BOUNDARY STRATEGIES FOR FIREFLY ALGORITHM ANALYSED USING CEC`17 BENCHMARK

Tomas Kadavy
Michal Pluhacek
Adam Viktorin
Roman Senkerik
Tomas Bata University in Zlin, Faculty of Applied Informatics
Nam T.G. Masaryka 5555, 760 01 Zlin, Czech Republic
{kadavy, pluhacek, aviktorin, senkerik }@utb.cz

KEYWORDS
Firefly Algorithm, FA, FFPSO, Boundary, CEC17

ABSTRACT

In this paper, we are presenting a comparison of few selected boundary strategies for two popular optimization algorithms. The Firefly Algorithm (FA) and its hybridized modification, the Firefly Particle Swarm Optimization (FFPSO). The problem of boundary constrained optimization was already extensively studied for well-known heuristic optimization Particle Swarm Optimization (PSO). This suggesting importance for similar research for other swarm-based algorithms, like FA. The recent CEC´17 benchmark suite is used for the performance comparison of the methods and the results are compared and tested for statistical significance.

INTRODUCTION

Firefly Algorithm (FA) (Yang 2008; Yang 2009) is one of the modern and versatile optimization algorithms developed by Yang in 2008. Over the years, the FA proved its robustness in performance on several problems (Oosumi et al. 2016; Yang 2013). Another proof of the importance of this modern swarm-based algorithm is supported by a number of its modification. For example, Lévy flights (Yang 2010) and chaos-driven FA (Coelho, Mariani 2012) show the large potential for future research and possible application.

The definition of typical optimization task contains boundary limits for optimized parameters. Due to nature of the metaheuristic optimization algorithm, the trial particle (in this case firefly) can emerge outside of the area of the feasible solution. The paper focuses on the question what to do with particle if it tries to violate the defined boundaries. The research in the area of possible border strategies and their influence on performance was already extensively done for another well-known optimization technique the Particle Swarm Optimization (PSO). As the provided studies suggesting, it could be a truly difficult task (Helwig et al. 2013; Kadavy et al. 2017).

Since there is a lack of such studies for FA available in the literature, we have decided to perform and present this original experimental research. In this paper, four relatively common borders strategies (or rather methods) are implemented and compared on CEC'17 benchmark set (Awad et al. 2016). Also, one modern hybrid optimization technique was tested with the mentioned boundary strategies. The Firefly Particle Swarm Optimization (FFPSO) (Kora, Rama 2016) was particularly selected due to its similarity with the PSO. The previous studies on PSO show the importance of careful selection of border strategy. The main research questions can be then summarized as follows:

- Could the selection of border strategy influence the performance of canonical FA or the FFPSO?
- What is the most suitable border strategy in general?

The paper is structured as follows. The FA and the FFPSO are described in details within the next two sections. The implemented and tested border strategies follow afterwards. Last two sections discuss the experiment setting and results.

FIREFLY ALGORITHM

This optimization nature-based algorithm was developed and introduced by Yang in 2008 (Yang 2008). The fundamental principle of this algorithm lies in simulating the mating behavior of fireflies at night when fireflies emit light to attract a suitable partner. The main idea of Firefly Algorithm (FA) is that the objective function value that is optimized is associated with the flashing light of these fireflies. The author for simplicity set a couple of rules to describe the algorithm itself:

- The brightness of each firefly is based on the objective function value.
- The attractiveness of a firefly is proportional to its brightness. This means that the less bright firefly is lured towards, the brighter firefly. The brightness depends on the environment or the medium in which fireflies are moving and decreases with the distance between each of them.
- All fireflies are sexless, and it means that each firefly can attract or be lured by any of the remaining ones.

