Modelling of the underwater targets tracking with the aid of pseudomeasurements Kalman filter

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KEYWORDS

TMA, passive tracking, bearing-only observation, fusion of passive and active measurements, pseudomeasurements Kalman filter

ABSTRACT

Target motion analysis of the underwater target tracking by the UUV (Unmanned underwater vehicle) usually based on the bearing-only observations including azimuth and elevation angles. However, low angular resolution of hydro acoustic sonars is not enough for the good quality of tracking. Moreover, angular observations lead to nonlinear filtering such as Extended Kalman Filtering (EKF) which usually produces estimations with unknown bias and quadratic errors. As it was mentioned long ago in a case of bearing-only observations target unobservability may take place, therefore, some special observer's motion become necessary. Other filters like the particle or unscented ones need the additional computer resources and also may produce the tracking loss. At the same time the pseudomeasurements Kalman filtering (PKF) method which transforms the estimation problem to the linear one and gives the current coordinates estimation with almost same accuracy could be modified to evaluate the moving target coordinates and velocities without bias. Since PKF gives unbiased estimate for the motion and the quadratic error it provides the good means for integration of various measurements methods such as passive (bearing-only) and active (range) metering. Using this filtering approach the good quality of target motion analysis (TMA) for randomly moving target may be achieved.

INTRODUCTION

Typical observations of underwater targets nowadays are based on so-called hydroacoustic imaging which simultaneously provides various modes of the observations such as passive ones (bearing-only) and/or active (range metering) [Sullivan (2015)]. In first works related to this class of observations it was mentioned that in the tracking of moving targets on the basis of bearing-only observations the unobservability may occur [Nardone and Aidala (1981)]. In the so-called target motion analysis (TMA) it is usually assumed that the motion of target is almost deterministic (just the initial position and velocity are unknown). This leads to the estimation procedure like the least square method on the basis of relatively long period of observations [Nardone et al. (1984)]. Most of results in this area demonstrate the difficulty of high quality TMA without observer's movement [Helbling (1988)] in [Chan (1988)]. Moreover, as was shown in [Jauffret et al. (2008)], [Pignol et al. (2010)] it is typical situation without observer's maneuvering. So the usage of so-called optimal moving observer performs to minimize the tracking error and even to solve some additional tasks such as minimizing the proximity between pursuer and evader [Rubinovich (2001)], [Andreev and Rubinovich (2016)]. There are plenty of filtering approaches on the basis of bearing-only observations, such as the particle filter [Fei et al. (2008)], cubature Kalman filter [Leong et al. (2013)], [Xin-Chun and Cheng-Jun (2013)], unscented filters [Barisic et al. (2012)], some versions of interpolation filtering [(Gupta et al., 2015)] and many others [Xin et al. (2004)]. The principal specific feature of all these nonlinear filters is the unknown bias and impossibility of correct estimation of the quadratic error which is one of the principal advantages of classical Kalman filtering. Some approaches which permit in principle to avoid the presence of bias lead to the serious increasing of necessary computational means since they lead to the usage of multiple filtering and dynamic equations like in the case of unscented Kalman filtering [Wan and van der Merwe (2000)]. Meanwhile, the comparison of various versions of nonlinear Kalman filters shows that in case of bearing-only observations most of existing filters give almost the same level of the estimation accuracy [Lin et al. (2002)]. At the same time [Lin et al. (2002)] the pseudomeasurements filter is easier for implementation and usually more stable than others. However, this filter, as it was mentioned long ago, also produces a bias due to the nonlinear dependence of the noise variance from unobservable parameters [Aidala and Nardone (1982)]. Recently we developed the version of pseudomeasurements Kalman filter (PKF) based on the idea of the best linear estimation such as used in the conditionally optimal filtering of V. S. Pugachev [Pugachev and I. N (1987)], [Miller and Pankov (2007)]. This filter does not have the bias and gives the unbiased estimation of current square error which is extremely important in the data fusion of optical measurements and inertial navigation system of the UAV. This filter serves as a basis for the UAV control in GPS denied environment [Amelin and Miller (2014)], [Miller and Miller (2014a)], [Miller (2015)] where it permits to develop the navigation system based on the observation of so-called singular points on the earth surface [Konovalenko et al. (2015)] and comparison of their position with template map uploaded to the UAV before flight. This method may be extended to the usage of 3-dimensional template map which increases the accuracy of the altitude evaluation [Karpenko et al. (2015a)], [Karpenko et al. (2015b)]. This PKF is used for control of the UAV in GPS denied environment and for landing with the aid of terrain bearing tools like

