

# PREDICTIVE CONTROL OF A SERIES OF MULTIPLE LIQUID TANKS SUBSTITUTED BY A SINGLE DYNAMICS WITH TIME-DELAY

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## KEYWORDS

Time-delay, Predictive control, flow liquid level control.

## ABSTRACT

The article focuses on control of a system consisting of a series of liquid tanks. Accumulation of individual dynamics causes, that the overall system exhibits high order behaviour. Another effect is a summation of slow responses of individual systems on an input signal leading to a significant time gap in reaction time of the whole system. In order to make control operations more straightforward and increase calculation speed, the mathematical description of gathered dynamics was approximated into a simplified form containing time-delay. The resulting form of the system is regulated by a predictive controller with time-delay compensation. The whole process is simulated in the Matlab environment.

## INTRODUCTION

Systems created by a serial connection of individual elements are often used in chemical and petrochemical industry. Connection of multiple subsystems often creates a complicated dynamic behaviour described by high-order differential equations. In order to control systems of this type, it is appropriate to apply procedures that are able to handle a complex dynamic with a sufficient precision. Nevertheless, these techniques very often require a noteworthy amount of computing power to function with sufficient speed. In order to minimize this negative phenomenon, the mathematical description of the controlled systems tends to be simplified, in exchange for its precision. In such case, a time-delay effect may represent a slow initial system response and it can be used for a simplified description of the slow dynamic.

The computation complexity of model predictive control (MPC) was a subject of several studies as (Morari and Lee, 1998), (Angeli et al., 2012). Even systems with seemingly simple dynamics may cause a significant decrease in performance during on-line optimization. Basic countermeasures like lowering the length of the horizon or the number of variables often lead to a drop in quality of control. Moreover, they limit

advanced functions that make use of MPC viable. Therefore, more elaborate techniques tend to be applied, such as an explicit predictive control (Kvasnica, 2010), faster optimization procedures (Wang et al., 2009) or just the simplification of mathematical description.

The utilization of time-delay effect in order to approximate the original complex behaviour with a simpler expression was a part of several studies (Richard, 2003), (Kubalčík and Bobál, 2012).

The article focusses attention on system formed by subsystems connected in series, which together create a complex dynamic. In this case, the representing subject of regulation is a set of eight liquid tanks. In order to achieve a sufficiently precise control, the mathematical description of the overall system is approximated into a simplified expression containing time-delay which replaces slow reactions accumulated from individual sections. The resulting form is then applied as a reference model for the predictive controller.

The article is organized as follows. The control technique of the predictive control is described in the first section, followed by an analysis of the series of liquid tanks. Next part describes the simplification process of the original system description. The Results section presents simulation outcomes of regulation processes.

## METHODS

The predictive control principle is based on using an optimization to determine the most suitable system input. The core element is an internal model of the controlled process, from which estimates of future output values are predicted. The control algorithm searches for such a vector of input values that will cause the system output value to reach the reference state, while the change of the control input is minimal. In order to achieve this function, several parameters are established to define optimization properties. The whole search for the optimal outcome happens on a time interval from the present to a defined point in the future. This interval is called prediction horizon. The distance of the output signal from the reference value is considered in the area limited by the minimal horizon  $N_1$  and the maximal horizon  $N_2$ . The change in the input signal  $\Delta u(k)$  influences computations from the current

time to the control horizon  $N_u$ . Due to possible inaccuracies in calculations and presence of noise, optimization repeats in every sampling step to minimize the influence of errors. This repetitive approach called receding horizon strategy is visualized on Figure 1.

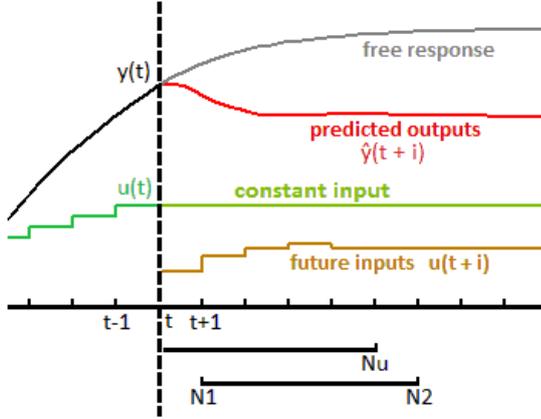


Figure 1: Receding horizon strategy

The estimation of the future outputs is based on the superposition principle and contains a sum of two calculated vectors. The first is a free response derived from the momentary system state and estimation that the input value will remain the same for the length of the horizon. The second is a forced response, a calculated outcome of the series of inputs suggested by the optimization applied on the internal model. Exchange of individual measurements and estimations is depicted in Figure 2.

