THE EFFECTS OF MODEL SELECTION ON THE GUARANTEES ON TARGET VOLATILITY FUNDS

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ABSTRACT

Target Volatility Funds are becoming a more and more popular asset class amongst Variable Annuity product designers. After the recent global crisis these funds provided a decent way to assure the guarantees that investors find so attractive. However, pricing of these guarantees highly depends on the modelling assumptions we use. Although this is an exciting and demanding problem, not much attention has been shed on this topic in the academic literature. In my work I extend some of the existing results to the Barndorff-Nielsen–Shephard model and to a Lévy-process with stochastic time.

INTRODUCTION

A Target Volatility Fund (TVF) is a portfolio of a risky and a risk-free asset dynamically rebalanced with the aim of maintaining a stable portfolio level. The price of the guarantees, also called guarantee cost, in question is the cost of providing a minimum payoff for the policyholder. Of course, when it comes to guarantees on TVFs, the guarantee cost equals the value of a European put option written on the TVF. The option price and the extent of how well TVFs can do what they are supposed to may highly depend on the stochastic model we use to simulate the price process of the risky asset.

In the past few years researchers have become more concerned in the pricing of derivatives written on TVFs, however, the earliest studies mainly looked at this asset class from a portfolio management and risk-return analysis point of view. Chew (2011) compares the performance of typical volatility-managed funds and other related market strategies and discusses the advantages of TVFs to investors and insurers. Stoyanov (2011) applies a Heston stochastic volatility model framework calibrated to Asian equity market data and shows that in the long run a target-volatility strategy significantly improves both the downside and the upside of the return distribution relative to a fixed-mix strategy. Contrary to previous researches, Xue (2012) claims that in an SVJD (stochastic volatility jump diffusion) model, because of the possible jumps, volatility targeting may not be superior to fixed allocation in terms of risk-return profile, and investors may favor one strategy over an other based on their own jump risk evaluation. Finally, in such a context, Hocquard et al. (2013) based on empirical researches also emphasizes the benefits of a constant volatility approach, as it can help investors obtain desired risk exposures over the short and long term, reduce tail-risk exposure, and increase the portfolio's risk-adjusted performance.

Morrison and Tadrowski (2013) considers the valuation of guarantees, that is European put options, written on target volatility funds, and the impact of some of the key modeling decisions on the derivative prices. The authors compare three popular stochastic models: 1. constant volatility Black-Scholes model, 2. Heston model which features stochastic volatility, and 3. a stochastic volatility model that incorporates jumps (SVJD), and they also examine the effect of different choices of rebalancing frequencies.

Kim and Enke (2016) proposed artificial neural networks for volatility forecasting to enhance the performance of an asset allocation strategy, and compared it to different volatility forecast methodologies. Last but not least, Torricelli (2017) was the first to develop a continuous-time finance mathematical model for target volatility strategies based purely on observable market inputs, which framework enables an efficient numerical valuation of certain derivatives on TVFs, e.g. of put options.

In my research I follow the path of Morrison and Tadrowski (2013) and expand the set of stochastic models under examination by evaluating European put option prices in the cases of Barndorff-Nielsen–Shephard model and Variance Gamma process with Gamma–Ornstein-Uhlenbeck stochastic clock (which is a Lévy-process with stochastic time), as well.

The rest of the paper is organized as follows. First, I introduce the analysis that Morrison and Tadrowski (2013) used. Afterwards, I describe the calibration methods that I used to determine the model parameters. Then, I interpret the result, and finally, I conclude.

ANALYSIS

To make the way of the analysis clear I briefly describe the method that Morrison and Tadrowski (2013) used.

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First, they derived implied volatilities. To do this, they started off with calibrating their models to the same risky asset that they used in the dynamically rebalanced TVF – they chose EuroStoxx 50 as an underlying product. After that, they applied Monte Carlo (MC) simulation for different maturities and strike prices. In each MC simulation, they generated 5000 paths for the risky asset and at the same time, according to the rebalancing procedure, they derived the price processes of the TVF. Then, they evaluated an option written on the TVF and an option written on the equity underlying the TVF, as well. Finally, they calculated the implied volatilities from the option prices and graphed the results for each model selection, and for three different maturities.

Secondly, they also compared the prices of at-the-money (ATM) put options on TVF and calculated percentage changes.

(As noted before, they examined the effects of rebalancing frequencies, as well, which is not included in this paper.)

