VALUATION OF THE PREPAYMENT OPTION IN THE BANKING BOOK

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ABSTRACT

One of the most important perspectives of interest rate risk in the banking book (IRRBB) is the valuation of so-called embedded options and the quantification of their impact on the value of bank portfolios. One unequivocal characteristic of mortgage portfolios is the option of prepayment, providing the borrower the possibility of redeeming their debt before maturity. We have created a theoretical model for valuing the prepayment option, based on which it can be demonstrated that, depending on the composition of the given bank portfolio (interest rate level, term to maturity), the prepayment option may have a significant effect on the sum of short-term interest income, as well as on the discount value of the bank portfolio via changing cash flows, and through this the value of economic capital. In this paper we analyse the impact of a possible model specification risk and the impact of changing the composition of banking portfolio under investigation.

INTRODUCTION

Mortgage based loans have become widely known for the public during the crisis of 2008. The importance of the issue and its widespread impact has reached even Hollywood – it is enough to mention only the film "Big short" which is excellent from professional point of view as well.

The paper we are about to present for the conference is related to the previously published analysis and results of Petra Kalfmann related to impact of loan prepayment on the value of the banking book (Kalfmann (2016)). The conclusions of the published model are based on simulation results of the stochastic Cox-Ingersoll-Ross (CIR) interest rate model, analysing the impact of prepayment on a mortgage portfolio in case of ascending and descending yield curves. We analyse the problem further in the following directions:

- we quantify the effect of a possible *model specification risk*;
- we analyse the impact of the changes in the *composition of the banking portfolio* on the original results, considering to change the composition in a way that the average interest rate level and maturity doesn't change, but its deviation.

THE PROBLEM AND THE BASIC MODEL

The logical framework of the original calculations is the following.

a) *Yield curve modelling*. Based on the CIR model, 30-year time horizon, monthly intervals.

b) *Determining the par yield curve*, as current refinancing interest rates.

c) *Determining the refinancing incentive*, based on a comparison between the par interest rate for the given remaining term to maturity and the average interest level of the loan portfolio, until the point the simulated par interest rate drops below the coupon value, when prepayment occurs.

d) *Determining the interest income effect*, assuming refinancing at the current interest rate, i.e. at the par interest rate, calculating the difference between the original cash flow and the altered cash flow, which is calculated based on the new interest rate.

Stochastic interest rate dynamics

The Cox-Ingersoll-Ross model is a continuous, one factor stochastic interest rate model, first proposed by Cox et al. (1985). According to the model, the dynamics of the instantaneous interest rate (r) in the risk-neutral world is given by the following stochastic differential equation

$$dr = a(b-r)dt + \sigma\sqrt{r}dW \tag{1}$$

where a, b and σ are constant, dW is normally distributed random variable with zero mean and dt variance. Mean

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reversion prevails in the model, and volatility is proportional to $\sqrt{r}$. This means that if the short rate increases, then so does its deviation as well. Parameter b is the long run equilibrium level of the interest rate, and the a parameter represents the speed of the mean reversion. Denoting with t^* the length of time, until the half of difference of the actual and equilibrium value disappears:

$$at^* = ln2 \quad (2)$$

The stylized mortgage portfolio of the bank

For the sake of simplicity, the loan portfolio comprises five elements, each representing a sub-portfolio. These sub-portfolios vary in average interest rate levels and remaining term to maturity, and we summarise their characteristics in Table 1.

Table 1: Composition of the examined loan portfolio

Sub-portfolios	**1.**	**2.**	**3.**	**4.**	**5.**
Weight in the portfolio	20%	20%	20%	20%	20%
Avg. interest rate level	4%	5%	6%	7%	8%
Avg. remaining term to maturity (years)	10	5	6	7	4

The aim of the portfolio settings is to obtain diversity in both interest levels and maturities, similarly to the composition of real-life portfolios.

The cited paper compares two different parameter settings for the CIR model (see Table 2) and consequently two scenarios in terms of the yield curve. Either of them results a declining yield curve, while the other an ascending one.

Table 2: Parameters of the CIR model

	Declining YC	*Ascending YC*
r_0	**6%**	**5%**
a	0.5	0.5
b	**4%**	**7%**
σ	5%	5%

One new portfolio was added to the original calculations: we calculated the results with one item considering the average interest rate level and time to maturity of the original sub-portfolios, if we had substituted the 5 sub-portfolios with one item (results are called "one CF" in Table 3).

The interest levels of the examined sub-portfolios and the prevailing interest rate environment, as well as assumptions of changes in the latter, have a significant influence on the results. As a consequence of the assumed effect of declining interest rates, the prepayment option had a significant effect in the case of sub-portfolio 5, while around 5% of the interest income on the portfolio as a whole was endangered. With these sub-portfolios the effects were concentrated in the first 12 months, so that the interest income effect was substantial within the first year.