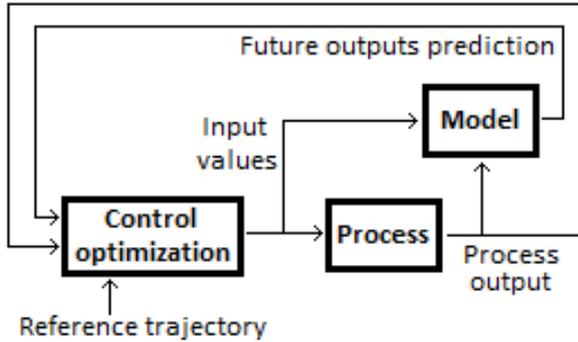


Figure 2: Basic structure of model predictive control

Conditions of the optimization procedure are stated in an objective function representing significance of individual signals involved in the control process. The algorithm aims to find a series of values  $\Delta u(k)$  to  $\Delta u(k+N_u-1)$  that results in the lowest result  $J$ .

$$J = \sum_{i=N_1}^{N_2} \delta(i) [\hat{y}(k+i) - w(k+i)]^2 + \sum_{i=1}^{N_u} \lambda(i) [\Delta u(k+i-1)]^2, \quad (1)$$

where  $\delta(i)$  and  $\lambda(i)$  are weighting values, usually constants representing a ratio of the minimization

between a divergence of output from the desired value and a change of the action value.

In case of time delay the control algorithm is extended with shifting the computed interval limited by the minimal and the maximal horizon by the size of time-delay and system estimations necessary for calculation of the free-response compute future sampling steps up to the size of delay (Normey-Rico & Camacho, 2007).

### System description

The controlled system is formed by eight identical water tanks connected in series, so the liquid flows directly from one tank to another. The goal of regulation is to control the height of water in the last tank by changing the inflow into the first tank.

Liquid levels are considered near an operating point and as such the physical relations between inflow, outflow and accumulation inside the tank can be described by the following equations:

$$F \frac{dh}{dt} = q_{IN} - q_{OUT} \quad (2)$$

$$q_{OUT} = K \cdot h \quad (3)$$

where  $F$  represents the surface area,  $h$  is the surface height,  $K$  is constant from the tank characteristics,  $q_{IN}$  is the flow of the liquid into the tank and  $q_{OUT}$  is the flow of the liquid out of the tank.

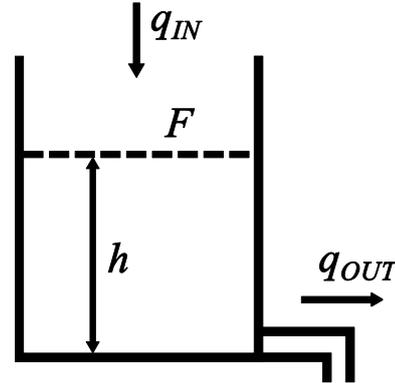


Figure 3: Illustration of a water tank

The transfer function describing the dependence of the surface height on the inflow is

$$h(s) = \frac{1}{\frac{F}{K}s + 1} q_{IN}(s) \quad (4)$$

If we consider that the parameter  $K$  is identical for every tank, then based on the equation (3) it is possible to deduce the surface height from the following relation:

$$h_n = \frac{K_g}{\left(\frac{F}{K}s + 1\right)^n} q_{IN}(s) = G(s)q_{IN}(s) \quad (5)$$

After performing of a substitution  $1/K = K_g$  and  $F/K = T$ , the transfer function of  $n$  tanks is described as

$$G(s) = \frac{K_g}{(Ts+1)^n}. \quad (6)$$

Individual tanks are connected in such way that outflow from one tank is inflow into another.