Implied volatility is an adaptable tool to compare the prices along the different models because it rescales the results to a more tractable form. We may think that if the rebalancing was perfect, i.e. the stochastic process of the equity was predictable and rebalancing was carried out in continuous time, the implied volatility curves of the options on TVFs would be completely flat. Indeed, seemingly it is the case for the Black-Scholes model but there is no theoretical proof of it. Moreover, it is not even necessarily true, especially for the models where the driving process is not similar to the one of the Black-Scholes model. However, we still can use implied volatilities to compare the differences of the resulted option prices. If we experience some deflection, that indicates the imperfection of volatility forecasting method or the characteristics of the stochastic models (or both).

Unfortunately, we cannot predict perfectly the future volatility, thus we need some kind of estimation. To this end, I use the same Exponentially Weighted Moving Average (EWMA) that Morrison and Tadrowski (2013) did:

$$\left(\hat{\sigma}_t^{equity}\right)^2 = \lambda\left(\hat{\sigma}_{t-\Delta t}^{equity}\right)^2 + (1-\lambda)\frac{1}{\Delta t}\log^2\left(\frac{S_t}{S_{t-\Delta t}}\right),$$

where $\hat{\sigma}_t^{equity}$ denotes the estimated volatility of the equity index over the next period, S_t is the value of the equity at time t, $\Delta t = 1$ business day is the time step for the discretization and λ is the rate of exponential decay. With a value of $\lambda = 0.99$ the mean age of the data being used is $\frac{\Delta t}{1-\lambda} \approx 0.4$ years.

The estimated volatility determines the weight of the equity in the dynamic rebalancing, which is given by

$$w_t^{equity} = \min\left(\frac{\sigma_{target}}{\hat{\sigma}_t^{equity}}, 100\%\right),$$

where σ_{target} is the prearranged target volatility level. This means that there is no leverage allowed in the risky asset if the estimated volatility is below the target volatility level.

To summarize, I examined the effects of modelling assumptions in the case of the models below.

- Black-Scholes (BS) model (Black & Scholes, 1973)
- Heston model (Heston, 1993)
- Bates model (as SVJD model; Bates, 1996)
- Barndorff-Nielsen–Shephard (BNS) model (Barndorff-Nielsen & Shephard, 1999)
- Variance Gamma process with Gamma–Ornstein-Uhlenbeck stochastic clock (VGGOU; discussed in general in Carr et al. (2003))

CALIBRATION

The risky asset I consider in this research is the S&P 500 Index and as an initial price for the simulation I use the spot price of this equity on October 9th, 2017, that is $S_0 = 2544.73$. The 1Y USD LIBOR interest rate at this date is $r = 1.809$ %.

For the calibration of the Black-Scholes model, i.e. to determine the constant volatility σ of the model, I used the 2-year historical data of the index.

To calibrate the other four models I applied a technique described in Kilin (2011). He investigated three different methods to ascertain which one of them accomplishes the fastest calibration of various stochastic models. The winner announced adapts the *direct integration method* which is based on calculating the prices of European call options using the characteristic function of the stochastic models. With this, we can calibrate our models to the underlying product's option surface observed on the market, with the parameters chosen form predefined intervals. The calibration itself consists of three runs of the Differential Evolution algorithm which applies the direct integration method, and with the results of the previous as initial guesses, three runs of the Levenberg-Marquardt algorithm. Because of the second phase, the resulting parameters can be out of the predefined intervals. If they seem reasonable we can let them be. As an object function, the algorithm uses the sum of the squared differences of the market option prices and the ones calculated with the actual parameter set. For some details and references of the algorithms mentioned here see Kilin (2011). The characteristic functions of the four models can be found in Schoutens et al. (2003) or Kilin (2011).

For the calibration I used 90 different options in total, along 10 maturities from 0.15 to 3.18 years, and 9 strikes from 1100 - deep in-the-money (ITM) call option - to 2800 – out-of-the-money (OTM) call option. Table 1 shows the resulted parameters of the calibrated models. We may notice some unexpected parameters. First, the high and positive correlation (ρ) in the cases of Heston and Bates models. Our presumption is a negative correlation between the volatility and stock price processes, thus further analysis is needed to determine the cause of this outcome. Next, in the Bates model the parameters λ, μ_J, σ_J, which are associated with jumps, are taken at the lower bounds. This corresponds to rare and small jumps, which is probably because the underlying equity is the S&P 500 Index. For this, to occur a jump the whole market should sustain a shock. At last, the parameter b of the BNS model is quite large and taken at the upper bound. The reason for this is that, the higher upper bound I chose for the initial interval for parameter b the better the model fitted to the actual data. However, at the same time the larger discretization error I received, especially in the long run and in the case of high strike prices. This upper bound seemed still reasonable, as over this the value of the goodness of fit didn't improve much and I didn't experience significant discretization error.

