

# NUMERICAL DISCRETE ELEMENT SIMULATION OF SOIL DIRECT SHEAR TEST

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## KEYWORDS

soil model, direct shear test, 3D DEM, discrete element simulation.

## ABSTRACT

One of the most common methods to measure soil mechanical properties (namely cohesion and internal friction angle) is direct shear box test. In this paper the development of a three-dimensional (3D) discrete element soil model for simulation of a cohesive soil's direct shear test is presented. The aim was to calibrate the properties of the Hertz-Mindlin with bonding contact model available in EDEM software to the results of real direct shear box test. The cohesion and internal friction angle were calculated from the equation of the Mohr-Coulomb line of the soil model. Results show that direct shear laboratory test can be simulated very well using discrete element method (DEM). The model's calculated cohesion and internal friction angle values and the corresponding mechanical properties of real cohesive soil were in good agreement with relative error of 4.44 percent and 4.66 percent, respectively.

## INTRODUCTION

In the last few decades the development of agricultural machines led to increase the size and mass of the machines and therefore the stress applied into agricultural soils as well. In soil-wheel interaction normal- and shear stress is generated in the soil by both driven and non-driven wheels. In designing agricultural machines and their equipment (e. g. running gear and tillage tools) engineers need to know the mechanical properties, namely cohesion and internal friction angle of the soils. These properties depend mostly on the soil's moisture content and bulk density (Sitkei 1967).

The most common methods to measure soil's shear strength parameters are direct shear and triaxial laboratory tests (McKyes 1985). Direct shear test can be easier to conduct and therefore is widely applied in agricultural researches (Dirgeliene et al 2014). From the results of the test cohesion and internal friction angle can be identified using the Mohr-Coulomb criterion.

In addition in the 20th-21st century the information technology has been evolved a lot allowing to create and use numerical simulations for modelling real materials.

The most known method is Finite Element Method (FEM) which is used mainly in simulation of homogeneous materials (e. g. steels and plastics). Efforts have been made for modelling soil with FEM (Mouazen and Neményi 1998; Chi and Kushwaha 1990) but FEM is not suitable for modelling granular materials (e. g. soils). Granular assemblies consist of individual elements and sub-assemblies with non-continuous displacements which cannot be simulated in FEM properly. In Discrete Element Method (DEM), developed by Cundall and Strack (Cundall and Strack 1979) the material is modelled as a group of individual particles therefore DEM can be a correct choice to simulate soil material. DEM has been used in several research works to study the dynamic motion of lunar (Nakashima et al 2010) and Mars wheel (Knuth et al 2012) or to simulate soil-tool interaction as well (Tamás et al 2013). Many researchers used DEM to simulate the direct shear test of real soil (Tamás et al 2013 and Sadek et al 2011) as well but non of them used the Hertz-Mindlin with bonding contact model in their simulations.

In this paper DEM was used to simulate cohesive soil's behaviour under direct shear test. The software used to perform the simulations was EDEM 2.7 Academic, available from DEM Solutions Ltd. Numerical direct shear test were performed and to calibrate the contact properties of the soil model a new process was used and will be presented in the paper.

## MATERIALS AND METHODS

In discrete element simulations the whole process is divided into small timestep of  $dt$ . The displacements of each elements are calculated from the forces and moments acting on them using Newton's 2<sup>nd</sup> law in every single timestep of  $dt$ . The forces and moments can be determined from the overlaps of the particles according to the used contact model. Therefore contact models play an important role in discrete element simulations because the material properties can be modelled properly by using the correct contact model with sufficient contact properties between the elements. In this paper the Hertz-Mindlin with bonding contact model available in EDEM 2.7 Academic software was chosen to simulate cohesive soil material. This contact

model is based on the work of Hertz (Hertz 1882), Mindlin (Mindlin 1949), Mindlin and Deresiewicz (Mindlin and Deresiewicz 1953) and Potyondy and Cundall (Potyondy and Cundall 2004). In this contact model the forces acting on an element are divided into normal and tangential (shear) directions. The normal force can be calculated as follows:

$$F_n = \frac{4}{3} \cdot E^* \cdot \sqrt{R^*} \cdot \delta_n^{\frac{3}{2}}$$

where  $\delta_n$  is the normal overlap of the contacting elements,  $E^*$  and  $R^*$  are the so-called equivalent Young's modulus and equivalent particle radius, respectively and can be calculated from the properties of the two contacting elements using Equation (1) and Equation (2).