The movement of one firefly towards another one is then defined by equation (1). Where x_i' is a new position of a firefly i, x_i is the current position of firefly i and x_j is a selected brighter firefly (with better objective function

Proceedings 32nd European Conference on Modelling and Simulation ©ECMS Lars Nolle, Alexandra Burger, Christoph Tholen, Jens Werner, Jens Wellhausen (Editors)
ISBN: 978-0-9932440-6-3/ ISBN: 978-0-9932440-7-0 (CD)

value). The α is a randomization parameter and *sign* simply provides random direction -1 or 1.

$$x_i' = x_i + \beta \cdot (x_j - x_i) + \alpha \cdot sign \quad (1)$$

The brightness *I* of a firefly is computed by the equation (2). This equation of brightness consists of three factors mentioned in the rules above. On the objective function value, the distance between two fireflies and the last factor is the absorption factor of a media in which fireflies are.

$$I = \frac{I_0}{1+\gamma r^m} \quad (2)$$

Where I_o is the objective function value, the γ stands for the light absorption parameter of a media in which fireflies are and the *m* is another user-defined coefficient and it should be set $m \geq 1$. The variable *r* is the Euclidian distance (3) between the two compared fireflies.

$$r_{ij} = \sqrt{\sum_{k=1}^{d} (x_{i,k} - x_{j,k})^2} \quad (3)$$

Where r_{ij} is the Euclidian distance between fireflies x_i and x_j. The d is current dimension size of the optimized problem.
The attractiveness β (4) is proportional to brightness *I* as mentioned in rules above and so these equations are quite similar to each other. The β_0 is the initial attractiveness defined by the user, the γ is again the light absorption parameter and the *r* is once more the Euclidian distance. The *m* is also the same as in equation (2).

$$\beta = \frac{\beta_0}{1+\gamma r^m} \quad (4)$$

Finally, the pseudocode below shows the fundamentals of FA operations.

```
FA initialization
while(terminal condition not met)
  for i = 1 to all fireflies
    for j = 1 to all fireflies
      if(Ij < Ii) then
        move xi to xj
        evaluate xi
      end if
    end for j
  end for i
  record the best firefly
end while
```

FIREFLY PARTICLE SWARM OPTIMIZATION

The typical example of a hybrid of the FA and PSO algorithms, the FFPSO (Kora, Rama 2016) introduced in late 2016 by Padmavathi Kora and K. Sri Rama Krishna. The basic idea behind such an approach is that the new hybrid strategy can share advantages from both algorithms and hopefully eliminate their disadvantages. The main principle remains the same as in the standard FA, but the equation for firefly motion (1) is slightly changed according to PSO movement (Eberhart, Kennedy 1995) and is newly computed as (5).

$$x_i' = wx_i + c_1 e^{-r_{px}^2}(pBest_i - x_i) + c_2 e^{-r_{gx}^2}(gBest - x_i) + \alpha \cdot sign \quad (5)$$

Where w, c_1, and c_2 are control parameters transferred from PSO and their values often depends on the user. Also, the *pBest* and *gBest* are variables originally belonging to PSO algorithm. They both represent the memory of the best position where *pBest* is best position of each particle and *gBest* is globally achieved best position so far. The remaining variables r_{px} (6) and r_{gx} (7) are distances between particle x_i and both $pBest_i$ and *gBest*.

$$r_{px} = \sqrt{\sum_{k=1}^{d} (pBest_{i,k} - x_{i,k})^2} \quad (6)$$

$$r_{gx} = \sqrt{\sum_{k=1}^{d} (gBest_k - x_{i,k})^2} \quad (7)$$

The pseudocode below shows the fundamentals of FFPSO operations. As it can be seen in given pseudocode, the main principle remains the same as in canonical FA.

```
FFPSO initialization
while(terminal condition not met)
  for i = 1 to all fireflies
    for j = 1 to all fireflies
      if(Ij < Ii) then
        calculate rpx and rgx
        move xi to xj
        evaluate xi
      end if
    end for j
  end for i
  record the best firefly
end while
```