Proceedings 31st European Conference on Modelling and Simulation ©ECMS Zita Zoltay Paprika, Péter Horák, Kata Váradi, Péter Tamás Zwierczyk, Ágnes Vidovics-Dancs, János Péter Rádics (Editors)
ISBN: 978-0-9932440-4-9/ ISBN: 978-0-9932440-5-6 (CD)

optic and radio-locators [Miller and Miller (2015)]. In the case of underwater navigation 3D measurement are one of the most important tools since modern hydro-locators (sonars) produce 3D image of the sea floor which may be used for own position and navigation estimation [Zhang et al. (2014)].

The aim of this work is to show that PKF may be used for underwater targets tracking with relatively high accuracy and without special maneuvering which permits to avoid the unobservability. The phenomenon of unobsrvability was first mentioned in [Nardone and Aidala (1981)] for the case of rectilinear target motion. In most successive works authors are using the special maneuvering of the observer to reduce the estimation errors [Rubinovich (2001)], [Andreev and Rubinovich (2016)]. Indeed, the unobservability is inherent to the case of almost deterministic target motion, where just measuring of the azimuth angle does not permit to distinguish the real and one of possible target's motions. However, in the case of the random perturbations in the target motion such situation could occur with null probability only. We observed that PKF provides good quality of the target tracking on the basis of bearing-only observations without special maneuvering [Miller and Miller (2014b)], here we apply this method to the tracking of the underwater targets. Of course, PKF demonstrates the randomness in the estimation error, however, one can observe good correspondence of the real tracking error and its mean quadratic error. The PKF can be easily extended to the case of additional range metering, which gives, of course, much better estimation's quality, though may be unacceptable in some special areas of applications. Meanwhile, the addition of range metering in a reasonable way can increase the estimation precision and prevent the tracking loss, inherent to the bearing-only tracking. Therefore, in the real case of possible obscurity observation constraints it may be used together with bearing-only observations in the optimal manner, providing the balance between obscurity and the precision of the TMA.

MODELS FOR PSEUDOMEASUREMENTS KALMAN FILTER (PKF)

Model of the observer motion

We assume the pursuer motion described by three coordinates $X(t_k)$, $Y(t_k)$, $Z(t_k)$, velocities $V_x(t_k)$, $V_y(t_k)$, $V_z(t_k)$. At times $t_k = k\Delta t, \quad k = 1, 2, ...$ pursuer state vector

$$\mathbf{X}(t_k) = (X(t_k), Y(t_k), Z(t_k), V_x(t_k), V_y(t_k), V_z(t_k))^T$$

satisfies the following dynamic equation:

$$\mathbf{X}(t_{k+1}) = F\mathbf{X}(t_k) + B\mathbf{A}(t_k), \tag{1}$$

where

$$\mathbf{A}(t_k) = (A_x(t_k), A_y(t_k), A_z(t_k))^T$$

is the vector of the pursuer accelerations, and the F, B matrix have the following form:

$$F = \begin{pmatrix} 1 & 0 & 0 & \Delta t & 0 & 0 \\ 0 & 1 & 0 & 0 & \Delta t & 0 \\ 0 & 0 & 1 & 0 & 0 & \Delta t \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix},$$

$$B = \begin{pmatrix} \frac{\Delta t^2}{2} & 0 & 0 \\ 0 & \frac{\Delta t^2}{2} & 0 \\ 0 & 0 & \frac{\Delta t^2}{2} \\ \Delta t & 0 & 0 \\ 0 & \Delta t & 0 \\ 0 & 0 & \Delta t \end{pmatrix}.$$

