CONTEXT-AWARE MULTI-OBJECTIVE VEHICLE ROUTING

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ABSTRACT
A vehicle routing deals with designing optimal delivery routes for a fleet of vehicles serving spatially distributed customers. There is a multitude of variants of this problem varying according to problem characteristics and assumptions. Routing is guided by multiple objectives and affected by different context factors. This paper formulates a vehicle routing optimization model allowing for incorporation of arbitrary selected context factors and objectives represented by KPI. In order to assess relative importance of the objectives and context factors, an adaptive approach is proposed. Experimental studies are conducted to illustrate application of the model and the adaptation method.

INTRODUCTION
Vehicle routing deals with finding a set of routes served by multiple vehicles that jointly traverse a number of customers (Breakers et al., 2016). Eksioglu et al. (2009) proposed the taxonomy of the vehicle routing problem. The taxonomy indicates that there are many variants of this problem. These variants differ by decision-making objectives, constraints, parameters and other factors considered. Koç et al. (2016) focus on structural variants routing models. They show derivation of different types of vehicle routing problem variants on the basis of the general model. The vehicle routing problem is inherently multi-objective problem (Current and Marsh, 1993). Optimization of costs, time and travelling distance are typical objectives (Jozefowiez et al., 2008). Additionally, environmental issues (Xiao and Konak, 2016), safety concerns (Carotenuto et al., 2007) and other factors are often mentioned as relevant. However, majority of models consider only one or two objectives. Execution of the routes obtained as a result of optimization is often affected by contextual factors such as traffic accidents (Psaraftis, 1995) and traffic intensity variations over time (Kok et al., 2010). Quality of routing results could be improved if all relevant contextual factors are taken into account already during the route optimization. If every context factor is treated individually, diversity of context factors might lead to a large number of highly specialized routing models. In order to streamline representation of multiple objectives and context, it is possible to treat these factors in a uniform manner.

The objective of the paper is to formulate an optimization model for multi-objective context-aware vehicle routing that supports usage of arbitrary selected objectives and context factors. Different decision-making objectives are represented by their measurements or Key Performance Indicators (KPI). The optimization is performed to minimize travel cost and deviations of actual values of KPI from their target values. KPI used in the objective function are selected for every specific case depending on decision-making needs. The travel cost is context dependent and is a composite of travel distance, time and context factors affecting route execution. Impact of the context factors and importance of KPI is not always known in advance, therefore that is evaluated and incorporated in the optimization model using an adaptive procedure. Application of the proposed model is illustrated using an example.

The rest of the paper is organized as follows. The next sections describe the vehicle routing problem considered. That is followed by model formulation and illustrative example. The paper completes with the concluding section.

PROBLEM STATEMENT
A company providing logistics services operates a fleet of vehicles. It receives customer service requests on the periodical basis. The customers should be visited within a specified time window. The vehicles should be routed to serve the customers at minimum cost where the cost can be expressed as a sum of multiple factors. The routes start and end at a depot. The main decision variables are vehicle allocation to customers and vehicle arrival time at the customer. The routing problem is formulated as a mathematical programming model and optimal routes are found by performing route optimization.

The company has multiple vehicles routing objectives including customer services level satisfactions, environmental impact reduction and ensuring a safe working environment. The objectives are measured by a set of KPI. Every KPI has a target value specified by management. The route optimization should be performed to take into account these specific KPI and their deviation from the target value. Actual values of KPI depend upon routing decisions made. The route
execution is affected by several case specific context factors such as weather, traffic accidents and calendar events. The context factors are beyond company’s control.

The aforementioned route optimization, execution and adaption process is shown in Figure 1.

**MODEL**

The vehicle routing model is formulated as a mathematical programming model. The formulation is based on traditional routing models (e.g., Kallehauge et al., 2005), which are augmented to include treatment of multiple objectives represented by KPI and to account for impact of different context factors.