Table 1: Parameters of the Models Calibrated Using Direct Integration Method. The Predefined Intervals Are Shown in Square Brackets below the Parameters.

BS			
$\sigma = 0.114$			
Heston			
$\sigma_0 = 0.067$	$\eta = 0.070$	$\kappa = 0.019$	$\theta = 0.015$
$\in [0.001, 1]$	$\in [0.001, 1]$	$\in [0.005, 6]$	$\in [0.001, 1]$
$\rho = 1.000$			
$\in [-1, 1]$			
Bates			
$\sigma_0 = 0.067$	$\eta = 0.256$	$\kappa = 0.005$	$\theta = 0.015$
$\in [0.001, 1]$	$\in [0.001, 1]$	$\in [0.001, 6]$	$\in [0.001, 1]$
$\rho = 1.000$	$\lambda = 0.050$	$\mu_J = 0.010$	$\sigma_J = 0.010$
$\in [-1, 1]$	$\in [0.05, 2]$	$\in [0.01, 0.5]$	$\in [0.01, 0.5]$
BNS			
$\sigma_0 = 0.004$	$a = 9.317$	$b = 1500$	$\lambda = 6.871$
$\in [0.001, 0.15]$	$\in [0.01, 100]$	$\in [5, 1500]$	$\in [0.01, 10]$
$\rho = -1.10e - 08$			
$\in [-50, 0]$			
VGGOU			
$C = 254.18$	$G = 423.80$	$M = 319.10$	
$\in [1, 100]$	$\in [30, 200]$	$\in [30, 200]$	
$\lambda = 1.896$	$a = 11.376$	$b = 11.990$	$y_0 = 1$
$\in [0.5, 20]$	$\in [2, 20]$	$\in [2, 20]$	$= 1$

RESULTS

I set the target volatility level to 5% by reason of choosing a (much) higher target volatility level would have made the TVF implied volatility curves equal to the equity implied volatility curves in the VGGOU case, and at (much) lower target volatility levels the implied volatility calculation algorithm does not always succeed, especially at lower strikes.

To investigate the effects of model selection on the guarantees on TVFs, first I present the ATM put option prices for maturities 2, 5 and 10 years in Table 2. To determine the prices, I generated 10000 scenarios.

After this, I inspect the implied volatilities calculated using another 10000 trajectories and I also show some sample scenarios (Figure 1-10). Particularly, to present these results I use the following two types of figures. The first one graphs the predefined target volatility level (dashed red line), the implied volatility for the equity put options (solid light blue line) and the TVF put option (solid dark blue line). The second type shows an example scenario for the model, which consists of four subfigures: a realization of the equity index (upper left corner), the model volatility and the one received by the EWMA estimator (upper right corner), the corresponding trajectory of the TVF (bottom left corner) and the TVF volatility with the target volatility level (bottom right corner).

ATM Put Options On TVF

Table 2 shows that for every maturity there is an increase in the put option price relative to the Black-Scholes model. However, in most cases it is below 10% so should be considered as an indicator rather than a decisive fact. To confirm the results a robustness analysis is needed. Yet, we can't let the VGGOU case go by which is rather convincing and where the percentage change increases from 60% to a level over 200%. Apart from VGGOU model, for every stochastic model the put option price decreases in time. Since in the VGGOU case the option price starts at a relatively high level and it even grows higher in the long run, it results in an extremely large percentage change for 10 years' maturity.

Table 2: Prices of ATM Put Options on TVF and Percentage Changes Relative to the Black-Scholes Model. Determined Using 10000 Scenarios.

Model \ years	2	5	10
BS	34.40	32.43	23.22
Heston	37.01	36.22	24.79
	7.58%	*11.67%*	*6.75%*
Bates	36.01	33.63	24.43
	4.68%	*3.67%*	*5.24%*
BNS	38.60	33.90	25.09
	12.21%	*4.51%*	*8.06%*
VGGOU	55.20	68.25	71.40
	60.49%	*110.42%*	*207.55%*

Black-Scholes Model

In the case of the Black-Scholes model the implied volatility on the TVF is almost equal to the target volatility for all strikes and maturities (Figure 1).

