TIME-DEPENDENT SIR MODELING FOR D2D COMMUNICATIONS IN INDOOR DEPLOYMENTS

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ABSTRACT

Device-to-device (D2D) communications is expected to become an integral part of the future 5G cellular systems. The connectivity performance of D2D sessions is heavily affected by the dynamic changes in the signal-to-interference ratio (SIR) caused by random movement of communicating pairs over a certain bounded area of interest. In this paper, taking into account the recent findings on the movement of users over a landscape, we characterize the probability density function (pdf) of SIR under stochastic motion of communicating D2D pairs on planar fractals. We demonstarte that the pdf of SIR depends on the fractal dimension and the spatial density of trajectories. The proposed model can be further used to investigate time-dependent user-centric performance metrics including the link data rate and the outage time.

INTRODUCTION

Fifth-generation (5G) mobile cellular systems are aimed at 1000x rate boost compared to the current 4G mobile networks (Andrews 2012). This dramatic increase in system capacity is expected to be achieved with a combination of additional bandwidth in the millimeter-wave bands, improved physical layer techniques, and fundamentally new networking mechanisms. Direct device-to-device (D2D) communications is considered as one of the techniques allowing to drastically improve the degrees of spatial reuse in 5G systems and thus enhance the overall system capacity (Asadi 2014).

The widely-accepted metrics for performance assessment of wireless connectivity in interference-limited cellular systems is the signal-to-interference ratio (SIR) defined as a ratio between the received signal strength and the aggregate interference. Using the Shannon-Hartley theorem (Proakis 1994), SIR can then be used to estimate the theoretically achivable capacity of a link. The latter serves as an upper bound for further development of the optimal modulation and coding schemes (MCSs).

The performance of 5G systems is conventionally assessed by utilizing the tools of stochastic geometry (Bacelli 2010, Haenggi 2012). Accordingly, communicating pairs over the same operating frequency are modeled with a stationary isopropic spatial point process, such as Poisson point process. The performance is then characterized with the standard methods of geometric probability that estimate SIR as a function of path loss models and random distances between points.

Even though the described approach allows to capture the spatially-averaged probability density function (pdf) of SIR, it effectively neglects the impact of movement of users over the landscape. At the same time, the movement of users is an inherent property that may affect the time-dependent SIR behavior. That is, even when the time-averaged behavior is satisfactory, there could still be occasional outages caused by positioning of the communicating pairs with respect to each other. The frequency of outages and their durations are critical parameters that affect user satisfaction of a service.

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In this paper, we formulate a new framework to assess time-dependent behavior of SIR in a moving field of mutually interfering D2D pairs. The movement of transmitters is modeled as random-walk trajectories on planar fractal sets. Based on a novel technique for non-stationary random walks originally reported in (Orlov 2014, Orlov and Fedorov 2016), we demonstrate that the shape of the pdf of SIR is heavily affected by the fractal dimension and the spatial density of trajectories.

Our proposed technique is based on solving non-stationary kinetic equations for the sample distributions of device coordinates while being able to describe spurious correlations. Over smaller sample sizes, these processes can be considered as stationary random walks with long-range dependences. Such walks are characterized by the pdfs with "fat tails" and often emerge in dynamic processes on fractal sets. In practice, such random walks can correspond to movement of people in large shopping malls, where relatively slow travel of customers in front of display windows is interchanged with rapid movement between them using e.g., elevators. The analysis of SIR under this type of mobility is an important theoretical problem in 5G wireless networks (Hesham et al. 2014, Samuylov at al. 2015, Orlov et al., 2016).

RANDOM WALKS ON FRACTAL SETS

The modeling methodology proposed in this study originates from (Orlov 2014, Orlov and Fedorov 2016). Particularly, a simulation technique for random-walk trajectories on Cantor set has been proposed in (Zenyuk et al. 2013). Further generalization to a broader class of fractal sets based on iterated function systems has been developed in (Zenyuk and Orlov 2016). Below, we briefly introduce the basic definitions and outline our proposed methodology.