**Notation**

\( i, j \) – client indices
\( k \) – route type index
\( l \) – vehicle index
\( h \) – KPI index
\( n \) – context element index

**Decision variables**

\( X^i_k \in [0,1] \) - 1 vehicle travels from client \( i \) to \( j \) client along \( k \) route

\( T^i_l \) - arrival time of \( l \) vehicle at customer \( i \)

\( P_h \) - penalty for not achieving \( h \) KPI

\( x_i \in [0,1] \) - is path \( i \) travel as a part of directions from one customer to another

**Parameters**

\( g_{ik} \) - cost of route type \( k \) from customer \( i \) to \( j \)

\( \tau_{ijk} \) - travel time of \( k \) route type from customer \( i \) to \( j \)

\( \mu_{ij} \) - route between customers \( i \) and \( j \)

\( d_i \) - length of path \( i \)

\( \sigma_n \) - weights indicating relative contribution of \( n \) context element

\( \omega \) - weights balancing distance, time and other context factors in composite costs calculation for \( k \) route type

\( \tau_{ijk} \) - visiting time window for customer \( i \)

\( w_n \) - weights balancing importance of composite cost and other KPI

\( \nu_h \) - weights indicating relative contribution of \( h \) KPI

\( \mu_{ii} \) - route between customers \( i \) and \( i' \)

\( \mu_{ij} \) - route between customers \( i \) and \( j \)

**Objective function**

The objective function (Eq. 1) minimizes a weighted sum of the total travel cost and the penalty function for failing to achieve target values of KPI (Eq. 2). The total travel cost \( g_{ik} \) parameter represents not only directs costs for covering some distance or spending time on the trip but also impact of contextual factors. The objective function combines traditional minimization of costs and minimization of non-performance penalty what allows to capture both structural and managerial context
decision-making characteristics of the vehicle routing problem.

\[
\min \ Z = w_1 \sum_{i=1}^{L} \sum_{j=1}^{N} \sum_{k=1}^{K} g_{ijk} X_{ijk}^{l} + w_2 P
\]

\[
P = \sum_{h=1}^{H} y_h P_h
\]

**Constraints**

The objective function is minimized subject to Eq. 9 imposes a penalty if target values of subsequent customer and traveling time between the depot (6). The customer service time windows should be observed (7). Eq. 8 relates arrival times and the flows are balanced by constraint (5). All trips end at the depot (referred as location zero). Incoming and outgoing visits are exactly once. Eq. 4 imposes that trips start at the depot (3).

For instance, one set of weights favors the shortest path while another set of weights favors the safest path. Additional constraints should be added for calculating estimated KPI values depending on the decision variables. The treatment of KPI and context factors is done in a uniform manner allowing to incorporate case specific KPI and context elements in the model with relative ease.

**ADAPTIVE PARAMETERS**

The routing model depends on a number of weighting parameters. The initial values of these parameters are specified in a judgmental manner. Subsequently, they are continuously updated to improve routing performance.

The relative importance of KPI in Eq.2 is determined by the weight coefficients \( v_h \). New values of these coefficients are calculated as

\[
v_{h}^{new} = v_{h} + v_{h}^{'}
\]

where \( v_{h}^{'} \) is the adjustment for the \( h \)th weight and \( v_{h}^{new} \) is the adapted value of the weight factor to be used in the next routing run. The adjustment is determined by maximizing the weighted total penalty (i.e., the biggest increment should be given to KPI with the largest penalty).

The sum of weights is required to be equal to one and the adjustment in a single step cannot exceed a specified threshold.

It is expected that there is a relationship between the context factors and observed values of KPI. This relationship is evaluated using a regression equation

\[
KPI_{h}^{current} = \alpha \cdot u + \gamma \cdot v
\]

where \( u \) and \( v \) are rankings and \( \alpha \) and \( \gamma \) are coefficients of the regression equation.

The weights characterizing relative importance of every context factor are obtained by ranking the context factors according to alpha and computing them as

\[
\sigma_{n} = \alpha \cdot u + \gamma \cdot v
\]

where \( u=1,2,...,r \) are rankings and \( \sum_{n=1}^{r} \sigma_{n} = 1 \)

It is important to note that the KPI values in the routing model are estimated values calculated during the route planning activity while adaptation is performed using the actual values evaluated during the route execution. The adaption is performed periodically once information about route execution is accumulated in the transportation planning application.

**APPLICATION EXAMPLE**

Application of the proposed model is demonstrated using an example. This example is aimed at illustrating the adaption of the model parameters. It is assumed that case specific KPI are KPI1) customer service measured as a percentage of the clients served during the specified time windows; KPI2) travel cost calculated as time spent on deliveries times hourly rate; KPI3) vehicle operating cost incurred for every vehicle used on a given day regardless of distance travelled; and KPI4) safety aimed at avoiding traversal of accident locations.
prone routes measured by an index characterizing frequency of the accidents. Case specific context elements affecting route execution: CTX1) route variability measured as variation of driving time from day to day; and CTX2) route safety measured as a number of accidents observed for the given route. Routing is performed for 20 client requests received for a single day. The optimization is performed using ILOG. The travel distance and time data are retrieved from OpenStreetMap (https://www.openstreetmap.org). The accident data are gathered from a web mapping service. For every pair of customers, three different paths are obtained by varying the context dependency weights in Eq. 10 (Table 1). The path type is referred as Short because the best route between two customers is found giving the most importance to the distance minimization. The path type is referred as Safe because the largest weight is given to the context factors including the safety context element CTX2.