Model of the motion of maneuvering target

The stochastic target's motion is given by three coordinates $x(t_k)$, $y(t_k)$, $z(t_k)$, velocities $v_x(t_k)$, $v_y(t_k)$, $v_z(t_k)$ and accelerations $a_x(t_k)$, $a_y(t_k)$, $a_z(t_k)$, which are forming the 9-dimensional vector of the target motion components (TMC). At times $t_k = k\Delta t, \quad k = 1, 2, ...$ the TMC vector satisfies the following equation:

$$\mathbf{x}(t_{k+1}) = Q\mathbf{x}(t_k) + \mathbf{W}(t_k), \tag{2}$$

where $\mathbf{W}(t_k)$ is a vector of motion current perturbations

$$\mathbf{W}(t_k) = (0, 0, 0, 0, 0, 0, \sigma_{\mathbf{x}} W_x(t_k), \sigma_{\mathbf{y}} W_y(t_k), \sigma_{\mathbf{z}} W_z(t_k))^T$$

and matrix Q:

$$Q = \begin{pmatrix} 1 & 0 & 0 & \Delta t & 0 & 0 & \frac{\Delta t^2}{2} & 0 & 0 \\ 0 & 1 & 0 & 0 & \Delta t & 0 & 0 & \frac{\Delta t^2}{2} & 0 \\ 0 & 0 & 1 & 0 & 0 & \Delta t & 0 & 0 & \frac{\Delta t^2}{2} \\ 0 & 0 & 0 & 1 & 0 & 0 & \Delta t & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & \Delta t & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & \Delta t \\ 0 & 0 & 0 & 0 & 0 & 0 & \alpha_x & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \alpha_y & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \alpha_z \end{pmatrix}.$$

We assume also that the target accelerations are random and satisfy the following equations:

$$\begin{pmatrix} A_x(t_{k+1}) \\ A_y(t_{k+1}) \\ A_z(t_{k+1}) \end{pmatrix} = \begin{pmatrix} \alpha_x A_x(t_k) \\ \alpha_y A_y(t_k) \\ \alpha_z A_z(t_k) \end{pmatrix} \Delta t + \begin{pmatrix} \sigma_{\mathbf{x}} W_x(t_k) \\ \sigma_{\mathbf{y}} W_y(t_k) \\ \sigma_{\mathbf{z}} W_z(t_k) \end{pmatrix}. \tag{3}$$

The coefficient $|\alpha| < 1$, therefore the target random accelerations constitute the stationary processes [Pugachev and I. N (1987)].

Model of bearing+range measurements

Here we give the general measurements model including asimuth, elevation and possible range metering in the universal form. At time t_k pursuer direction finder (DF) produces a set of three target position measurements (see Fig. 1). It measures azimuth angle $\phi(t_k)$:

$$I(t_k)\frac{y(t_k) - Y(t_k)}{x(t_k) - X(t_k)} = I(t_k)(\tan\phi(t_k) + \varepsilon_k^{\phi}), \tag{4}$$

elevation angle $\lambda(t_k)$:

$$I(t_k)\frac{z(t_k)-Z(t_k)}{y(t_k)-Y(t_k)}\sin\phi(t_k) = I(t_k)(\tan\lambda(t_k)+\varepsilon_k^{\lambda}) \tag{5}$$

and range $l(t_k)$:

$$I(t_k)\frac{z(t_k)-Z(t_k)}{\sin\lambda(t_k)} = I(t_k)(l(t_k)+\varepsilon_k^{l}), \tag{6}$$

where one can assume that $\varepsilon_k^{\phi} \sim \mathcal{WN}(0,\sigma_{\phi}^2)$, $\varepsilon_k^{\lambda} \sim \mathcal{WN}(0,\sigma_{\lambda}^2)$, $\varepsilon_k^{l} \sim \mathcal{WN}(0,\sigma_{l}^2)$ are uncorrelated random variables with zero means and variances $\sigma_{\phi}^2, \sigma_{\lambda}^2, \sigma_{l}^2$, defined as errors in measurements of DF and forming the white noise ($\mathcal{WN}$) sequences. $I(t_k)$ is an indicator function which is equal to 1 if at time t_k target is in the coverage area of the pursuer DF and zero otherwise.