The simplest way to construct a one-dimensional Cantor C_λ set is to iteratively eliminate open middle intervals of length λ from the set of closed intervals. Specifically, one starts by discarding the middle interval of $[0,1]$, thus leaving two line segments; then, the middle part of the remaining segments is removed, hence leaving four line segments. The process is continued *ad infinitum*. At the *n*-th iteration, 2^{n-1} open intervals each having the length of β^{n-1} are eliminated, where $\beta=(1-\lambda)/2$. Therefore, the total length, l_n, of the removed interval is given by

$$l_n=\lambda(2\beta)^{n-1},\quad \beta=\frac{1-\lambda}{2}, \tag{1}$$

which implies that C_λ has zero Lebesgue measure. The Hausdorff dimension of Cantor set is $-\log_\beta 2$.

Furhter, every point of Cantor set can be represented as

$$x=(1-\beta)\sum_{k=1}^{\infty}b_k\beta^k,\quad b_k\in\{0,1\}. \tag{2}$$

The popular version of C_λ is a ternary Cantor set with $\lambda=\beta=1/3$ characterized by a closed-form expression

$$C_{1/3}=[0,1]\setminus\bigcup_{m=1}^{\infty}\bigcup_{k=0}^{3^{m-1}-1}\left(\frac{3k+1}{3^m};\ \frac{3k+2}{3^m}\right). \tag{3}$$

From (2), it follows directly that the expansion of base 3 of every point in the ternary Cantor set does not contain a unit. The form (2) is also a bijection from Cantor set onto the set of all infinite binary sequences enabling an explicit synthesis of random walk on C_λ.

Consider a random process c_t in a discrete time $t\in N$,

$$c_t=(1-\beta)\sum_{k=0}^{\infty}\xi_t^{(k)}\beta^k, \tag{4}$$

where $\xi_t^{(k)}$ are the discrete random variables having Bernoulli distribution. The simplest type of a random walk is obtained, when $\xi_t^{(k)}$ are mutually independent for every fixed *t* and every $\xi_t^{(k)}$ is a time-homogeneous Markov chain. Without the loss of generality, assume that the process starts from zero, $\forall k\ P\left(\xi_1^{(k)}=0\right)=1$ for every *k*. We also introduce the following notaton

$$\begin{aligned}&P(\xi_t^{(k)}=0)=u_t,\\&P(\xi_t^{(k)}=1)=w_t,\\&u_t+w_t=1.\end{aligned} \tag{5}$$

The state probabilities satisfy a system of equations

$$\begin{pmatrix}u_{t+1}\\w_{t+1}\end{pmatrix}=\Pi^T\begin{pmatrix}u_t\\w_t\end{pmatrix},\quad \Pi=\begin{pmatrix}p_{00}&p_{01}\\p_{10}&p_{11}\end{pmatrix}, \tag{6}$$

where p_{ij} are the transition probabilities from $\{\xi_t^{(k)}=i\}$ to $\{\xi_{t+1}^{(k)}=j\}$. If $p_{00}=p_{11}=1$, this system has the only trivial solution $u_t=1, w_t=0$, which implies that $c_t\equiv 0$. Otherwise, the solution of (6) is

$$\begin{aligned}u_t&=\frac{p_{10}}{p_{01}+p_{10}}+\frac{p_{01}}{p_{01}+p_{10}}(\mathrm{Tr}\Pi-1)^t,\\w_t&=\frac{p_{01}}{p_{01}+p_{10}}-\frac{p_{01}}{p_{01}+p_{10}}(\mathrm{Tr}\Pi-1)^t,\end{aligned} \tag{7}$$

and the process c_t is strictily stationary when $\mathrm{Tr}\Pi=1$.

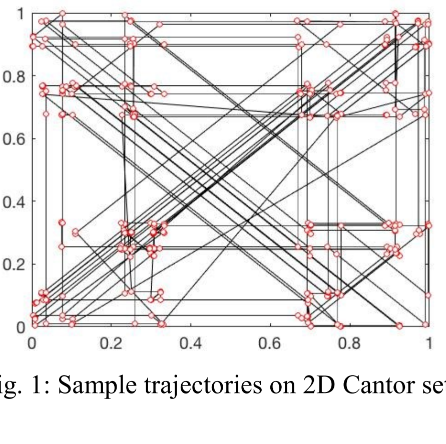

Fig. 1: Sample trajectories on 2D Cantor set.

