ik} = 1$. The bargaining power between the two players is denoted by $\theta_{ij} \in (0, 1)$. Greater values of $\theta_{ij}$ mean larger payoffs to player $i$ assuming non-negative profits. $d_{ij}$ is a signed variable, which means that player $i$ may as well be the one paying to player $j$.

The money transacted between players $i$ and $j$ is made of two components. First, player $i$ pays $q_{ij}$ amount to player $j$, where $q$ is the commonly known fair value of the asset on the internal market. Second, player $i$ receives $a_{ij}$ signed amount from player $j$ depending on their bargaining powers. Therefore the cash flow between players $i$ and $j$ becomes $c_{ij} = -q_{ij} + a_{ij}$. 


Player $i$ intends to trade a net amount of $y_i$ and receive a net (signed) amount of $\sum_{k=1}^{K} c_{ik}$ in return. The utility function of player $i$ considering the transactions is:

$$U_i(y_i) = \mu (x_i + y_i) - \frac{1}{2} \lambda_i \sigma^2 (x_i + y_i)^2 + \sum_{k=1}^{K} c_{ik} (y_i)$$  \hspace{1cm} (5)$$

Under a certain $q$ internal market fair value, the net demand of player $i$ is the quantity $y_i$ where the marginal utility of player $i$ is zero, and at the same time maximizes the utility function

$$y_i^*(q) = \left\{ y_i \bigg| \frac{\partial U_i}{\partial y_i} (y_i, q) = 0, \frac{\partial^2 U_i}{\partial y_i^2} (y_i, q) < 0 \right\}$$  \hspace{1cm} (6)$$

where $u_i (y_i)$ is the $i$th element of the explicit utility vector. The net demand does not specify the trading partners, only the intended amount of transactions for the specific player. On a certain market defined by $(x, \lambda, \Gamma, \theta)$ tuple, the net demand function of player $i$ becomes:

$$y_i^*(q) = -x_i + \frac{1}{\lambda_i \sigma^2} \mu - \frac{1}{\lambda_i \sigma^2} q$$  \hspace{1cm} (7)$$

By summing up these demands, one can derive the $y^*$ equilibrium allocation for player $i$, resulting

$$y_i^* = \frac{1}{\lambda_i \sigma^2} \sum_{k=1}^{K} x_k - x_i$$  \hspace{1cm} (8)$$

The fair value that clears the market (sum of the net demands becoming zero) is:

$$q^* = \mu - \frac{\sum_{k=1}^{K} x_k}{\sum_{k=1}^{K} \lambda_k \sigma^2}$$  \hspace{1cm} (9)$$

With the help of the market clearing fair value, one can easily express the price of all bilateral transactions as $p_{ij} = q - d_{ij}/h_{ij}$.

**Examples**

We show some basic examples for illustrating how the market model works. Let us assume the following setup:

- Number of players is $K = 4$;
- The expected value is $\mu = 1$ and the variance of the asset equals to $\sigma^2 = 1$;
- We suppose the same risk aversion parameter for each dealer, $\lambda = 2$;
- We suppose that the bargaining power is $\theta = 1/2$ for each player, hence they have equal power in the bilateral bargains;
- We define $\Gamma$ trade preference matrix as it captures some special network structures: a full graph, star-like network (where player 2 is a middleman), and a circle structure.

Table 1. summarizes the initial distribution of the $x_i$ assets, the initial level of individual $U_i(0)$ utilities, and the net trade amounts in equilibrium. In these settings, the market clearing fair value equals to $q = 1$.

Figure 1 illustrates the trade in equilibrium. On the first network, the asset transfers from player 4 to player 1 in many ways: once directly, then through player 2 or player 3 itself, or through both of these actors.

![Elementary examples of the markets](image-url)
Considering the first case, all market players split their trade. Dealers 1 and 4 increase their utilities through risk mitigation, while dealers 2 and 3 attain some benefits by helping them to trade.

The second case illustrates a player who acts a purely intermediary role. Dealer 2 receives the assets from player 4 and she passes it along to player 1. However, dealer 2 does not modify her starting inventory at the end of the day, she gains from connecting to the others. Furthermore, player 3 is able to reach player 2, but they have no motives for trading.

The third case is about a possible long trade chain. Although, dealer 4 is able to connect to dealer 1 directly, she chooses to share some of her trade between players 1 and 3, because of her trade preferences (e.g. counterparty limit considerations). Hence, dealers 2 and 3 realize gains from trading as intermediary players. The literature refers to this phenomenon of multiple exchanges as hot potato trading.

Comparing the increments of the end of day utilities, dealers gain different profits according to their initial inventories and positions in the network structure. Players may obtain benefits by diminishing their risks through trading in the opposite direction of their initial positions. However, they have to share these gains in order to motivate the others for cooperation. Distribution of the prices in the bilateral trades depends on the inventories and the network topology as well.

4 SIMULATION METHODOLOGY

In this section we briefly present the improvements to the original model as well as the model settings that we employ. We detail the building blocks of the simulated networks, and expose the concept of the asymmetric bargaining powers.