At given time t_k either angular or angular+range measurements may be used together. However, due to the obscurity constraints the range measurements may be used in case of necessity to get more precise measurements when the estimated quadratic error is less than given threshold or with larger period than angle measurement.

DEVELOPMENT OF UNBIASED PKF

Using the pseudomeasurements method [Aidala and Nardone (1982)], [Lin et al. (2002)] we separate in (4),(5,)(6) observable and unobservable values which gives the system of linear measurement equations

$$\mathbf{m_k} = \begin{pmatrix} m_k^{\phi} \\ m_k^{\lambda} \\ m_k^{l} \end{pmatrix} =$$

$$I(t_k)\begin{pmatrix} x(t_k)\sin\phi(t_k) - y(t_k)\cos\phi(t_k) \\ +\varepsilon_k^{\phi}\cos\phi(t_k)(x(t_k)-X(t_k)) \\ \\ y(t_k)\sin\lambda(t_k) - z(t_k)\sin\phi(t_k)\cos\lambda(t_k) \\ +\varepsilon_k^{\lambda}\cos\lambda(t_k)(y(t_k)-Y(t_k)) \\ \\ z(t_k) - l(t_k)\sin\lambda(t_k) - \varepsilon_k^{l}\sin\lambda(t_k) \end{pmatrix}. \tag{7}$$

We assume that at the moment t_k we have unbiased estimates of target position

$$\hat{\mathbf{x}}(t_k), \hat{P}(t_k) = E(\mathbf{x}(t_k)-\hat{\mathbf{x}}(t_k))(\mathbf{x}(t_k)-\hat{\mathbf{x}}(t_k))^T,$$

where $\hat{x}(t_k)$ is such that

$$E(\hat{\mathbf{x}}(t_k)) = \mathbf{x}(t_k). \tag{8}$$

Using the prediction-correction filter algorithm [Amelin and Miller (2014)], [Miller (2015)], [Miller and Pankov (2007)] we receive estimates $(\hat{\mathbf{x}}(t_{k+1}), \hat{P}(t_{k+1}))$ at times t_{k+1}, which satisfy the unbiasedness condition (8), using the estimates received at time t_k, measurements m_k, known pursuer coordinates and target dynamics (2).

Thus the prediction is obtained by assuming that at the moment t_{k+1} the value of $m(t_{k+1})$ will be known

$$\begin{aligned} &\tilde{\mathbf{x}}(t_{k+1}) = Q\hat{\mathbf{x}}(t_k), \\ &\tilde{\mathbf{P}}(t_{k+1}) = Q\hat{P}(t_k)Q^T + E\mathbf{W}(\mathbf{t_k})\mathbf{W^T}(\mathbf{t_k}), \\ &\tilde{\mathbf{m}}_{k+1} = \begin{pmatrix} \tilde{m}_{k+1}^{\phi} \\ \tilde{m}_{k+1}^{\lambda} \\ \tilde{m}_{k+1}^{l} \end{pmatrix}. \end{aligned} \tag{9}$$