The optimization is performed by allowing to select any of the paths (EXP1), only the shortest path (EXP2), only the safe path (EXP3) and only the balanced path (EXP4). The values of KPI obtained for these four experiments are reported in Table 2. These values are reported relative to EXP1 or the optimal case but Z, which is the actual objective value observed and is a weighted sum of all criteria. It can be observed that EXP2 yields the best result in term of actual costs but neglects the impact of context factors (high value of the cost) and delivers weak customer service performance. Similarly, EXP3 selects safe paths and scores the best according to the safety KPI4. Selecting balanced path (EXP4) expectedly yields results close to the optimal. None of the experiments yields satisfactory customer service performance (KPI1). The actual values recorded were 65 to 75% while the KPI target was 100%. It is important to emphasize that different routes were obtained in different experiments indicating that route selection is indeed context dependent.

Table 1: The weights used to find the best path between two customers.

<table>
<thead>
<tr>
<th>Path type</th>
<th>$\omega_1$</th>
<th>$\omega_2$</th>
<th>$\omega_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Short</td>
<td>0.8</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>Safe</td>
<td>0.1</td>
<td>0.1</td>
<td>0.8</td>
</tr>
<tr>
<td>Balanced</td>
<td>0.34</td>
<td>0.33</td>
<td>0.33</td>
</tr>
</tbody>
</table>

As mentioned before KPI1 did not yield satisfactory performance. The adaption is performed to alter balance among KPI in the objective function, i.e., by changing the weights $\mathbf{\omega}$. Five adaption cycles are performed for EXP1. The same set of customer requests is used in all five cycles though different customer requests would be expected in real life situations. Figure 3 shows the adaption results. KPI values are reported relative to the target values. Values above one indicates that the KPI target value has been achieved. In the first cycle the set of weights $\mathbf{\omega}$ has values (0.25,0.25,0.25,0.25). Given these parameters, the target values are not achieved for KPI1 and KPI2. Adaption allows to reach the target

Table 2: Values of KPI

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Z</th>
<th>Cost</th>
<th>KPI1</th>
<th>KPI2</th>
<th>KPI3</th>
<th>KPI4</th>
</tr>
</thead>
<tbody>
<tr>
<td>EXP1</td>
<td>0.28</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>EXP2</td>
<td>0.40</td>
<td>1.76</td>
<td>0.86</td>
<td>0.75</td>
<td>0.50</td>
<td>0.95</td>
</tr>
<tr>
<td>EXP3</td>
<td>0.72</td>
<td>3.41</td>
<td>1.00</td>
<td>1.85</td>
<td>1.50</td>
<td>0.38</td>
</tr>
<tr>
<td>EXP4</td>
<td>0.30</td>
<td>1.10</td>
<td>1.07</td>
<td>1.08</td>
<td>1.00</td>
<td>0.97</td>
</tr>
</tbody>
</table>
value for KPI2 already after the third cycle with \( v = (0.32, 0.16, 0.36, 0.16) \). The value of KPI1 changes from 0.65 to 0.75 though the target value cannot be achieved. The final set of weights is \((0.277, 0.102, 0.518, 0.103)\).

**CONCLUSION**

The vehicle routing model has been elaborated. The distinctive features of this model are the uniform treatment of decision-making objectives and context elements what allows to include arbitrary or case specific objectives and context elements in the model. It is shown that routes developed are context dependent and the adaptation allows for balancing impact of different objectives.

The vehicles routing model is computationally demanding and an algorithm for improving computational efficiency should be elaborated. The adaptation procedure currently deals with the weights assigned to individual KPI. This procedure should be made more comprehensive to cover all judgmental parameters included in the optimization model. Properties and behavior of the adaption procedure also should be explored in more details.

The model is developed as a part of the industrial research project where the customer service level has been identified as a major vehicle routing concern. Simulation could be used to better evaluate relationships among context elements and the customer service level. It also could be used to analyze sensitivity and stability of the adaption process.

**REFERENCES**


**AUTHOR BIOGRAPHIES**

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