Observe that many fractal sets, including the Cantor set, are attractors of the iterated systems of contraction mappings, see e.g., (Hutchinson 1981). Random walks on such fractal sets can be constructed in a similar way as described above. Sample paths of trajectories for several fractal sets inside a unit square are depicted in Fig. 1-3. A sample trajectory of Brownian motion is given in Fig. 4 for the sake of comparison.

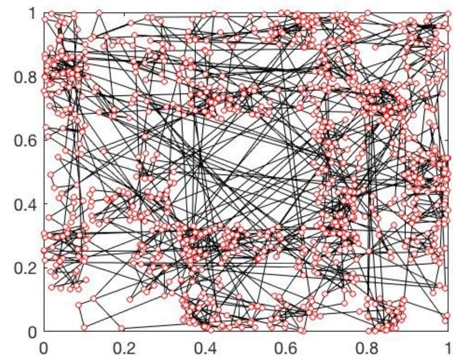

Fig. 2: Sample trajectory on Sierpinski square.

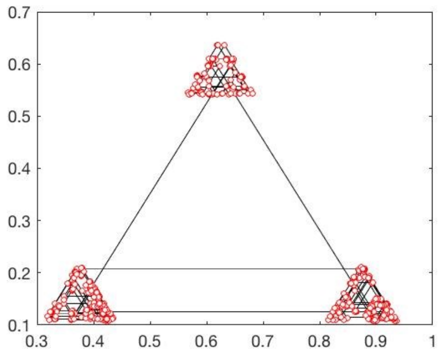

Fig. 3: Sample trajectory on Sierpinski triangle.

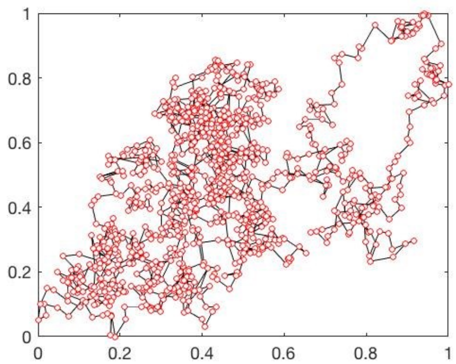

Fig. 4: Sample trajectory of Brownian motion.

As one may observe, the random walks on fractale sets allow to represent a number of practical situations when users are moving in certain indoor areas. A characteristic example is the movement of users inside shopping malls.

In the following sections, we investigate a dependence of SIR on the geometric properties of the underlying set. We demonstrate that in case of random walks on fractals, the SIR is affected not only by the Hausdorff dimension of the set, but also by its shape.

SIR DISTRIBUTION DENSITY

Our system of interest consists of $N+2$ moving devices. Their trajectories are assumed to be mutually independent. A pair $(x_i(t), y_i(t))$ defines the position of the i-th device at time step t. Squared distance between two particular devices at time step t equals to

$$r_{ij}^2(t) = (x_i(t) - x_j(t))^2 + (y_i(t) - y_j(t))^2 . \quad (8)$$

The path loss value is chosen to be proportional to the inverse square of distance between the devices i.e., $\varphi_{ij} \equiv \varphi(r_{ij}) = 1/r_{ij}^2$. The SIR for every pair of devices is calculated according to

$$S(\mathbf{r}_1, \mathbf{r}_2; t) = \frac{\varphi_{12}(t)}{\sum_{j=3}^{N+2} \varphi_{1j}(t)} , \quad (9)$$

where the sum in the denominator is the aggregate interference at the device.

The mean interference is then

$$U(\mathbf{r}, t) = \int_V \varphi(|\mathbf{r} - \mathbf{r}'|, t) \rho(\mathbf{r}') d\mathbf{r}' , \quad (10)$$

where $\rho(\mathbf{r}')$ is the joint pdf of distances.

We now set i=1, j=2, $\mathbf{r} = \mathbf{r}_{12}$, and calculate all distances in the moving system of coordinates associated with the second device. Observe that (9) can be rewritten as

$$S \equiv S(\mathbf{r}, t) = \frac{\varphi(r, t)}{NU(\mathbf{r}, t)} . \quad (11)$$

We are interested in the mean SIR over the ensemble of trajectories e.g., defined as,

$$q(t) = \int_V S(\mathbf{r}, t) \rho(\mathbf{r}) d\mathbf{r} . \quad (12)$$

The time series of SIR observaions, $q(t)$, obtained over one experiment with N=100 is shown in Fig. 5-8. According to these illustrations, the values of SIR on different fractal sets show a similar behavior. However, as we demonstarte in what follows they have drastically different statistical properties.