After getting the measurements at the moment t_{k+1} one can obtain the estimate of the target position and velocity at this moment and the matrix of the mean square errors:

$$\begin{aligned} &\hat{\mathbf{x}}(t_{k+1}) = \tilde{\mathbf{x}}(t_{k+1}) \\ &+\tilde{\mathbf{P}}^{\mathbf{xm}}(t_{k+1})(\tilde{P}^{mm}(t_{k+1}))^{-1}(\mathbf{m}_{k+1}-\tilde{\mathbf{m}}_{k+1}), \end{aligned} \tag{10}$$

$$\begin{aligned} &\hat{P}(t_{k+1}) = \tilde{P}(t_{k+1}) \\ &-\tilde{\mathbf{P}}^{\mathbf{xm}}(t_{k+1})(\tilde{P}^{mm}(t_{k+1}))^{-1}\tilde{\mathbf{P}}^{\mathbf{xm}}(t_{k+1})^T, \end{aligned} \tag{11}$$

where the innovation process has a form

$$(\mathbf{m}_{k+1}-\tilde{\mathbf{m}}_{k+1}) =$$

$$I(t_{k+1})\begin{pmatrix} (x(t_{k+1})-\tilde{x}(t_{k+1}))\sin\phi(t_{k+1}) \\ -(y(t_{k+1})-\tilde{y}(t_{k+1}))\cos\phi(t_{k+1}) \\ +\varepsilon_{k+1}^{\phi}\cos\phi(t_{k+1})(x(t_{k+1})-X(t_{k+1})) \\ \\ (y(t_{k+1})-\tilde{y}(t_{k+1}))\sin\lambda(t_{k+1}) \\ -(z(t_{k+1})-\tilde{z}(t_{k+1}))\sin\phi(t_{k+1})\cos\lambda(t_{k+1}) \\ +\varepsilon_{k+1}^{\lambda}\cos\lambda(t_{k+1})(y(t_{k+1})-Y(t_{k+1})) \\ \\ z(t_{k+1})-\tilde{z}(t_{k+1}) - l(t_{k+1})\sin\lambda(t_{k+1}) \\ -\varepsilon_{k+1}^{l}\sin\lambda(t_{k+1}) \end{pmatrix}.$$

Quadratic characteristics of elements $x(t_{k+1}) - X(t_{k+1})$ may be evaluated via representation

$$\begin{aligned} &x(t_{k+1}) - X(t_{k+1}) \\ &= [x(t_{k+1})-\tilde{x}(t_{k+1})] + [\tilde{x}(t_{k+1})-X(t_{k+1})], \end{aligned}$$

where the first difference can be evaluated via $\tilde{P}^{xx}(t_{k+1})$ and the second one is known at time instant t_{k+1}. Here we use the orthogonality of the best linear estimate and the linear space spanned on observations $\mathbf{m}_i, i = 1..k$. Thereby the elements of matrix

$$\tilde{\mathbf{P}}^{\mathbf{xm}}(t_{k+1}) = (\tilde{P}^{xm}(t_{k+1}), \tilde{P}^{ym}(t_{k+1}), ..., \tilde{P}^{a_z m}(t_{k+1}))^T$$

are given by relations

$$\left[\tilde{P}^{xm}(t_{k+1})\right]^T =$$

$$E\left[(x(t_{k+1}) - \tilde{x}(t_{k+1}))(\mathbf{m}_{k+1} - \tilde{\mathbf{m}}_{k+1})\right]$$

$$= \begin{pmatrix} \tilde{P}^{xx}(t_{k+1})\sin\phi(t_{k+1}) - \tilde{P}^{xy}(t_{k+1})\cos\phi(t_{k+1}) \\ \\ \tilde{P}^{xy}(t_{k+1})\sin\lambda(t_{k+1}) \\ -\tilde{P}^{xz}(t_{k+1})\sin\phi(t_{k+1})\cos\lambda(t_{k+1}) \\ \\ \tilde{P}^{xz}(t_{k+1}) \end{pmatrix} \tag{12}$$

and similarly for $\tilde{P}^{ym}(t_{k+1}), ..., \tilde{P}^{a_z m}(t_{k+1})$.