Network construction

There are many definitions of core-periphery networks. According to a stylized, edge-based definition provided by van der Leij et al. (2016), a core-periphery network satisfies three properties: the core agents form a completely connected clique; there are no links between periphery agents; each core agent is connected to at least one periphery agent and vice versa.

We use a more general approach suggested by Rombach et al. (2012) (on page 9): “a CP(N,d,p,k) network has N nodes, where dN of the nodes are core nodes, (1 – d)N of the nodes are peripheral nodes, and d ∈ [0,1]. The edges are assigned independently at random. The edge probabilities for periphery-periphery, core-periphery, and core-core pairs are p, kp, and k²p, respectively, where p ∈ [0,1] and k ∈ [1, (1/p)²].”

We generate the Γ matrix of our model by elements as ϑij = gij / ∑gij, by producing g for all i > j such as

\[ g_{ij} = \begin{cases} 1 & \text{if } \xi > \omega_{ij} \\ 0 & \text{otherwise} \end{cases} \]

and for i ≤ j gii = gji, where \( \xi \sim U(0,1) \) is a uniformly distributed random variable. Furthermore, \( \omega_{ij} \) thresholds are defined by

\[ \omega_{ij} = \begin{cases} k^2p & \text{if } i \in C \land j \in C \\ kp & \text{if } i \in C \land j \in P \\ p & \text{if } i \in P \land j \in P \end{cases} \]

by using notations \( C \) and \( P \) for the set of nodes in the core, and in the periphery respectively. Thus, the matrix of \( g_{ij} \) is symmetric, with zero diagonal and one or zero non-diagonal elements.

For illustration of the characters of the core-periphery networks, we plot graphs of two networks: a core-periphery network with weak and large core (\( d = 0.5, p = 0.4, k = 1.2 \)), and a core-periphery network with strong and small core (\( d = 0.2, p = 0.1, k = 1.8 \)). Figure 4 shows the traded amount among the nodes. The larger nodes have more edges. Green nodes possess positive, red nodes possess negative initial positions. Blue nodes have near to zero initial inventories.

a) Less centralized (weak, large core)
b) More centralized (strong, small core)

Figure 2: Trade networks (simulated)
Negotiation technology revised

The bargaining power in the model was exogenously specified in the bilateral bargains. We extend this approach for examining the bargaining effects on the transactional prices and gains. Let us assume that the counterparties play a bargaining game according to the alternating-offers model of Rubinstein (1982), but with certain modifications.

According to this game, the two players are bargaining across several periods of time on the same day. If they cannot make a deal, the counterparty can always make another offer after some $\Delta > 0$ time, and so on. We assume that after $\Delta$ units of time player $i$ loses $\psi_i \in \mathbb{R}^+ \in [0,1]$ portion of their payoff, which is the cost of skipping a bargain and entering a new one with the same counterparty. This $\psi_i$ cost of accessing the market can also be interpreted as the impatience attitude of a dealer.

Accordingly, $\Delta$ periods of delay discounts the present value of the original payoff of player $i$ by a factor of $(1 - \psi_i\Delta)$. Muthoo (2001) shows that, in a subgame perfect equilibrium, the bargaining power of player $i$ against player $j$ in an infinitesimally short period of time (when $\Delta \to 0$) follows:

$$\theta_{ij} = \frac{\psi_j}{\psi_i + \psi_j}$$

This means that in such a bargaining process the player with cheaper access to the market (the more liquid or the more patient one) may claim a larger portion of the overall profit on a certain day.

We suppose furthermore that there are economies of scale on the costs of accessing market. The average cost of entering a bargain is a decreasing function of the number of edges. It represents that a player with many nodes has a cost advantage of accessing its neighbours on the network. The cost is

$$\psi_i(n) = \psi_i n_i^{\beta - 1}$$

for player $i$, where $\psi$ is the cost of entering into a bargain if the player has only one neighbour, $n_i$ is the number of edges for player $i$, and $0 < \beta < 1$ is the scale parameter.

We remark that $\psi$ is the same for everyone, thus it does not appear in the $\theta$ coefficients.

Setup

We use the following setup for the simulations.

- There are $K = 30$ players on the market;
- Asset value follows $v \sim N(\mu = 1, \sigma^2 = 1)$;
- Homogeneous risk aversion, $\lambda = 2$ for all players;
- Random initial inventories, $x_i \sim N(\mu_i = 1, \sigma^2 = 1)$;
- There are two networks investigated:
  - core-periphery network with weak and large core: $d = 0.5, p = 0.4, k = 1.2$;
  - core-periphery network with strong and small core: $d = 0.2, p = 0.1, k = 1.8$;
- No particular preference for trade. We define the $\Gamma$ trade preference matrix so that a player who has more nodes is willing to trade with all its neighbours in equal amounts. Hence, we determine $\gamma_{ij} = 1/n_i$ for all $j$.
- We set the individual utility increment allocation rule as $\phi_{ij} = \gamma_{ij}$. It means that the utility increment that player $i$ reaches by trading with player $j$ is proportional to the degrees.
- We use three cases for the scale parameters on the cost of bargaining: $\beta = 0, 0.5, 1$ (strong, weak and no economic of scales).