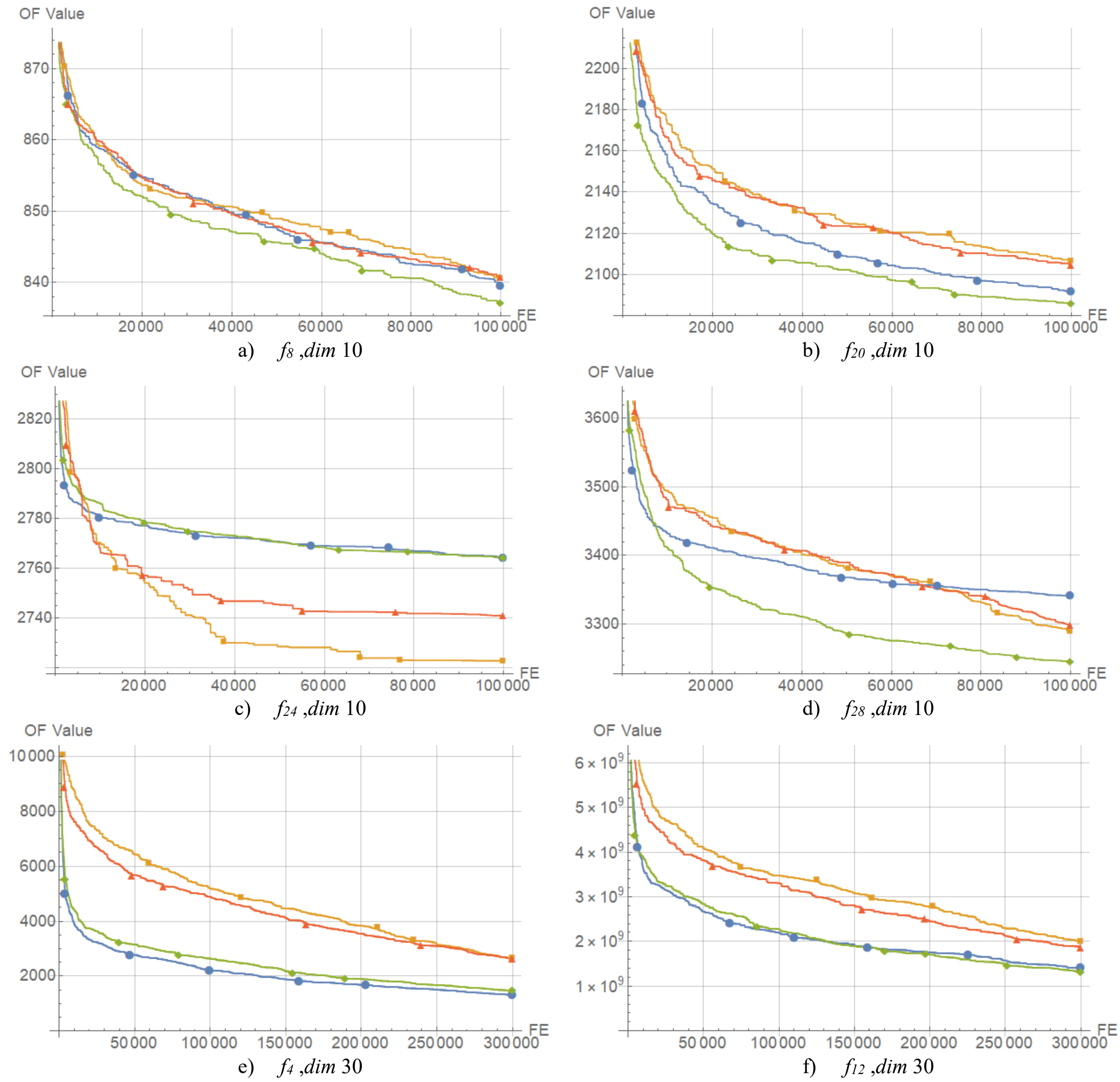


Figure 1: Convergence graphs of canonical FA The clipping strategy is represented as blue line with circle marks. The random strategy is represented as orange line with rectangle marks. The reflection strategy is represented as green line with rhombus marks. Finally, the periodic strategy is represented as red line with triangle marks.