Matrix $\tilde{P}^{mm}(t_{k+1})$ has the following form

$$\tilde{P}^{mm}(t_{k+1}) = \begin{pmatrix} E[a^2] & E[ab] & E[ac] \\ E[ab] & E[b^2] & E[bc] \\ E[ac] & E[bc] & E[c^2] \end{pmatrix}, \tag{13}$$

where

$$\begin{aligned} a &= m^{\phi}_{k+1} - \tilde{m}^{\phi}_{k+1}, \\ b &= m^{\lambda}_{k+1} - \tilde{m}^{\lambda}_{k+1}, \\ c &= m^{l}_{k+1} - \tilde{m}^{l}_{k+1}. \end{aligned} \tag{14}$$

Method of the expectation calculation in (13) had been presented in [Amelin and Miller (2014)], [Miller and Miller (2014b)], [Miller (2015)], and as an example we give

$$\begin{aligned} &E[a^2](t_{k+1}) = I(t_{k+1})\Big[\tilde{P}_{yy}(t_{k+1})cos^2\phi(t_{k+1}) \\ &-\tilde{P}_{xy}(t_{k+1})sin2\phi(t_{k+1}) + \tilde{P}_{xx}(t_{k+1})sin^2\phi(t_{k+1}) \\ &+\sigma^2_{\phi}cos^2(t_{k+1}) \\ &\times\left(X(t_{k+1}) - \hat{x}((t_k) - \hat{v}_x(t_k)\Delta t - \hat{a}_x(t_k)\frac{(\Delta t)^2}{2}\right)^2 \\ &+\tilde{P}_{xx}(t_{k+1})\Big] \end{aligned} \tag{15}$$

other terms in (13) may be obtained similarly.

MODELLING OF THE FUSION OF BEARING-ONLY AND RANGE METERING

Generally equations (10), (11) give the dependence of current estimate accuracy in terms of the observer motion and the observation strategy, that is possible integration of bearing-only and range metering. In this article we wish to find the answer on the question how to combine passive and active metering. So we use three different strategies

- bearing-only, so the estimation is based on the angle measurements only,
- bearing-only and range metering in various combination, such as range measurements with fixed period which is greater or equal to the period of bearing measurement,
- bearing-only and range metering, where the range metering is made when the current quadratic error is greater than prescribed threshold.

For each type of strategies we are performing the Monte-Carlo simulation of random target motion under the same parameters and the same observer's motion. The observer motion and position are supposed to be known without errors, while the target motion is random with stationary accelerations satisfying equations (2) with perturbations having:

$$\begin{aligned} \sigma_{\mathbf{x}} &= 0.025 * 9.8, \\ \sigma_{\mathbf{y}} &= 0.025 * 9.8, \\ \sigma_{\mathbf{z}} &= 0.005 * 9.8. \end{aligned}$$

The following conditions were used for the duration of tracking and the accuracy of angle and range measurements, where we give the standard deviations:

- $T = 120$;
- $\sigma_{\phi} = \sigma_{\lambda} = 0.01 \approx 0.5^o$.
- $\sigma^2_l = 5.0$
- average velocity of observer ≈ 15, average velocity of target ≈ 10;
- the radius of observer's sonar sensitivity is 1600,

here the time and metric figures are in arbitrary units.

Examples of the observer motion and the target motions with and without the observer maneuver are shown on Fig. 1. The target position tracking are given on Fig. 2 for bearing-only, and on Fig. 3 and 4 for bearing and maneuvering observer, for the target position and velocities, respectively. Corresponding examples of the target tracking with range metering ($\Delta T = 10$) and with threshold strategy with the threshold $SD_{pos} = 10$ are given on Fig. 5 and Fig. 6. One can observe that in bearing-only case possible tracking loss may occur. The joint bearing and range metering provides much more reliable tracking but may be unacceptable due to the energy and obscurity constraints. So the possible solution may be the rare range measurement with either fixed period or with feedback law when error of tracking is less than desired. One should stress that PKF gives reliable means for such feedback since it provides the unbiased estimate of quadratic error.