The effect within one year appears much more forcefully. The results obtained within one year are a potential maximum, since we assumed an optimal decision-making mechanism, while not reckoning with prepayment and transaction charges. Accordingly, assuming a declining interest rate environment for the hypothetical portfolio, at a 95% confidence level 6% of the planned one-year interest income is potentially endangered. The interest income effect is considerably smaller than this, since the declining interest rates are also apparent in declining funding costs, so that the net effect must be considerably more favourable than the theoretical maximum determined for interest income.

Table 3: Statistics of interest income effect – declining yield curve

Sub-portfolios	**1.**	**2.**	**3.**	**4.**	**5.**	**Overall effect**	**One CF**
Coupon rate	*4%*	*5%*	*6%*	*7%*	*8%*		*6%*
Remaining term to maturity	*10*	*5*	*6*	*7*	*4*		*6.4*
Average	-0.28%	-4.27%	-5.93%	-6.54%	-12.37%	-5.03%	-5.65%
Volatility	0.62%	0.46%	0.28%	0.18%	0.41%	0.16%	0.23%
95% confidence level	-1.68%	-5.01%	-6.38%	-6.81%	-13.02%	-5.35%	-6.03%
99% confidence level	-1.73%	-5.34%	-6.58%	-6.95%	-13.35%	-5.44%	-6.18%

In case of an increasing interest rate trajectory, the potentially endangered interest income is also considerable, meaning that the effect remains significant even when assumptions of the interest rate environment theoretically do not favour prepayment. The scale and nature of the effect are fundamentally influenced by the composition of the examined loan portfolio, since the effect appears in the case of sub-portfolios with a high coupon rate, where – as interest rates start from a low level compared to the coupon rate, and thus rising interest rates can be assumed – it makes sense to prepay. Naturally the result thus obtained can also be regarded as a potential maximum.

The Income-based approach vs Capital value-based approach

In the original paper the cash flow effect was also determined both without a discount and based on discounted cash flow. The cash flow effect is useful for examining the impact of the income-based approach, the goal of which is to estimate the interest income effect. The goal of the discounted cash flow effect is to estimate the change in asset value, and to calculate the impact of the economic capital-based approach accordingly. The impact on capital value was lower than on the cash flows only.

Inclusion of early prepayment fee

In the original paper the calculations were performed also with the incorporation of early repayment fee. In respect of the cost 2% fixed fee was considered to be payable in the case of early prepayment. The cost element affects cash flows through the refinancing incentive. Refinancing has taken place in the model if the par rate pertaining to the given residual maturity and the amount of the annualised value of the early prepayment fee distributed for the residual maturity were even lower than the coupon. In certain cases the incorporation of the early prepayment fee diverts the refinancing decision made merely on the basis of the par rate level since refinancing is not worth any more if the fee is taken into account.

The implementation of the early repayment fee further deteriorates the interest income impact. The reason for this is that due to the fee, early prepayment takes place fewer times but when the loan is refinanced according to the model, on average it takes place at an interest rate that is lower than that in the case when there was no early repayment fee in the model.

SUMMARY OF ORIGINAL CALCULATION RESULTS

The model examines the effect of the optimal prepayment option on the cash flow of bank portfolios and the value of economic capital. Based on the model's results, it can clearly be stated that the prepayment option can have a significant impact on both short-term, one-year total interest income and, via changing cash flows, on the bank portfolio's discounted value and through this the value of economic capital.

The results are largely influenced by the portfolio's interest rate structure (coupon rates) and how it relates to changes occurring in the interest rate environment (declining/ascending yield curve). Considering extending the original model with new yield curve calculation method and changing portfolio composition we expect that portfolio composition change shall have more adequate impact on the original results.

Based on the results of the model, it can be asserted that the interest income effect, depending on assumptions made about the interest rate environment, can be very considerable with respect to expected interest income both in the short term and throughout the loan duration. Change in capital value we attribute in the model to the result of change in the present value of cash flows. This approach also allows long-term effects to be quantified, since it determines a theoretical bond price, as well as any change occurring therein. Methodologically, this approach fits into the logic of determining the capital requirement, on which long-term capital management decisions can be based.

EXTENSION 1

In this section we quantify the effect of a possible *model specification risk*. Our question is the following. What happens if we can observe the yield curve, but we do not know the actual dynamics of the interest rate? How serious is the model specification error if we use the CIR model but the 'reality' is driven by a Hull-White (HW) interest rate model with constant volatility? This model enables us to fit an exactly matching yield curve to the one, which has been obtained by the CIR model. In a specific case of the HW model, namely the Vasicek model we can achieve a close fit to the CIR yield curve just changing the 3 parameters of the model: a, b and σ.