$$E^* = \left( \frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2} \right)^{-1} \quad (1)$$

where  $\nu_1$  and  $\nu_2$  are the Poisson ratio and the  $E_1$  and  $E_2$  are the Young's modulus of the first and second contacting element, respectively.

$$R^* = \left( \frac{1}{R_1} + \frac{1}{R_2} \right)^{-1} \quad (2)$$

where  $R_1$  and  $R_2$  are the radius of the first and second contacting element, respectively.

In addition tangential force can be transmitted from one particle to another. The magnitude of this force is calculated by Equation (3).

$$F_t = -S_t \cdot \delta_t \quad (3)$$

where  $\delta_t$  is the tangential overlap of the contacting elements and  $S_t$  is the tangential stiffness calculated as follows:

$$S_t = 8 \cdot G^* \cdot \sqrt{R^*} \cdot \delta_n \quad (4)$$

In Equation (4) the  $G^*$  is the equivalent shear modulus which is determined from the shear modulus and the Poisson ratio of the contacting elements,  $G_1$ ;  $G_2$  and  $\nu_1$ ;  $\nu_2$ , respectively.

$$G^* = \left( \frac{1-\nu_1}{G_1} + \frac{1-\nu_2}{G_2} \right)$$

The tangential force is limited by its maximum value which can be determined from the Coulomb friction:

$$F_t \leq F_n \cdot \mu_s$$

where  $\mu_s$  is the static frictional coefficient between the elements.

There are additional damping forces acting in normal,  $F_n^d$  and tangential directions,  $F_t^d$  as well. These can be calculated with the following formulas:

$$F_n^d = -2 \cdot \sqrt{\frac{5}{6}} \cdot \beta \cdot \sqrt{S_n \cdot m^*} \cdot v_n^{rel} \quad (5)$$

$$F_t^d = -2 \cdot \sqrt{\frac{5}{6}} \cdot \beta \cdot \sqrt{S_t \cdot m^*} \cdot v_t^{rel} \quad (6)$$

In Equation (5) and Equation (6)  $m^*$  is the equivalent mass,  $S_n$  is the normal stiffness,  $\beta$  is given below,  $v_n^{rel}$  and  $v_t^{rel}$  are the relative normal and tangential velocities of the contacting elements, respectively. These can be determined by using the following equations:

$$m^* = \left( \frac{1}{m_1} + \frac{1}{m_2} \right)^{-1}$$

$$S_n = 2 \cdot E^* \cdot \sqrt{R^*} \cdot \delta_n$$

$$\beta = \frac{\ln e}{\sqrt{\ln^2 e + \pi^2}}$$

where  $e$  is the so-called coefficient of restitution.

In addition there are bond forces and moments in the contacts to bond the particles together and to simulate the soil cohesive behaviour. The following force- and moments are summed to the corresponding Hertz-Mindlin components:

$$\Delta F_n = -S_n^B \cdot A \cdot \Delta \delta_n \quad (7)$$

$$\Delta F_t = -S_t^B \cdot A \cdot \Delta \delta_t \quad (8)$$

$$\Delta M_n = -S_n^B \cdot J \cdot \Delta \theta_n \quad (9)$$

$$\Delta M_t = -S_t^B \cdot \frac{J}{2} \cdot \Delta \theta_t \quad (10)$$

In Equation (7) to Equation (10) the  $S_n^B$  and  $S_t^B$  are the bond normal- and tangential stiffness,  $\Delta \delta_n$  and  $\Delta \delta_t$  are the relative normal- and tangential displacements,  $\Delta \theta_n$  and  $\Delta \theta_t$  are the relative normal- and tangential rotations of the contacting elements, respectively and:

$$A = \pi \cdot R_B^2$$

$$J = \frac{1}{2} \cdot \pi \cdot R_B^4$$

are the area and the polar moments of inertia of the bond's cross section, respectively ( $R_B$  is the radius of the bond).  $\Delta\delta_n$ ,  $\Delta\delta_t$ ,  $\Delta\Theta_n$  and  $\Delta\Theta_t$  are calculated from the bond formation time of  $t_{Bond}$  when the bond forces and moments are set to zero. In addition these bonds can be broken when the normal- and tangential bond stresses exceed their limits (bond normal- and tangential strength):