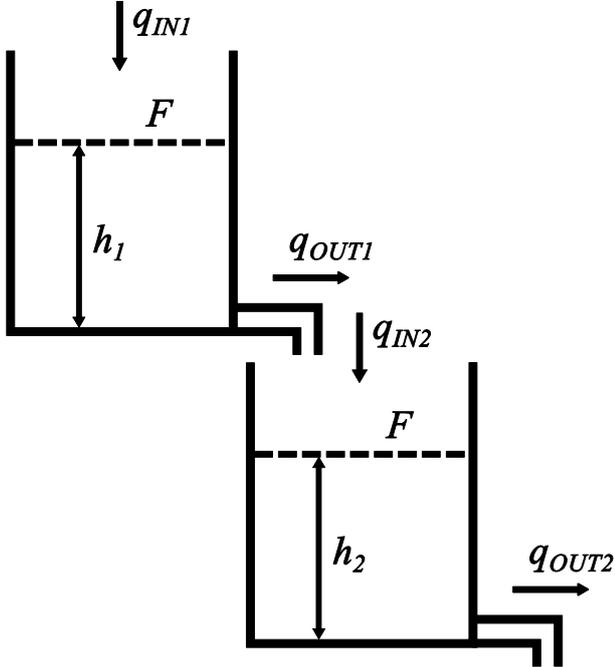


Figure 4: Illustration of two water tanks connected in series

In case of this experiment we consider the following:

Maximal tank height  $h_{\max} = 1.5$  m,

tank diameter  $d_T = 1$  m,

water surface area  $F = \frac{\pi \cdot d_T^2}{4} = 0.785$  m<sup>2</sup>,

time constant  $T = 2$  min,

constant  $K = \frac{F}{T} = 0.3925$  m<sup>2</sup> min<sup>-1</sup>,

system gain  $K_g = 1/K = 3.08$  m<sup>-2</sup> min.

And the system description is therefore given as

$$G(s) = \frac{3.08}{(2s+1)^8} \quad (7)$$

After a transformation of (7) into the numerical description, this system would have a general form expressed as

$$G(z^{-1}) = \frac{b_1 z^{-1} + b_2 z^{-2} + \dots + b_8 z^{-8}}{1 + a_1 z^{-1} + \dots + a_8 z^{-8}} \quad (8)$$

Applying an 8<sup>th</sup> order internal model into the predictive controller as in form (8) would increase the already abnormal computation time of the applied predictive controller (Bobál et al., 2016).

## System approximation

In order to decrease the complexity of the system control, a simplification was performed to express its dynamic as a 2<sup>nd</sup> order system. Additionally, an accumulation of several low level dynamics may lead to a slow response of the overall system which may be interpreted as time-delay.

With aim to achieve a system description in the following form

$$G(z^{-1}) = \frac{b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}} z^{-d} \quad (9)$$

As an identification algorithm was selected the least square method (LSM) aiming for a system with a possible time delay value. This was achieved by repeating the identification, each time with a different value of time-delay. All system responses related to every time-delay in a defined interval were compared with the original 8<sup>th</sup> order system and the most precise outcome was determined by the integrated square error criterion.

The sampling period  $T_0$  was set to 1 minute.

Based on results of the method the final system parameters were determined as

$$G(z^{-1}) = \frac{-0.01638z^{-1} + 0.06371z^{-2}}{1 - 1.815z^{-1} + 0.8314z^{-2}} z^{-5} \quad (10)$$

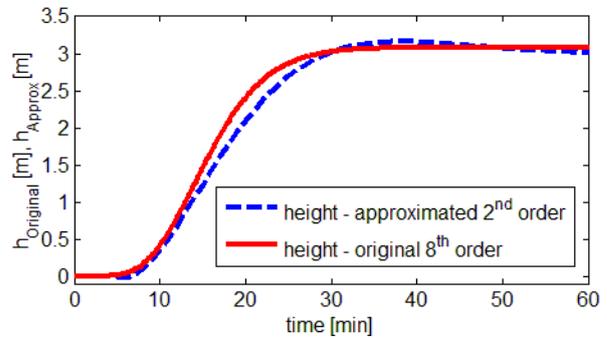


Figure 5: Comparison of transfer functions of original 8<sup>th</sup> order and approximated 2<sup>nd</sup> order systems

Figure 5 illustrates the similarity between the original 8<sup>th</sup> order system and the newly approximated 2<sup>nd</sup> order system. As can be seen the description received from the LSM manages to provide a system with a very similar behaviour with a slight difference in the area where the signal begins to settle. Furthermore, the approximated system exhibits more oscillating performance.

## RESULTS

The approximated system was used as the internal model for the Generalized predictive controller, in order to provide estimated responses on the series of input signals.