BORDER STRATEGIES

Every time, when a single objective function optimization problem has defined a range where the best value is being found by the metaheuristic algorithm, one of the many difficult tasks could arise to an operator or a user. After each step of an algorithm, in this case after position update of a firefly, the new position should be checked if it lies in the appropriate range or boundaries (inside space of feasible solution). In case that the new position of the particle is outside this allowed region a certain correction has to be made. Several possible correction methods or strategies could do the trick. However, select the most appropriate is not an easy task since each of them could have a very different effect on the algorithm ability to achieve a good solution (Helwig et al. 2013). For this paper, the most common ones were selected and compared together to show how they could affect the FA or FFPSO on different benchmark functions.

Clipping strategy

The first selected strategy is rather simple in principle. The particle (or in this case firefly) cannot cross the given boundaries in each dimension. This strategy is very simple to implement and is described as (8).

$$x_i' = \begin{cases} x_i = b^u, if\ x_i > b^u \\ x_i = b^l, if\ x_i < b^l \\ x_i, otherwise \end{cases} \tag{8}$$

Where x_i is the position of i firefly before boundary check, the x_i' is a newly updated position after the boundary check and the b^u and b^l are the upper and lower boundary given to each dimension.

Random strategy

If a firefly violates the boundary in any dimension, the new position for this firefly for a particular dimension is created between the lower and upper boundary (with a pseudo-random uniform distribution). Again this strategy is rather simple and very easy to implement, as the equation (9) shows.

$$x_i' = \begin{cases} x_i = U(b^l, b^u), if\ x_i > b^u\ OR\ x_i < b^l \\ x_i, otherwise \end{cases} \tag{9}$$

Where U stands for uniform distribution in range from b^l (lower boundary limit) to b^u (upper boundary limit).

Reflection strategy

The reflection strategy (Helwig et al. 2013) reflects the particle back to feasible space of solution if it tries to violate the defined borders. This strategy tries to emulate the reflection characteristic of for example a mirror. For violated dimension, the correction of a position of a particle is computed as (10). Where again the b^u and b^l are the upper boundary limit and lower boundary limit.

$$x_i' = \begin{cases} x_i' = b^u - (x_i - b^u), if\ x_i > b^u \\ x_i' = b^l + (b^l - x_i), if\ x_i < b^l \\ x_i, otherwise \end{cases} \tag{10}$$

Periodic strategy

The possible solution to prevent the infeasibility could lie in the method of infinite copies of the optimized hyperspace (Zhang et al. 2004). This strategy involves only mapping the particle back to the space of available solution using only the modulo function (11).

$$x_i' = b^l + \left(x_i\ MOD\ (b^u - b^l)\right) \tag{11}$$

EXPERIMENT SETTING

The experiments were performed on a set of well-known benchmark functions CEC'17 which are detailly described in (Awad et al. 2016). The tested dimensions were 10 and 30. The maximal number of function evaluation was set as 10 000 · *dim* (dimension size). The lower and upper boundary was as $b^l = -100$ and $b^u = 100$ according to CEC'17. The number of fireflies was set to 40 for both dimension sizes. Every test function was repeated for 51 independent runs and, the results were statistically evaluated. The benchmark itself includes 30 test functions in four categories: unimodal, multimodal, hybrid and composite types. The global minimum of each function is easy to determine as it is $100 \cdot f_i$ where i is an order of test function.
The parameters of FA were set as $\alpha = 0.5$, $\gamma = 1$, $\beta_0 = 0.2$ and $m = 1$ according to author (Yang 2008). The parameters of FFPSO were set the same as FA including the parameters borrowed from PSO ($c = 1.49445$, $w = 0.729$).