$$\sigma_{\max} = \frac{-\Delta F_n}{A} + \frac{2 \cdot M_t}{J} \cdot R_B$$

$$\tau_{\max} = \frac{-\Delta F_t}{A} + \frac{M_n}{J} \cdot R_B$$

There is another important parameter, namely the contact radius ( $R_{\text{contact}}$ ) which determines the point where two particles become in contact from. While using bonds between the particles this parameter have to been set up larger than the real radius of the particle in order to allow to transmit tensile force between the elements while they are no more in contact physically.

### The laboratory direct shear tests

Direct shear tests were conducted in the Szent István University of Gödöllő to measure the frictional mechanical properties of real cohesive soil. The soil samples were collected with core cylinders near Mohács, Hungary in the November of 2015 and were transported later to the Szent István University where ELE 26-2112/01 direct shear apparatus - shown in Figure 1 - was used to measure the soil mechanical properties.



Figure 1: The used ELE 26-2112/01 type direct shear apparatus

The shear box with dimension of  $\text{Ø}6.4\text{e-}02 \times 2.54\text{e-}02$  m was filled with soil samples and the vertical load of 200 N, 400 N and 600 N was added to the samples, respectively. After that the horizontal speed of  $5 \text{ mm min}^{-1}$  which is equal to  $8.3\text{e-}02 \text{ mm s}^{-1}$  was set up

and the shear process was started. During the measurements the horizontal displacement of the top section and the shear force (is necessary to push out the top section on the bottom section of the shear assembly) were measured by the built in ASCCELL TC type load cell. Note that the vertical displacement of the samples were not measured during the measurements.

Finally the results were evaluated from the shear force-horizontal displacement diagrams. In case of each normal load of  $N$  there is a maximum shear force of  $T$  as it can be seen in the results section of the paper. From this data the Mohr-Coulomb line of the soil can be drawn and the mechanical properties of the soil were calculated using the following equation (Terzaghi 1943):

$$T = c \cdot A + N \cdot \tan \varphi$$

where  $c$  means the cohesion,  $A$  is the sheared area and  $\varphi$  is the soil's internal friction angle.

### Development of the 3D discrete element model

A 3D discrete element model was developed to simulate direct shear tests of cohesive soil. First the same geometry was created virtually to that of real direct shear apparatus. Therefore two cylinders with radius of  $3.2\text{e-}02$  m and height of  $1.27\text{e-}02$  m each were set into the model and than were filled with spherical elements. After the elements settled down by Earth gravity and reached the equilibrium state (the maximum velocity of the elements was under the value of  $1\text{e-}05 \text{ m s}^{-1}$ ) the vertical force was added to the model by a disk made of spherical elements as well. Between these particles there were no contacts calculated during the simulations they are only for allowing to add the vertical load to the soil model. Finally the top cylinder was moved horizontally with the speed of  $8.3\text{e-}01 \text{ mm s}^{-1}$  which is 10 times higher than the speed used in real direct shear tests. This was done to minimize the calculation time because in case of lower horizontal speeds the value of the shear force was not change significantly according to our earlier results but it takes for weeks to complete the simulations.

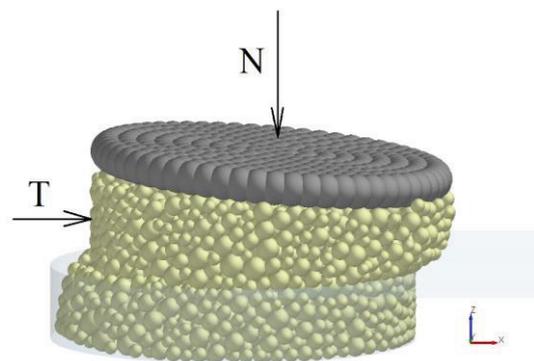


Figure 2: The 3D discrete element model of soil direct shear test with the disk

The timestep of  $1e-05$  was chosen which is approximately 11.7 % of the Rayleigh timestep. According to the EDEM user manual the timestep have to set up 20 % or lower to the Rayleigh timestep to guarantee good results from the simulations. During each direct shear simulations the horizontal force and the displacement of the top section and the vertical force of the disk (e. g. the normal load of the sample) were calculated and saved in each  $5e-02^{\text{th}}$  timestep. Finally the results were evaluated using Microsoft Excel 2016 software by drawing the corresponding shear force-horizontal displacement diagram and from that the Mohr-Coulomb line of the soil model as well.