According to the Vasicek (1977) model, the dynamics of the instantaneous interest rate (r) in the risk-neutral world is given by the following stochastic differential equation

$$dr = a(b - r)dt + \sigma dW \tag{3}$$

where a, b and σ are constant, dW is normally distributed random variable with zero mean and dt variance. Mean reversion prevails in this model as well, but now the volatility is constant.

Since the formulas of the yield curves (both spot and forward) are known in both interest rate models, we can use numerical approximation to find various parameter settings that result very similar yield curves in the two models. (Of course, there are numerous 'matching' pairs of parameter settings.) We picked the declining yield curve of the original paper (see Table 2) and searched for corresponding Vasicek parameters. Four possible results are summarised in Table 4.

Table 4: Parameter settings resulting in similar yield curves

	CIR	*Vas1*	*Vas2*	*Vas3*	*Vas4*
r_0	**6%**	**6%**	**6%**	**6%**	**6%**
a	0.5000	0.5147	1.0356	0.5000	0.500
b	**4.00%**	**4.11%**	**4.00%**	**4.47%**	**4.00%**
σ	5.00%	2.66%	5.00%	5.00%	1.01%

Figure 1 shows the difference in the forward yield curves derived from the different models summarised in Table 4. On the same figure, we plotted also the forward yield curve arising from Vasicek model if we use exactly the same parameters as in the CIR model. In this case the resulting two yield curves are different on the long end (see the curves VasCIR and CIR on Figure 1).

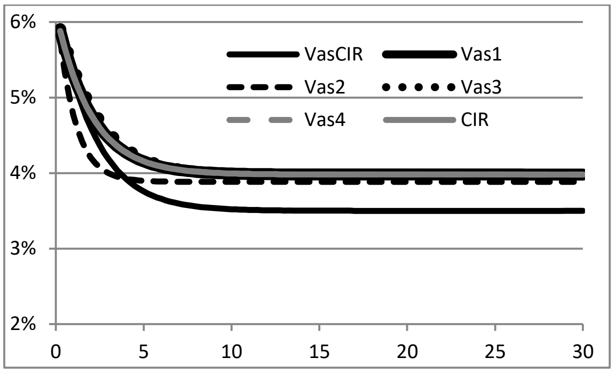

Figure 1: Forward yield curves in different parameter settings

We can achieve a nearly perfect fit to the forward yield curve from CIR by alternating the parameters in the Vasicek. The greatest improvement can be achieved by changing the sigma parameter. Table 5 shows the relative errors related to the different Vasicek parameter settings compared to CIR model.

Table 5: Relative errors related to different Vasicek parameter settings

	Relative error
VasCIR	11,13%
Vas1	5,04%
Vas2	0,87%
Vas3	0,39%
Vas4	0,03%

Doing so, the yield curve is the same, but the short term interest rate dynamics are quite different: unlike in the CIR model, the effect of the random component in the level of the interest rate is now independent from the interest rate itself.

Now we recalculate the interest income effect with our new Vasicek-parameters and compare it to the original results. The results are summarised in Table 6.

Table 6: Results on overall effect in different parameter settings

	CIR	*Vas1*	*Vas2*	*Vas3*	*Vas4*
Avr.	-5,03%	-5,00%	-5,61%	-4,73%	-4,99%
Vol.	0,16%	0,29%	0,28%	0,46%	0,13%
95%	-5,35%	-5,50%	-6,04%	-5,47%	-5,27%
99%	-5,44%	-5,67%	-6,23%	-5,75%	-5,40%

The impact of changing the model with comparable parameters with lowest relative error (*Vasicek1*) is twofold:

1. the average of the interest income effect is somewhat lower compared to the original results;
2. while its volatility is higher, that is true also for the values of the percentiles.

The other calculations compared the results with the original CIR results ceteris paribus:

1. *a*: with higher mean reversion the interest income effect is also higher, while the volatility is also higher;
2. *b*: with higher equilibrium interest rate level the average of the interest income effect is lower, while the percentiles are higher;
3. *volatility*: with lower volatility the effect is almost the same as in case of the original model.

Considering *Vasicek2* results, higher mean reversion means that prepayment happens more often, this is the reason for higher interest income effect. In case of

Vasicek3 the equilibrium interest rate level was increased, which means that the new interest rate at which prepayment happens is also higher, so the final result is also more favourable. In this model the volatility of the interest income effect was the highest from all the cases, also causing higher percentile values at the same time. The *Vasicek4* yield curve was approximating the original yield curve the most, with changing the σ parameter. In this case the result was the closest to the original results, just somewhat lower because of the lower σ.