RESULTS

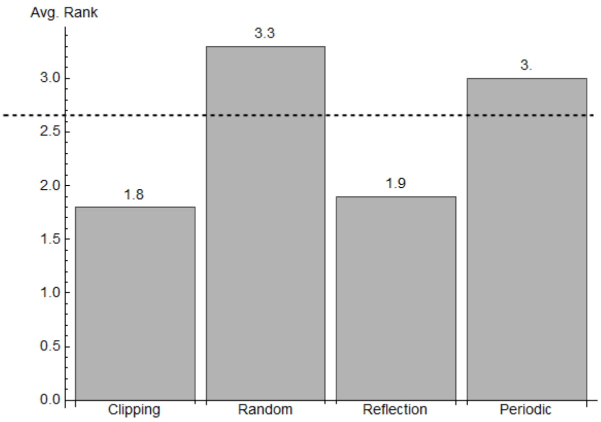
The results of performed experiments are presented in this section. Firstly, the examples of convergence behavior of the compared methods are given in Figure 1.
Furthermore, the results were tested for statistical significance using the Friedman Rank test (Demšar 2006). The null hypothesis that the mean is equal is rejected at the 5 percent level based on the Friedman Rank test. The corresponding p-values of Friedman Rank test are presented in Table 1. If the p-value is lower than 0.05, the further Friedman rankings are relevant (these values are given by bold numbers in Table 1). In Figure 2, the Friedman ranking for both algorithms (FA and FFPSO) on both dimension sizes is shown. The lower the rank is, the better is the performance of the strategy labeled strategy. Furthermore, the presented Friedman ranks are accompanied with critical distance evaluated according to the Nemenyi Critical Distance post-hoc test for multiple comparisons. The dashed line represents the critical distance from the best boundary method (the lowest mean rank).
The critical distance (CD) value for this experiment has been calculated as 0.656757; according to the definition given in (12) and value $q_a = 2.56892$; using $k = 4$ boundary methods and a number of data sets $N = 51$ (51 repeated runs).

$$CD = q_a\sqrt{k(k+1)/(6N)} \tag{12}$$

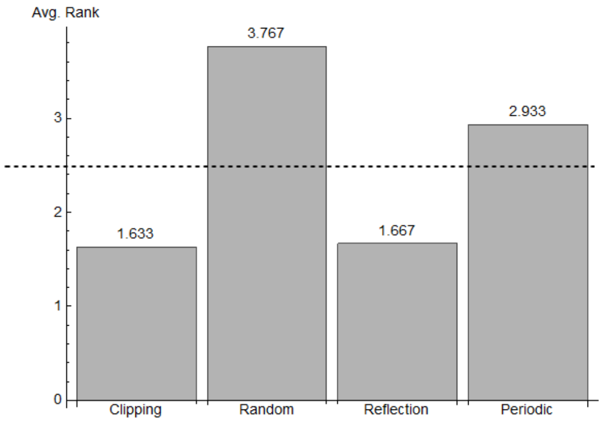
Table 1: P-values of Friedman Rank tests

Algorithm	Dimension size 10	Dimension size 30
FA	**3.757E-08**	**1.208E-19**
FFPSO	0.788E-00	0.268E-00

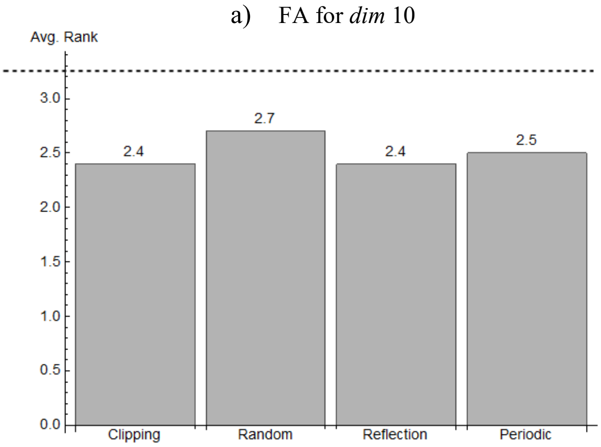
According to this evaluation and the ranking shown in Figure 2, the significant impact of the selected border strategies is mostly observed on canonical FA. For the hybrid method FFPSO the results suggesting some sort of insignificance for the used method. For dimension sizes 10 and 30, the most favorable strategies seem to be clipping and reflection methods. Despite the fact that at first sight, the used strategies are unique to each other, some similarities are shared among them. The clipping and reflection strategies forcing the fireflies to move only in the feasible space of solution without loss of information of the previous position. On the contrary, the two remaining strategies have in common the loss of the previous position and their behavior resembling the random search optimization (Bergstra, Bengio 2012).