Results of comparison of various observation strategies are summarized in the following table. These results are obtained with Monte-Carlo modelling by 100 samples of the target motions. where

$$SD_{pos} = \sqrt{\hat{P}^{xx} + \hat{P}^{yy} + \hat{P}^{zz}}$$

mean square error of the target position estimation,

$$SD_{vel} = \sqrt{\hat{P}^{V_xV_x} + \hat{P}^{V_yV_y} + \hat{P}^{V_zV_z}}$$

calculated in PKF during the tracking of target and averaged over Monte-Carlo modelling. SD_x, SD_y, SD_z are the standard deviations of the tracking errors calculated along with tracking of target with the aid of PKF, and $SD_{V_x}, SD_{V_y}, SD_{V_z}$ are the corresponding standard deviations of the velocity estimation errors. On each of

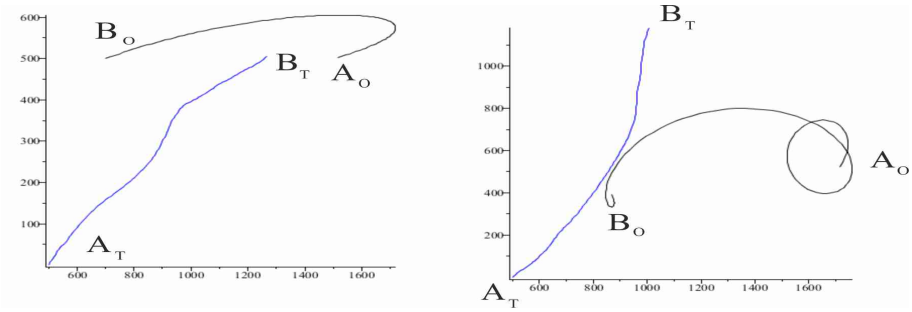

Figure 1: The Observer (black) and Target (blue) motion, A_O, B_O, A_T, B_T initial and final points of the observer and target positions. Left - without observer maneuvering, right - with observer maneuvering.

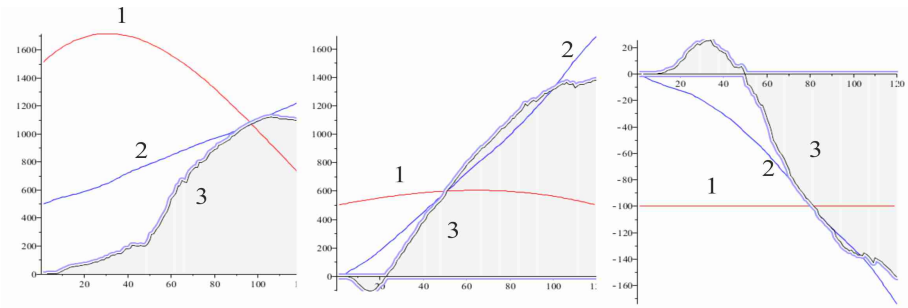

Figure 2: Tracking of the target position, bearing-only measurements without maneuvering. Left - X, center - Y, right - Z. 1 - observer position, 2 - (blue) real target position, 3 - target position estimation. There is a tendency to the tracking loss.

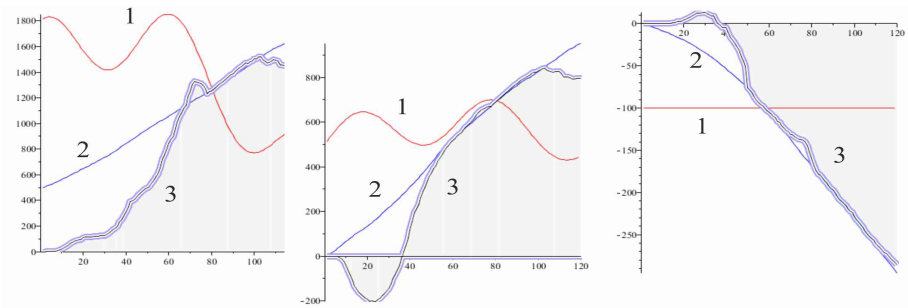

Figure 3: Tracking of the target position, bearing and maneuvering. Left - X, center - Y, right - Z. 1 - observer position, 2 - (blue) real target position, 3 - target position estimation. Even if in general the tracking is better than on Fig. 2, one can observe the tendency to the tracking loss.

the figures by gray color period of time when the target is in the scope of the pursuer DF is designated.