The EWMA estimator seems to work well in estimating the constant model volatility, as it is close to it in Figure

2, upper right corner. The TVF process varies on a smaller scale than the equity index (Figure 2, subfigures on the left) and the TVF model volatility fluctuates around the target volatility level (Figure 2, bottom right corner). Overall, the method works well in this case.

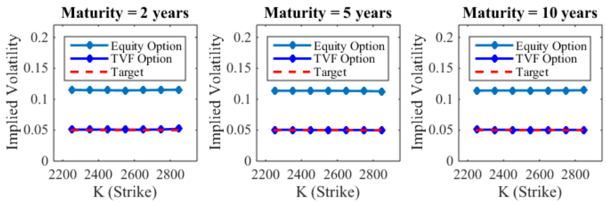

Figure 1: Implied Volatilities for the Black-Scholes Model. Calculated Using 10000 Scenarios.

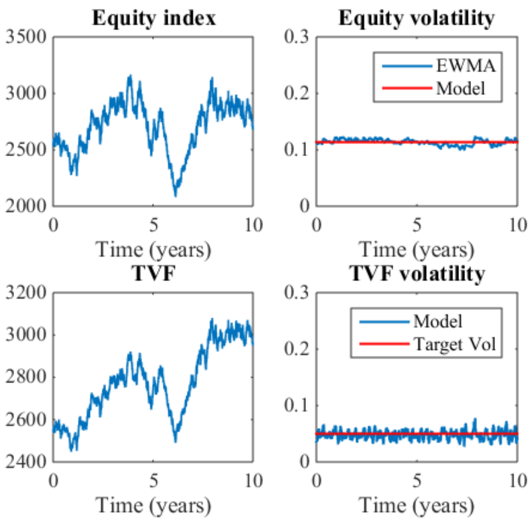

Figure 2: Example Scenario for the Black-Scholes Model.

Heston Model

As for the Heston model, the equity implied volatility curve has a large skew at 2 years' maturity that flattens over time, and the level of the curve increases at the same time (Figure 3).The TVF implied volatility curve tends to fit quite well, although in the short run it has a small skew that seems to disappear over time, as well.

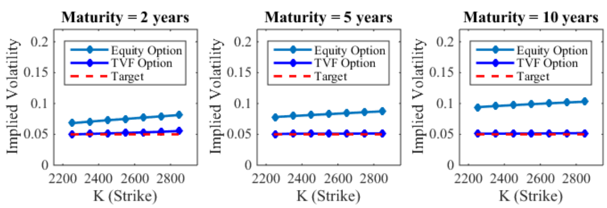

Figure 3: Implied Volatilities for the Heston Model. Calculated Using 10000 Scenarios.

The equity volatility grows towards the long variance as it is higher than the spot variance. The EWMA estimator works well in following it, however, we can see some distortions (Figure 4, upper right corner). The TVF model volatility moves around the target with a small deviation (Figure 4, bottom right corner). This suggests that it can maintain an almost constant volatility level over time, but according to Table 2 the method results in higher guarantee costs.

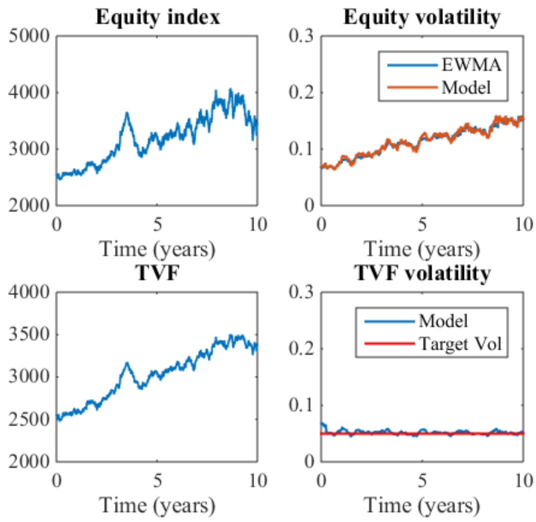

Figure 4: Example Scenario for the Heston Model.

Bates Model

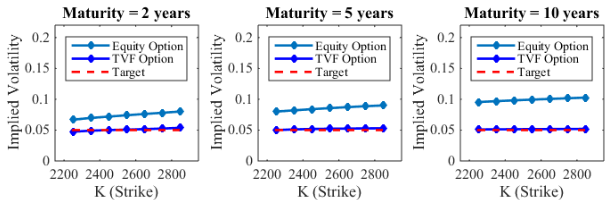

Figure 5: Implied Volatilities for the Bates Model. Calculated Using 10000 Scenarios.