While using the Hertz-Mindlin with bonding contact model the contact properties in Table 1 have to been set up before the simulation. To obtain these data the following steps were used:

- Step 1: Determination of the elements' Young's modulus. In this paper the Young's modulus was chosen according to the gradient of the shear force-displacement diagram in case of normal load of 200 N (see Figure 4).
- Step 2: Determination of the bond strength. In this paper the bond tangential strength was chosen to be approximately equal to soil's cohesion and the bond normal strength was chosen to be twice as much than the tangential strength presenting the shear stress is twice as dangerous as the tensile stress.
- Step 3: Determination of the elements' contact radius. In this paper the contact radius was chosen to be 1.3 times greater than the real particle's radius.
- Step 4: Calculation of the bond stiffness from the bond strength, contact radius and other geometrical properties of the elements.
- Step 5: Calibration of the bond radius to the maximum shear force.

The aim of this process is to determine or calculate the contact properties so that the bond breaks between two elements because it exceed its stress limit values and not because the two contacting elements get too far from each other so their contact radiuses do not overlap each other. Therefore in step 4 the stiffnesses of the bond should be calculated with Equation (11).

$$S_n^B = S_t^B = \frac{\tau_{\max}}{\Delta u} \quad (11)$$

where  $\Delta u$  can be determined as follows:

$$\Delta u = 2 \cdot R_{\text{contact}} - NO$$

where  $NO$  is the average normal overlap of the elements at the bond formation time ( $t_{\text{Bond}}$ ). With the data given in Table 1 the bond normal- and tangential stiffness were calculated with Equation (11) and were exactly  $1.193e+07 \text{ Pa m}^{-1}$ . Because of the computational error of the discrete element simulation this value was rounded up to  $1.2e+07 \text{ Pa m}^{-1}$ .

Table 1: The properties of the discrete element model derived from the 3D direct shear simulations

Parameter	Value
<i>Geometrical properties</i>	
Particle radius distribution (m)	2e-03...4.5e-03
Contact radius ( $R_{\text{contact}}$ ) (m)	2.67e-03...6e-03
Initial porosity (before the normal load applied) (%)	0.415
<i>Properties of the Hertz-Mindlin with bonding contact model</i>	
Bulk density ( $\text{kg m}^{-3}$ )	1800
Shear modulus (Pa)	2.88e+06
Poisson ratio (-)	0.3
coefficient of restitution (e) (-)	0.5
Friction coefficient between ball and ball ( $\mu_{\text{ball}}$ ) (-)	0.6
Friction coefficient between ball and walls ( $\mu$ ) (-)	0.5
Bond radius ( $R_B$ ) (m)	1.2e-03
Bond normal stiffness ( $S_n^B$ ) ( $\text{Pa m}^{-1}$ )	1.2e+07
Bond shear stiffness ( $S_t^B$ ) ( $\text{Pa m}^{-1}$ )	1.2e+07
Bond normal strength (Pa)	7.738e+4
Bond shear strength (Pa)	3.869e+4

The data in Table 1 are the results of many calibrational simulations.

## RESULTS AND DISCUSSION

The results of real direct shear tests and 3D discrete element simulations will be presented together in this section.

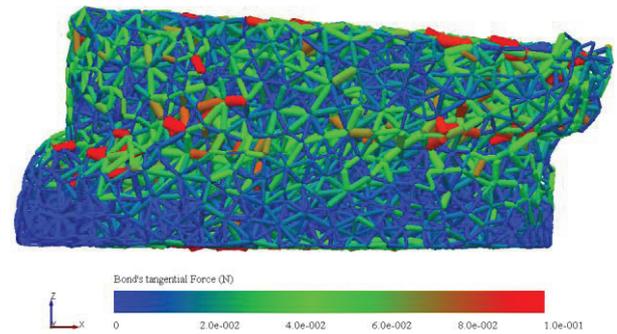


Figure 3: The bonds between the elements with the calculated bond tangential forces in case of 3D discrete element simulation of normal load of 200 N

In Figure 3 the bonds between the particles are presented and are coloured according to the bond tangential force in case of simulation of normal load of 200 N. It can be seen that the higher bond forces arise in the middle of the model (near the shear zone) and at the

top of the model where the disk with the normal load is contacting with the soil particles. This observation can be experienced in case of real soils as well where the highest stresses arise close to the shear zone of the sample.