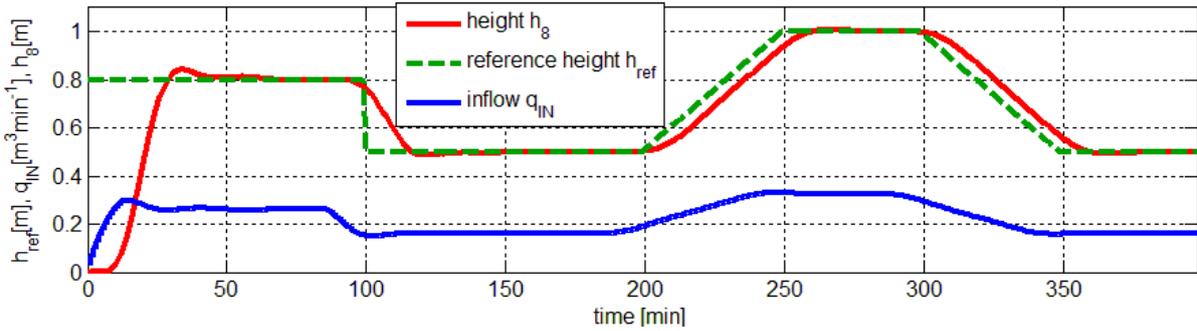


Figure 6: Predictive control of the 8<sup>th</sup> order process approximated by a 2<sup>nd</sup> order model

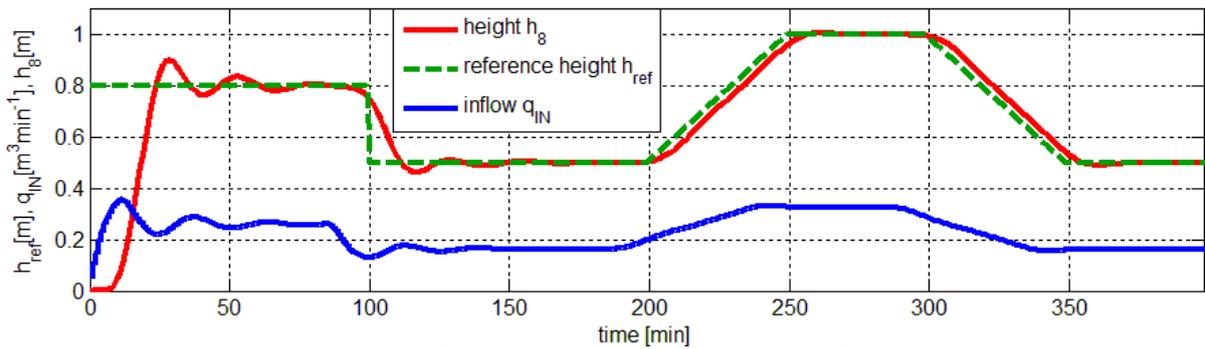


Figure 7: Predictive control of the 8<sup>th</sup> order process approximated by a 2<sup>nd</sup> order model with a decreased optimization of changes in the input signal

The reference trajectory was shaped to contain sudden steps, static section as well as gradual linear changes in positive and negative direction. Moreover, the duration of the simulation was set to enable stabilization after each change.

Figure 6 shows how the predictive controller was able to follow the reference trajectory with precision despite the simplified description of the controlled system.

Variability of the predictive control also offers an option of a faster transition in exchange for the precision of control. By decreasing the weight parameter for the optimization of changes in the input signal, the overall process gains a faster performance and it is able to follow ramp changes more closely, on the other hand the stabilization during step changes in the reference trajectory exhibit a significant increase in oscillations. Results of regulation altered in this way are illustrated in Figure 7 and displays an improvement in areas of smaller but frequent changes of the desired value. The disadvantage of lost precision can be seen at around 30 minutes and 120 minutes of simulation time.

## CONCLUSION

The paper presented an experiment where instead of a complex high-order system a simplified interpretation was performed, consequently decreasing demands for the control algorithm and computation time.

The original series of eight 1<sup>st</sup> order systems was tested by excitation signals. Based on gained responses, the whole process was identified as a 2<sup>nd</sup> order system. Furthermore, an accumulation of slow initial increases was replaced with time-delay effect. Consequently, this approximation managed to decrease computation demands of the applied predictive controller.

The control approach was verified by a subsequent simulation, which has demonstrated its applicability without significant errors in control.

Additional experiment has shown that it is possible to increase the speed of transitions in the output variable and its precision during linear changes by changing weighting parameters.

A series of low-order systems, creating a single high-order system can be controlled as a low-order system with time-delay.

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