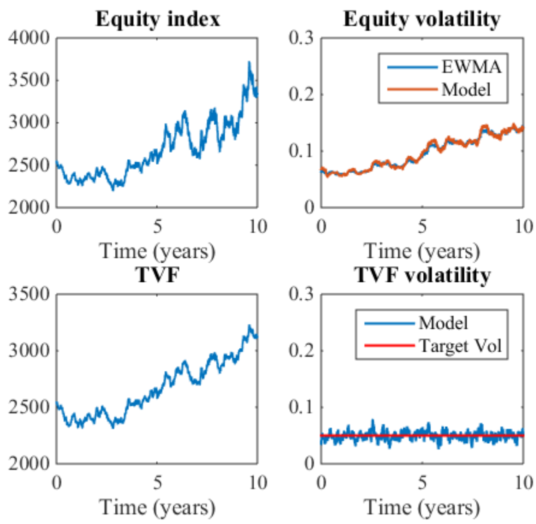

Figure 6: Example Scenario for the Bates Model.

In the case of the Bates model, in many aspects we get similar results to the ones seen at the Heston model

(Figures 5 and 6), the only significant difference seems to be the larger deviation of the TVF model volatility around the target (Figure 6, bottom right corner).

BNS Model

Moving on to the BNS model - the first one that Morrison and Tadrowski (2013) did not consider – we see distinctive results. The implied volatility curves seem to be completely flat, however, the level of the equity implied volatility curve slightly increases over time, but still remains under the implied volatility on equity index of the Black-Scholes model. The implied volatility curves of the TVF are a little bit above the target, and this difference is not likely to disappear over time (Figure 7).

Again, the EWMA estimator tends to do a good job (Figure 8, upper right corner) and the TVF model volatility fluctuates around the target (Figure 8, bottom right corner). The rebalancing method seemingly makes the TVF price process less volatile than the equity price process (Figure 8, subfigures on the left), but yet again, as Table 2 suggests we arrive at higher put option prices.

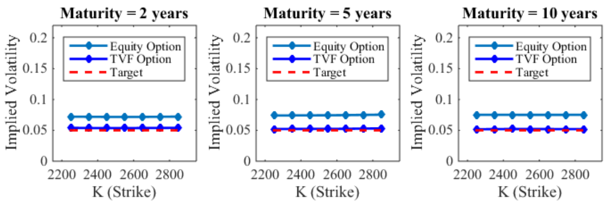

Figure 7: Implied Volatilities for the BNS Model. Calculated Using 10000 Scenarios.

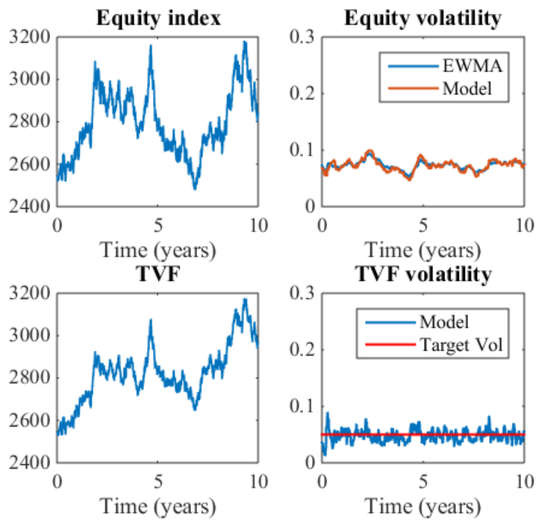

Figure 8: Example Scenario for the BNS Model.

VGGOU Model

Lastly, we investigate the case of the VGGOU model, which is the second one that extends the previous analysis of Morrison and Tadrowski (2013). The implied volatility on the TVF has a skew and is considerably high for all strikes, and even increases over time (Figure 9). This is in connection with the large percentage change in ATM put option prices seen in Table 2.

Although the EWMA estimator seems to work well (Figure 10, upper right corner), the TVF model volatility fluctuates around the target with a much higher volatility than that we have seen in any other case before (Figure 10, bottom right corner). As this figure suggests, the rebalancing method cannot maintain a stable constant volatility for the TVF, but the volatility of the TVF price process seems to be less than the one of the equity price process (Figure 10, subfigures on the left). Furthermore, the smaller target volatility level we chose the lower the level of the implied volatility on TVF will be, however, it will never reach the target level.