In Figure 4 the results of real direct shear test as dotted lines and results of discrete element simulations as continuous lines are presented. The gradient of the shear force-displacement curve of simulation is similar to that of measurement in case of normal load of 200 N and in range of displacements higher than  $2e-03$  m. This was expected according to the step 1 of the calibration process. Small difference between these curves can be seen in range of displacements smaller than  $2e-03$  m. This can be experienced probably because there are sudden changes in the shear force values in range of displacements smaller than  $2 \dots 3e-03$  m in case of each test which can be related to the error of direct shear measurements. Figure 4 shows greater differences between the simulation and measurement curves in case of higher normal loads especially in range of small displacements but comparing the maximum values of the shear force in the simulations and in the measurements they are close to each other in case of all normal loads. This means that the failures of the soil models in the simulations are very similar in point of view of shear force value to the failures of real soils. The maximum shear forces were summarized in Table 2 as well. These data can be used to draw the Mohr-Coulomb line of the soil models and real soils as well.

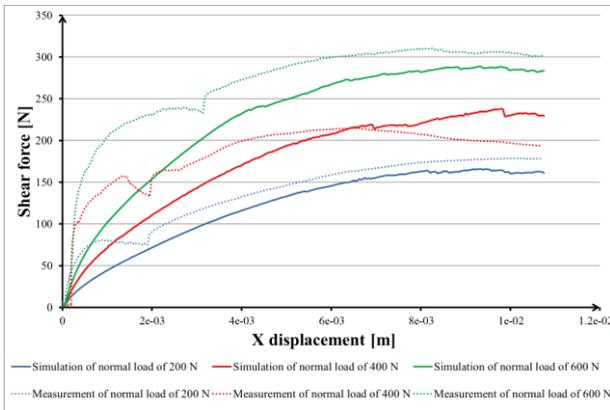


Figure 4: Results of the direct shear simulations

Table 2: The maximum of the shear force in the three different simulations and direct shear tests

Normal load (N)	Maximum of the shear force in simulation (N)	Maximum of the shear force in measurement (N)
200	165.99	179.03
400	237.56	215.23
600	288.72	310.54

These lines can be seen in Figure 5 where the results of the simulations are illustrated as red points and red

dotted line and the results of the measurements are illustrated as blue points and blue dotted line.

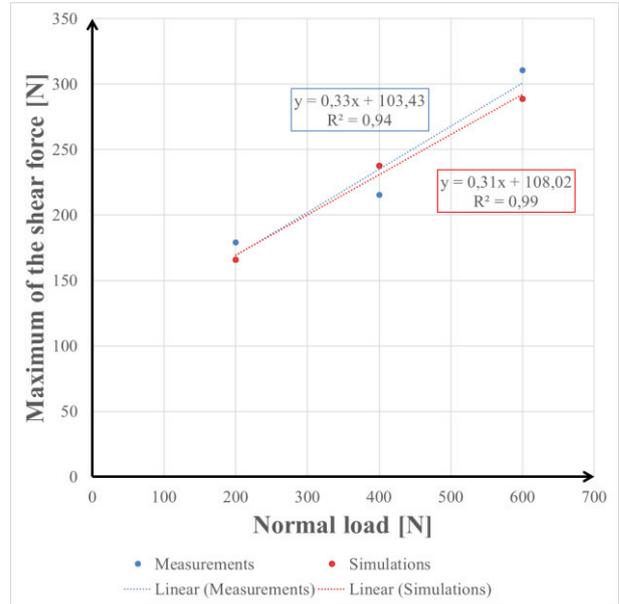


Figure 5: The calculated Mohr-Coulomb line of the real soil and of the discrete element soil model

Linear trendlines were fitted to the measurement and to the calculated simulation values with high  $R^2$  value of 94% and 99%, respectively using the Ordinary Least Squares available in Microsoft Excel 2016 software. Figure 5 shows very small differences between the Mohr-Coulomb lines of simulation and measurement therefore similarities in the values of cohesion and internal friction angle can be expected as well. The relative error of these properties ( $RE$ ) was calculated using the following formula:

$$RE = \frac{N_{simulation} - N_{measurement}}{N_{measurement}} \cdot 100$$

where  $N_{simulation}$  is the value of the mechanical properties from the simulation and  $N_{measurement}$  is the value of the properties from the measurement. The results were summarized in Table 3.