Figure 2 shows the distribution of results of different parameter settings. As stated above the distribution of the results also show the difference in the general statistics of the results. Since *Vasicek4* yield curve was approximating the original yield curve the most the distribution of the results is almost the same as in case of CIR model.

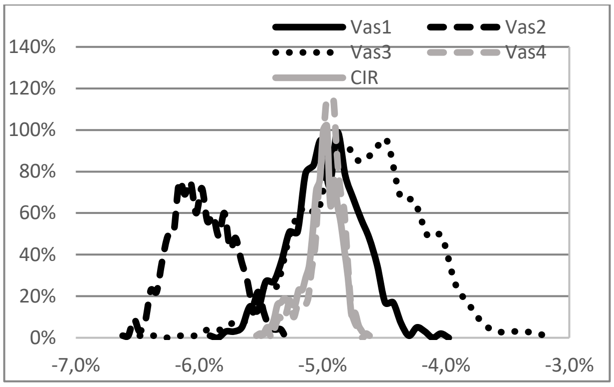

Figure 2: Distribution of results of different parameter settings (CIR and Vasicek)

EXTENSION 2

In this section we quantify the effect of *changes in the bank portfolio composition*. We examine portfolios that have the same *average* interest rate levels and the same *average* maturities, but the standard deviation of these parameters are different.

We consider 4 different compositions:

1. both interest rates and maturities are more diversified compared to original composition;
2. both interest rates and maturities are less diversified compared to original composition;
3. interest rates are unchanged, but maturities are more diversified;
4. maturities are unchanged, but interest rates are more diversified.

The compositions used for the calculations are summarised in Table 7.

Table 7: Composition of the examined loan portfolios

Sub-portfolios	**1.**	**2.**	**3.**	**4.**	**5.**
Weight in the portfolio	20%	20%	20%	20%	20%
Original					
Interest rate level	4%	5%	6%	7%	8%
Maturity (years)	10	5	6	7	4
Case1					
Interest rate level	3%	4%	6%	8%	9%
Maturity (years)	12	4	7	6	3
Case2					
Interest rate level	5%	5.5%	6%	6.5%	7%
Maturity (years)	9.5	5.5	6	6.5	4.5
Case3					
Interest rate level	4%	5%	6%	7%	8%
Maturity (years)	12	4	7	6	3
Case4					
Interest rate level	3%	4%	6%	8%	9%
Maturity (years)	10	5	6	7	4

As a result we can see that changing the interest rate level and the maturities have different impact on the interest income. The results show that if we consider a portfolio composition where interest rates can vary on a wide scale ceteris paribus (*Case4*) it causes higher possible interest rate risk. While changing the maturities in the same direction, i.e. considering a portfolio composition where maturities can vary on a wide scale ceteris paribus (*Case3*), the impact is opposite, it causes lower interest rate risk.

In that case when both interest rates and maturities were put on a less wide scale (*Case2*) the final result actually hasn't changed compared to the original one.

When analysing the results it must be stated that all the above results come from a hypothetical simulation, the portfolio compositions should be varied on a more wide scale to reach a more comprehensive result that could be comparable with real banking portfolios. The results are summarised in Table 8.

Table 8: Statistics of interest income effect of different compositions

Sub-portfolios	Original	Case1	Case2	Case3	Case4
Average	-5.03%	-6.67%	-5.15%	-4.92%	-5.47%
Volatility	0.16%	0.19%	0.10%	0.17%	0.15%
95% confidence level	-5.35%	-7.08%	-5.32%	-5.25%	-5.78%
99% confidence level	-5.44%	-7.22%	-5.38%	-5.35%	-5.88%

CONCLUSION

We analysed the impact of changing model specification and banking portfolio composition of a theoretical model aiming at analysing the effect of prepayment option on a stylized mortgage portfolio. As a result we can state that it makes sense to alternate the yield curve models used for modelling interest rate risk in the banking book. We analysed the results with CIR and a special HW model with constant equilibrium interest rate (Vasicek). In case of Vasicek we used parameter settings causing nearly perfectly fitting yield curve with the originally used CIR model. Even with almost perfectly fitting yield curves we received different results from the original CIR model. We changed parameters a, b and σ. We received the best fit with changing sigma parameter. In this case interest income effect was very close to the original results, but somewhat lower because of the lower sigma.

We also analysed the impact of different banking portfolio compositions. We diversified the portfolio changing the interest rates and maturities of the sub-portfolios, but considering the same average interest rate level and maturity of the original portfolio composition. An interesting outcome was that if we diversify the portfolio along the interest rates we received higher interest income effect, while diversifying the portfolio along the maturities resulted in opposite effect. It must be stated that we used a simple portfolio composition, more robust results can be achieved with analysing more comprehensive portfolio composition closer to real banking portfolios.

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