The main observation is that maneuvering target is rather difficult to track with typical hydro acoustic DF. But even if we assume less perturbations in the target motion the results of the estimation accuracy become worse. It is not surprising, but in accordance with the possibility of unobservability during the mission the tracking loss may occur. One can see this tendency to the tracking loss on Fig. 2 and on Fig. 6 at time instants $t = 50$ and $t = 100$. Meanwhile, the addition of range metering even rather rare (last line of the table and Fig. 7) could prevent the tendency to the tracking loss, which may be observed at time instants $t = 50$ and $t = 100$.

CONCLUSIONS

The modelling results show that PKF provides the reliable means for data fusion of bearing-only observations which are inherent to various passive tracking and measurement method with either active or passive measurements. In general:

- PKF has been developed for underwater targets tracking, the method may be used either in passive or active modes.

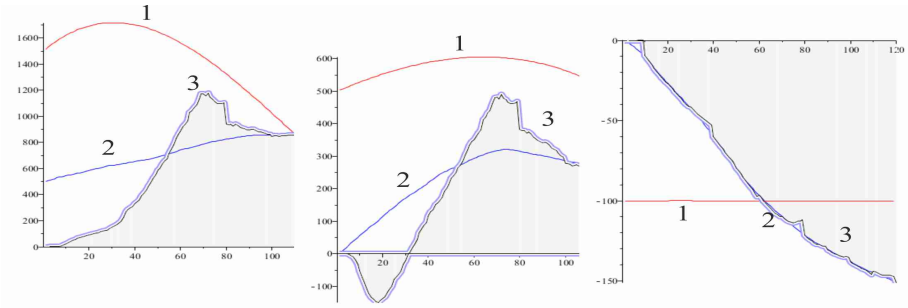

Figure 4: Tracking of the target position, bearing and range measurements with period $DT = 10$. Left - X, center - Y, right - Z. 1 - the observer position, 2 - (blue) the real target position, 3 - the target position estimation.

Mean squared error of tracking and velocities estimation

Method of measurements	SD_{pos}	SD_{vel}	SD_x	SD_y	SD_z	SD_{V_x}	SD_{V_y}	SD_{V_z}
Bearing-only (azimuth and elevation)	63.42	2.79	26.86	21.57	3.55	1.19	0.62	0.086
Bearing + the observer maneuvering	58.5	2.6	40	27	7.3	1.33	0.92	0.29
Bearing + range measurements	33.23	2.21	6.71	6.88	0.038	0.299	0.239	0.012
Bearing + range (threshold 20)	35.47	2.28	8.9	7.73	0.82	0.41	0.26	0.032
Bearing + range (threshold 10)	33.63	2.24	8.36	5.97	0.125	0.34	0.21	0.016
Bearing + range ($\Delta t = 10$)	34.63	2.23	9.12	5.54	0.73	0.35	0.19	0.018
Bearing + rare range ($\Delta t = 50$)	32.67	2.32	71.26	15.23	11.41	2.98	1.1	0.31

Figure 5: Mean squared error of tracking and velocities estimation in various modes of measurements

- Active modes provide essentially higher level of accuracy, however the combination of both modes may be rather effective in the case of obscurity constraints.
- Special maneuvering amends relatively to the bearing-only case, but not radically. Much better results give even very rare range metering which, however, must be coordinated with the obscurity constraints.

ACKNOWLEDGEMENTS

This research was conducted at the A. A. Kharkevish Institute for Information Transmission Problems, RAS, IITP RAS 127994, 19, Bolshoy Karetny per., Moscow, Russia E-mails: amiller@iitp.ru, bmiller@iitp.ru and partially supported by the Russian Basic Research Foundation grant 16-31-60049.

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