In total, we have 6 different cases (two networks and three scale parameters). We generate 50 runs for each setting and analyze the results, which seems to be enough for describing the asymptotic behavior of our model.

5 EVALUATION OF THE RESULTS

Inter-dealer prices

First, we investigate the distribution of the transactional prices. Let us define the average spread as the difference between the volume weighted average price of buys and the volume weighted average price of sells as the following:

$$s_i = \sum_j \left( \frac{t_{ij}^b}{\sum_j t_{ij}^b} p_{ij} \right) - \sum_j \left( \frac{t_{ij}^s}{\sum_j t_{ij}^s} p_{ij} \right)$$

For identifying the players who are only motivated in the one-sided trade and the players who rather transmit the assets, we introduce the trade balancedness measure. This indicator calculates the relative net trade position to the total amount of individual sells and buys:

$$tb_i = \frac{\left| \sum_j t_{ij}^b - \sum_j t_{ij}^s \right|}{\sum_j t_{ij}^b + \sum_j t_{ij}^s}$$

In Figure 3, we explain the average spreads by the trade balancedness indicators. Combining the results of the 50 independent runs of a particular setup, we plot the average spreads on the less and more centralized networks. Each average spread indicator belongs to a player in a particular run. $Beta 0$ indicates that the bargaining power is proportional to the number of neighbours on the network. $Beta 1$ cases describe the situations where the bargaining powers are symmetric. On the legend, $c.bt.w$ stands for betweenness centrality measure and the colors of the plots indicate the individual betweenness centrality of the actors.
Considering the distribution of the average spreads, one can spot some interesting differences among the effects caused by the network structures.

First, on the less centralized network the core is larger (half of the players) and the core-to-core interlinkage is less dense. The average spread is positive even for some of the players who have unbalanced trade positions. The dispersion of the average spread is higher when the unbalancedness is higher. These actors usually possess large positions in one direction of the trade. Sometimes their spread goes negative, which suggests that they are willing to pay more for trading in a given direction. We remark that these players are able to cover this trade from their benefits from the risk mitigation.

Second, on the more centralized network, the core members are able to enforce more favorable transactional prices, thus they can calculate with higher average spread. Unbalanced trade positions are punished intensely by the market, because the periphery players have less linkage to look for an appropriate counterparty.

We capture the centrality positions of the players by the degree centrality measures, and use a simple colour scheme to identify the network positions. The centrality does not play a crucial role in the spreads, they are rather determined by the initial positions of the actors and their neighbours.

**Profit distributions**

Second, we focus on the benefits from the risk mitigating actions and the gains from the dealer-to-dealer intermediary businesses. To separate these benefits from each other, we introduce a benchmark theoretic utility level that lacks the benefits or losses of the bargains. We define the total utility of trading at fair price as the utility of player $i$ if the agent performs all of her transactions at price $q$.

$$\bar{U}_i(y_i) = \mu(x_i + y_i) - \frac{1}{2}\lambda \sigma^2(x_i + y_i)^2 - q y_i$$

The $U_i(y_i) - U_i(0)$ difference shows the gains from the sum of the activities, and $U_i(y_i) - \bar{U}_i(y)$ indicates the bargaining gains or costs related to the intermediation. The difference $\bar{U}_i(y_i) - U_i(0)$ refers to the gross gain of entering the market with non-zero initial position (in other terms, the gross gain of the hedge). Although, the gross gain is important to understand the motives of the players, they sometimes fail to reach this theoretic level, because of the incurring costs of the exchange. Hence, we apply the concept of the pure (or net) benefits of the hedge and the pure benefits of the intermediary activities. The net benefits of the hedge equal to

$$B_{\text{Hedge}}^i = \frac{1}{2}(\bar{U}_i(y_i) + U_i(0)) - U_i(0)$$

In other terms, it is the non-negative gain that a player surely realizes by entering the market. We remark that dealers with non-zero initial assets have some monopolistic power in this model. The net benefits of risk mitigation do not depend directly on the network structure, rather on the initial asset positions. We define the net benefits of the intermediation as

$$B_{\text{Inter}}^i = U_i(y_i) - \frac{1}{2}(\bar{U}_i(y_i) + U_i(0))$$

which is the non-negative gain that a player can realize by its transmission provided by its own position in the dealer network.
Figure 4 shows the net benefits from passing through the assets by the players’ degree centrality. We calculated the average gain over the 50 independent simulations, and the average degree centralities. Hence, we are able to draw 30 points (players) for each setup by their degree centrality (c.deg). Three settings of the less centralized and three settings of the more centralized networks are presented on the two subplots. The most obvious implication of the model is that the core members acquire more benefits from the trade if the core in the network is heavily connected and the number of core members is relatively low. Moreover, different costs of bargaining imply further diversity of gains, this effect is higher on the more centralized markets.

6 SUMMARY

We explored the relationship between the possible benefits of trading on decentralized markets and two attributes of the traders: status in the network and the initial asset position. Based on a theory of Havran and Szűcs (2016), we constructed an extended model of the decentralized markets, which is able to represent the core-periphery network structure, and offers a bargaining mechanism describing asymmetric relationships in bilateral deals.

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