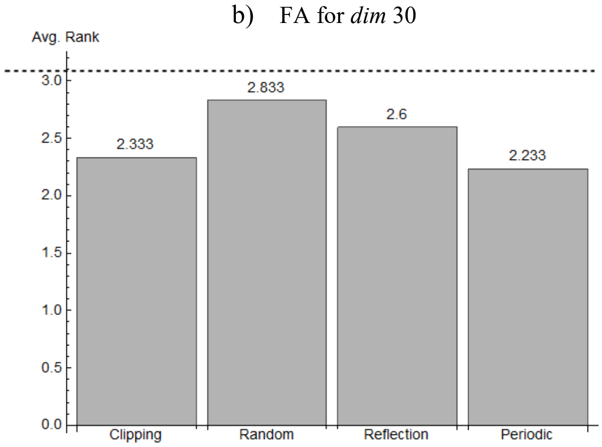
a) FA for *dim* 10



b) FA for *dim* 30



c) FFPSO for *dim* 10



d) FFPSO for *dim* 30

Figure 2: Friedman Rank tests for selected border strategies.

CONCLUSION

In this original study, the impact of various border strategies on the performance of the FA and FFPSO is tested. The topic is actual due to the increasing variety and complexity of optimization problems. As a benchmark for the performance comparisons, the CEC 2017 set was used. It represents the most recent collection of artificial optimization problems that vary in terms of modality and other characteristics of the fitness land scape.

It may be concluded, that according to statistical data, the clipping and reflection strategies seem to be favorable over the other two (random and periodic strategies). As the Friedman ranks showed, the slightly better performance is achieved by the clipping method. However, the same four strategies strangely seem to have no significant impact on the hybrid FFPSO algorithm. Moreover, all the observation suits for both dimension setting with almost the same results.

To answer the research question:

- Could the selection of border strategy influence the performance of canonical FA or the FFPSO?
- From the achieved results, the border strategy has a significant impact on the performance for only the canonical FA. For FFPSO, the influence is negligible.
- What is the most suitable border strategy in general?
- In general term of speaking, the most suitable strategies are two. Specifically, the clipping and reflection strategy.

Despite that, the results of this study are useful as an empirical study for researchers dealing with firefly algorithm. This research will continue in the future with exploring the performance of firefly algorithm with different boundary strategies on different fitness landscape models and real-world problems, especially with a focus on the algorithm setup to achieve the best performance.

ACKNOWLEDGEMENT

This work was supported by the Ministry of Education, Youth and Sports of the Czech Republic within the National Sustainability Programme Project no. LO1303 (MSMT-7778/2014), further by the European Regional Development Fund under the Project CEBIA-Tech no. CZ.1.05/2.1.00/03.0089 and by Internal Grant Agency of Tomas Bata University under the Projects no. IGA/CebiaTech/2018/003. This work is also based upon support by COST (European Cooperation in Science & Technology) under Action CA15140, Improving Applicability of Nature-Inspired Optimisation by Joining Theory and Practice (ImAppNIO), and Action IC1406, High-Performance Modelling and Simulation for Big Data Applications (cHiPSet). The work was further supported by resources of A.I.Lab at the Faculty of Applied

Informatics, Tomas Bata University in Zlin (ailab.fai.utb.cz).

REFERENCES

X. S. Yang, Nature-Inspired Metaheuristic Algorithms, Luniver Press, UK, (2008).

X. S. Yang, Firefly algorithms for multimodal optimization, Proc. 5th Symposium on Stochastic Algorithms, Foundations and Applications, Lecture Notes in Computer Science, 5792: 169-178 (2009).

Oosumi R., Tamura K., Yasuda K. Novel single-objective optimization problem and Firefly Algorithm-based optimization method. 9-12 Oct. 2016; IEEE; 2016.