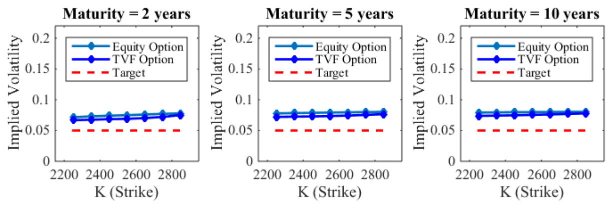

Figure 9: Implied Volatilities for the VGGOU Model. Calculated Using 10000 Scenarios.

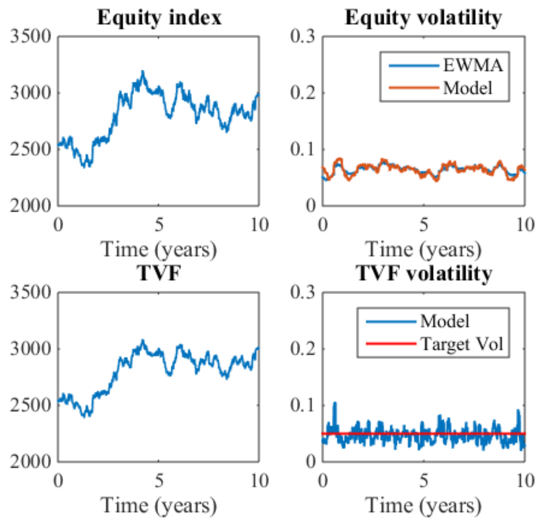

Figure 10: Example Scenario for the VGGOU Model.

CONCLUSIONS

In this paper, I extended the analysis of Morrison and Tadrowski (2013) to the Barndorff-Nielsen–Shephard model and to a Lévy-process with stochastic time, which was the Variance Gamma process with Gamma–Ornstein-Uhlenbeck stochastic clock.

The rebalancing method performs well assuming Black-Scholes model, but probably results in higher guarantee costs if we change to more complex, stochastic volatility models. The case of the VGGOU is the most convincing with extremely dear guarantees relative to the Black-Scholes model. These results are rather interesting, however, some further examination is needed to investigate the robustness of them. Also, it is necessary to prosecute a more detailed analysis to examine if the results are robust using other, hopefully more volatile underlying products and various target volatility levels. Finally, it is a possibility to inspect the effect of using different volatility estimators like Kim and Enke (2016) did.

REFERENCES

Barndorff-Nielsen, O. E., & Shephard, N. (1999). *Non-Gaussian OU based models and some of their uses in financial economics*. Oxford: Nuffield College.

Bates, D. S. (1996). Jumps and stochastic volatility: Exchange rate processes implicit in deutsche mark options. *The Review of Financial Studies*, *9*(1), 69-107.

Black, F., & Scholes, M. (1973). The pricing of options and corporate liabilities. *Journal of political economy*, *81*(3), 637-654.

Carr, P., Geman, H., Madan, D. B., & Yor, M. (2003). Stochastic volatility for Lévy processes. *Mathematical Finance*, *13*(3), 345-382.

Chew, L. (2011). Target volatility asset allocation strategy. *Society of Actuaries International News*.

Heston, S. L. (1993). A closed-form solution for options with stochastic volatility with applications to bond and currency options. *The review of financial studies*, *6*(2), 327-343.

Hocquard, A., Ng, S., & Papageorgiou, N. (2013). A constant-volatility framework for managing tail risk. *The Journal of Portfolio Management*, 39(2):28-40.

Kilin, F. (2011). Accelerating the calibration of stochastic volatility models. *Journal of Derivatives*, *18*(3), 7.

Kim, Y. és Enke, D. (2016). Using neural networks to forecast volatility for an asset allocation strategy based on the target volatility. Procedia Computer Science, 95:281-286.

Morrison, S. & Tadrowski, L. (2013). Guarantees and target volatility funds. *Moody's Analytics B&H Research series*.

Schoutens, W., Simons, E., & Tistaert, J. (2003). A perfect calibration! Now what?. *The best of Wilmott*, 281.

Stoyanov, S. (2011) Structured Equity Investment Strategies for Long-Term Asian Investors. *EDHEC Risk Institute Publication*.

Torricelli, L. (2017). Assessing target volatility investment strategies using stochastic delayed differential models. SSRN. URL http://dx.doi.org/10.2139/ssrn.2902063.

Xue, Y. (2011). Target volatility: an effiective risk management tool for VA? *Society of Actuaries*.

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