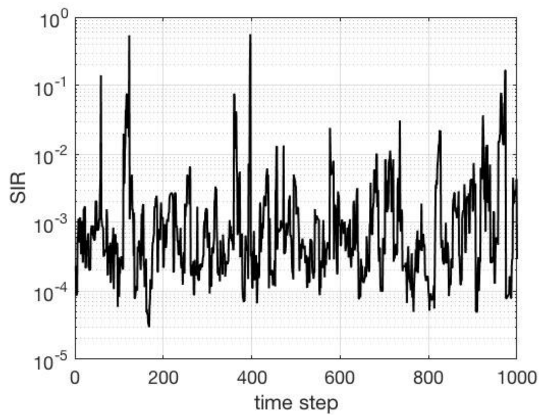

Fig. 5. Time series of SIR for 2D Cantor set.

Time series of SIR obtained in one experiment with N=100 is shown in Fig. 9. As one may observe, the structure of time series for random walks on fractals is different from that observed for Brownian motion.

The sample pdfs of SIR with $N = 10$ are depicted in Fig. 10. We specifically enforced a special non-uniform division of the value sets to obtain pdfs with coinciding modes. Note that this procedure does not affect the qualitative structure of the pdfs.

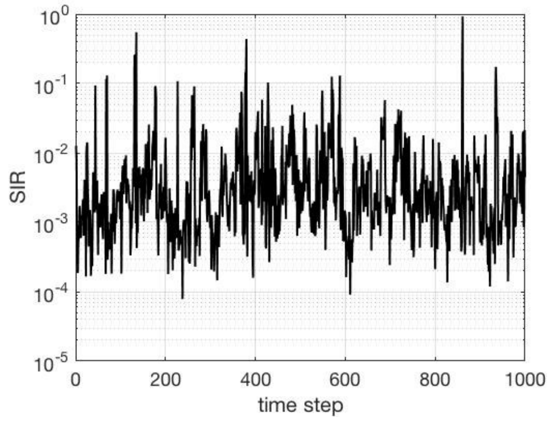

Fig. 6. Time series of SIR for Sierpinski square.

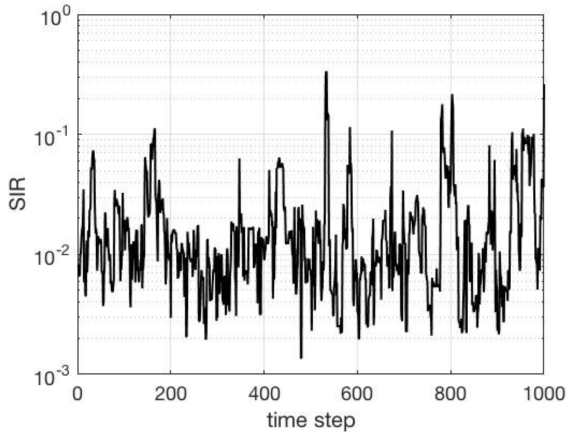

Fig. 7. Time series of SIR for Sierpinski triangle.

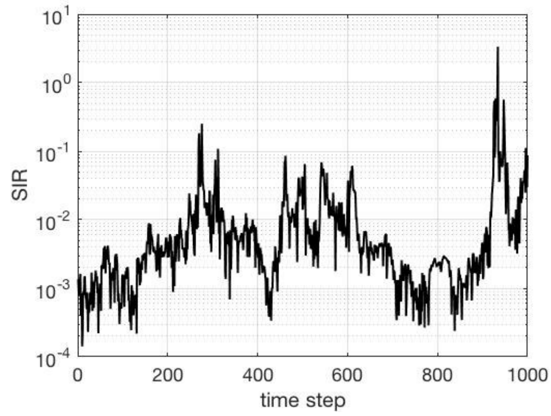

Fig. 8. Time series of SIR for Brownian motion.