Yang X. Multiobjective firefly algorithm for continuous optimization. Engineering with Computers. 2013; 29 (2):175-84.

X. S. Yang, Firefly algorithm, Lévy flights and global optimization, in: Research an Development in Intelligent Systems XXVI (Eds M. Bramer, R. Ellis, M. Petridis), Springer London, pp. 209-218 (2010).

Coelho LdS, Mariani VC. Firefly algorithm approach based on chaotic Tinkerbell map applied to multivariable PID controller tuning. Computers & Mathematics with Applications. 2012 October 1; 64(8):2371-82.

Helwig S, Branke J, Mostaghim S M. Experimental Analysis of Bound Handling Techniques in Particle Swarm Optimization. TEVC. 2013; 17(2):259-71.

Kadavy T, Pluhacek M, Viktorin A, Senkerik R. Comparing Border Strategies for Roaming Particles on Single and Multi-swarm PSO. In: Artificial Intelligence Trends in Intelligent Systems: Proceedings of the 6th Computer Science On-line Conference 2017 (CSOC2017), Vol1. Cham: Springer International Publishing; 2017. p. 528-36.

N. H. Awad, et al. Problem Definitions and Evaluation Criteria for CEC 2017 Special Ses-sion and Competition on Single-Objective Real-Parameter Numerical Optimization, 2016.

Kora P, Rama Krishna KS. Hybrid firefly and Particle Swarm Optimization algorithm for the detection of Bundle Branch Block. International Journal of the Cardiovascular Academy. 2016 March 1;2(1):44-8.

Eberhart R, Kennedy J. A new optimizer using particle swarm theory. 1995.

Helwig, S., Branke, J., & Mostaghim, S. (2013). Experimental analysis of bound handling techniques in particle swarm optimization. IEEE Transactions on Evolutionary computation, 17(2), 259-271.

W.-J. Zhang, X.-F. Xie, and D.-C. Bi, "Handling boundary constraints for numerical optimization by particle swarm flying in periodic search space," in Proc. IEEE Congr. Evol. Comput., Jun. 2004, vol. 2, pp. 2307–2311.

Demšar, J. (2006). Statistical comparisons of classifiers over multiple data sets. Journal of Machine learning research, 7(Jan), 1-30.

Bergstra, J., & Bengio, Y. (2012). Random search for hyper-parameter optimization. Journal of Machine Learning Research, 13(Feb), 281-305.

AUTHOR BIOGRAPHIES

TOMAS KADAVY was born in the Czech Republic, and went to the Faculty of Applied Informatics at Tomas Bata University in Zlín, where he studied Information Technologies and obtained his MSc degree in 2016. He is studying his Ph.D. at the same university and the fields of his studies are: Artificial intelligence and evolutionary algorithms. His email address is: kadavy@utb.cz

MICHAL PLUHACEK was born in the Czech Republic, and went to the Faculty of Applied Informatics at Tomas Bata University in Zlín, where he studied Information Technologies and obtained his MSc degree in 2011 and Ph.D. in 2016 with the dissertation topic: Modern method of development and modifications of evolutionary computational techniques. He now works as a researcher at the same university. His email address is: pluhacek@utb.cz

ADAM VIKTORIN was born in the Czech Republic, and went to the Faculty of Applied Informatics at Tomas Bata University in Zlín, where he studied Computer and Communication Systems and obtained his MSc degree in 2015. He is studying his Ph.D. at the same university and the fields of his studies are: Artificial intelligence, data mining and evolutionary algorithms. His email address is: aviktorin@utb.cz

ROMAN SENKERIK was born in the Czech Republic, and went to the Tomas Bata University in Zlín, where he studied Technical Cybernetics and obtained his MSc degree in 2004, Ph.D. degree in Technical Cybernetics in 2008 and Assoc. prof. in 2013 (Informatics). He is now an Assoc. prof. at the same university (research and courses in: Evolutionary Computation, Applied Informatics, Cryptology, Artificial Intelligence, Mathematical Informatics). His email address is: senkerik@utb.cz