Table 3: The calculated mechanical properties of the soil model and of the real soil.

	Discrete element soil model	Real soil	Relative error (%)
Cohesion (c) (Pa)	3.82e+4	3.66e+4	4.44
Internal friction angle ( $\phi$ ) ( $^\circ$ )	17.06	17.89	4.66

According to Table 3 the mechanical properties of real soils and the same properties of soil models are close to each other. The relative errors are under the value of 5% which means that the presented calibrational process can be used to calibrate the contact properties of the Hertz-Mindlin model in case of direct shear simulations.

In the future additional calculations should be performed to obtain more similar shear force-displacement curve in simulations to that of real measurements especially in range of small displacements. To do this it is necessary to investigate to effect of changing the Poisson ratio on the value of shear force.

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## REFERENCES

- Chi, L. and Kushwaha, R. L. 1990. "A non-linear 3-d finite element analysis of soil failure with tillage tools." *Journal of Terramechanics*, 27(4), 343-366.
- Cundall, P. A. and Strack, O. D. L. 1979. "Discrete numerical model for granular assemblies." *Geotechnique*, 29(1), 47-65.
- Dirgéliené, N.; Amsiejus, J.; Norkus, A. and Skuodis, S. 2014. "Comparison of sandy soil shear strength parameters obtained by various construction direct shear apparatuses." *Archives of Civil and Mechanical Engineering* 14, 327-334.
- Hertz, H. 1882. "On the contact of elastic solids." *J. reine und angewandte Mathematik* 92, 156-171.
- Knuth, M. A.; Johnson, J. B.; Hopkins, M. A.; Sullivan, R. J. and Moore, J. M. 2012. "Discrete element modeling of a Mars Exploration Rover wheel in granular material." *Journal of Terramechanics*, 49, 27-36.
- McKyes, E. 1985. *Soil Cutting and Tillage*. Elsevier, New York, USA.
- Mindlin, R. D. 1949. "Compliance of elastic bodies in contact." *Journal of Applied Mechanics* 16, 259-268.
- Mindlin, R. D. and Deresiewicz H. 1953. "Elastic spheres in contact under varying oblique forces." *ASME*, September, 327-344.
- Mouazen, A. M. and Neményi, M. 1998. "A review of the finite element modelling techniques of soil tillage." *Mathematics and Computers in Simulation*, 48 (1), 23-32.
- Nakashima, H.; Fujii, H.; Oida, A.; Momozu, M.; Kanamori, H.; Aoki, S.; Yokoyama, T.; Shimizu, H.; Miyasaka, J. and Ohdoi, K. 2010. "Discrete element method analysis of single wheel performance for a small lunar rover on sloped terrain." *Journal of Terramechanics*, 47, 307-321.
- Potyondy, D. O.; Cundall, P. A. 2004. "A bonded-particle model for rock". *International Journal of Rock Mechanics & Mining Sciences*, 41, 1329-1364.
- Sadek, M. A.; Chen, Y. and Liu, J. 2011. "Simulating shear behavior of a sandy soil under different soil conditions." *Journal of Terramechanics*, 48, 451-158.
- Sitkei, Gy. 1967. *Mezőgazdasági gépek talajmechanikai problémái (The soil mechanics problems of the*

*agricultural machines)*. Akadémiai Kiadó, Budapest, Hungary (in hungarian).

Tamás, K.; Jóri, J. I. and Mouazen, A. M. 2013. "Modelling soil-sweep interaction with discrete element method." *Soil & Tillage Research*, 134, 223-231.

Terzaghi, K. 1943. *Theoretical Soil Mechanics*. John Wiley and Sons, New York, USA.

## AUTHOR BIOGRAPHIES



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