Analyzing Fig. 9, one may observe that the geometry of the underlying set plays an important role affecting the structure of the pdf of SIR. Particularly, Sierpinski triangle set, whose sample paths are illustrated in Fig. 3, generates a distribution that significantly differs from other sets. The Hausdorff dimension of the underlying set also affects the statistical properties of SIR. In order to analyze this effect, we introduce the so-called *consistent stationary level* (Orlov 2016). According to its definition, this level is obtained as a solution to the following transcendent equation

$$1 - \varepsilon = K\left(\sqrt{\frac{T}{2}\varepsilon}\right) \qquad (13)$$

with respect to ε, where T is the overall number of observations and $K(z)$ is the Kolmogorov distribution function. The solution of (13) is equal to the limiting error rate of the statistical test under the null hypothesis. It consists in that the two samples of length T are drawn from the same distribution with the significance level of $\varepsilon(T)$, as the number of independent experiments tends to infinity.

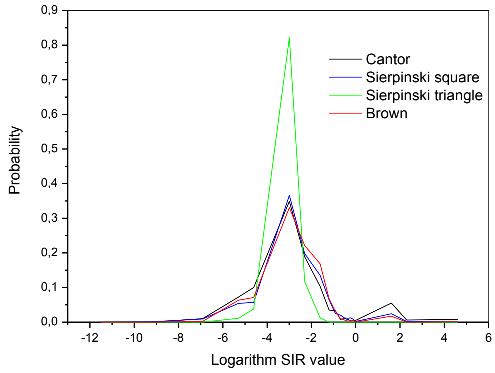

Fig. 9. Empirical SIR pdfs for different underlying sets

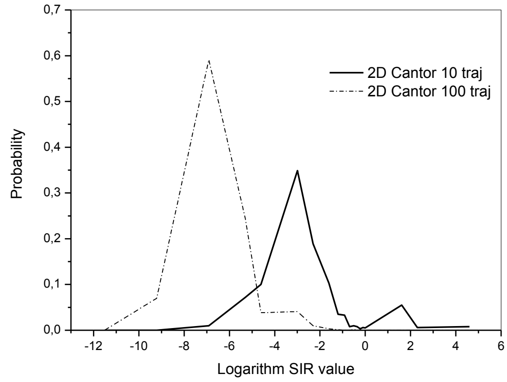

Fig. 10. Empirical SIR pdfs for Cantor square.

For example, for $T = 1000$ the consistent stationary level is $\varepsilon_0 = 0.059$. We define the consistent stationary level for the distance statistics similarly, that is, it is a solution to the following equation

$$G_T(\rho) = 1 - \rho \qquad (14)$$

with respect to $\rho^*(T)$, where $G_T(\rho)$ is the cumulative distribution function of $\rho_T = \sup_x |F_{1,T}(x) - F_{2,T}(x)|$

that captures the distance between two samples in C-norm. Combining these two values, the non-stationary index can be defined as

$$J(T) = \frac{\rho^*(T)}{\varepsilon(T)}. \quad (15)$$

When $J(T) \le 1$, the time series is considered stationary. Otherwise, it is non-stationary, as the null hypothesis of homogeneity at different time steps is rejected.

Our results show that the non-stationary indices for SIR samples on different sets with a square shape (2D Cantor set, Sierpinski square, Brownian motion case) are all greater than 1 and distinct. More importantly, even for the random walks on the same fractal set, the SIR samples differ more than those in the stationary case: for 2D Cantor set $J(100)=2.5$, for Sierpinski square $J(100)=2.7$, and for Sierpinski triangle $J(100) = 3.1$. As the number of observations grows, the index of non-stationarity decreases monotonically. The stationary statistics is achieved when T exceeds 10000.
Finally, we note that SIR is inversely proportional to N. The results of our numerical simulation are in good agreement with this fact, as one may observe in Fig. 10.

CONCLUSIONS

In this paper, we analyzed the time-dependent behavior of SIR in D2D environments for different random walks on fractal sets. We demonstrated that the pdf of SIR depends on the fractal dimension and the spatial density of trajectories. We also showed that the pdfs of SIR in case of fractale random-walk models are indistinguishable from those for the non-stationary random-walk model.
Our proposed model can be used to investigate the time-dependent user-centric performance metrics including the link data rate and the outage time in complex indoor environments, such as shopping malls, where the movement of users is not purely stochastic but rather is modulated by dedicated attractor points.

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