BALL & PLATE MODEL FOR ROBOTIC SYSTEM

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KEYWORDS

YuMi, robotics, LQ polynomial control, LQR state-space control, PD controller.

ABSTRACT

There are many solutions to control Ball & Plate model, ranging from hobby projects to more advanced control. This paper brings a new idea of control using robotic manipulator. This is quite challenging because industrial robots are not originally designed as a motion system for relatively fast and unstable system, which the Ball & Plate certainly is. This paper compares 3 controller designs to better comprehend the situation - a general LQR state-space control, LQ polynomial control and a basic PD controller. Results are also compared for a range of reference values to better understand advantages and disadvantages of chosen controllers, which will lead to future work and implementation for the real system. Data presented in this paper serve as a valuable background for next steps of the research and implementation.

INTRODUCTION

The Ball & Plate model is a well-known representative of fast and unstable systems ideal for designing and testing control algorithms. It is possible to find many designed Ball & Plate models on the Internet (mostly as hobby projects) with 2DoF separated control. There are of course more advanced structure with redundant degrees of freedom (Bruce et al. 2011) and even solution with 6DoF Stewart platform, which offers additional movement in the space.

Many control strategies are developed for Ball & Plate model starting at simple PID control (Jadlovska et al. 2009) and ending with fuzzy supervision (Moarref et al. 2008). This paper is aimed at the comparison of 2DoF LQ polynomial controller (Bobal et al. 2005), classic PD controller and LQR state-space controller. This comparison is also done for multiple reference values to better understand designed controllers. The quality of control is not the key aspect of this paper, but quality criteria for errors and controller effort are introduced in a table to better see the differences.

The paper is organized as follows. The first chapter deals with the theory and background behind Ball & Plate system and used robotic manipulator. It is divided into 3 subsections dealing with Ball & Plate, robot and identification separately. The next chapters describe 2DoF LQ polynomial controller and LQR state-space controller. The paper is closed with a chapter presenting obtained results and conclusion.

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BALL & PLATE ROBOTIC SYSTEM

Ball & Plate

A great model for testing control algorithms for unstable systems is Ball & Plate model. This model has relatively fast dynamics and with its instability provides quite a challenging task for controller design. Its general simplified and linearized mathematical description can be described by (1) and (2). This was more closely described in the previous work of authors (Spacek et al. 2017).

$$\ddot{x} = K\alpha \quad \rightarrow \quad G_x(s) = \frac{K}{s^2} \tag{1}$$

$$\ddot{y} = K\beta \quad \rightarrow \quad G_y(s) = \frac{K}{s^2} \tag{2}$$

where x,y are ball coordinates from the center of the plate ($\ddot{x}$, $\ddot{y}$ are respective 2nd time derivatives), α, β are angles of the plate, K is constant dependent on the gravitational acceleration g and the momentum of the ball and $G_x(s)$, $G_y(s)$ are continuous-time transfer functions with complex variable s. It is obvious these functions are symmetric in nature and thus only one-dimensional solution (Fig. 1) will be presented in simulations in this paper.

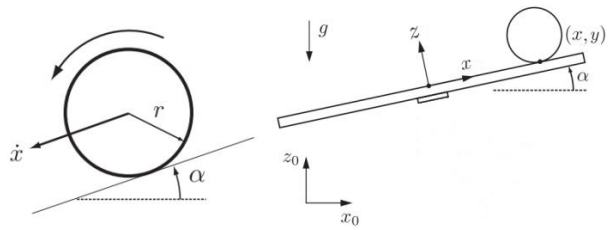

Fig. 1. Ball & Plate setup (Nokhbeh et al. 2011)

Collaborative Robot YuMi

The industrial robotic manipulator ABB IRB14000 YuMi (Fig. 2) was used as the motion structure for the simulations as a background to the real implementation. It has two manipulators with 7DoF and 0.02 mm repeatability each, which offers more possibilities and opportunities as the classic 2DoF solution with two servo motors or even 6DoF parallel manipulator commonly known as Stewart platform.

The YuMi is a collaborative robot, which means it can operate without external caging, which is safer and more efficient during implementation (Fryman and Matthias 2012). It would not be practical to have the

robot in a cage during control because it is possible to interact with the ball externally. The advantage is also that YuMi has two arms, which extends this application to the cooperative solution of more manipulators.

Fig. 2. ABB IRB 14000 YuMi

The development and simulation environment from ABB called RobotStudio is also a great advantage, as it excels in the deployment of similar applications and supports virtual sensors for measurements. The 7DoF manipulator has certainly complicated dynamics, but only last three joints actively influence the angle of the plate. The plate angle dynamics can be approximated by 2nd order transfer function, as can be seen from the change of the plate angle from 0 to 10 degrees in Fig. 3. This paper presents a solution, where the dynamics of the plate angle are approximated by 1st order transfer function. It is not the usual practice nowadays to simplify 2nd order dynamics to 1st order, but because Ball & Plate model consists of both the Ball & Plate dynamics and motor dynamics, it is the necessary step to reduce complexity. The identification chapter will show that it is not a very serious issue, as combined dynamics will negate this effect.

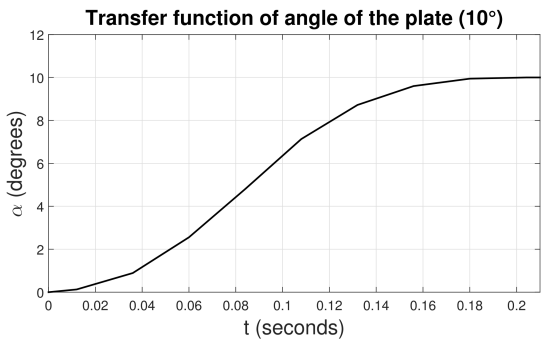

Fig. 3. Plate angle dynamics

Identification

The Ball & Plate model was identified for one dimension for both input-output (3) and state-space (4) models. Note that all measurements were in RobotStudio which is still the virtual environment, thus the identified system is still an ideal (semi-real) representation of the real system.

$$G(s) = \frac{K}{s^2\left(Ts+1\right)} = \frac{K}{Ts^3+s^2} \tag{3}$$

where $G(s)$ is continuous transfer function with complex variable s, K is velocity gain and T is the time constant of the system.

$$\begin{bmatrix}\dot{x}\\ \ddot{x}\\ \dot{\alpha}\end{bmatrix} = \begin{bmatrix}0 & 1 & 0\\ 0 & 0 & K_s\\ 0 & 0 & T_s\end{bmatrix}\begin{bmatrix}x\\ \dot{x}\\ \alpha\end{bmatrix} + \begin{bmatrix}0\\ 0\\ 1\end{bmatrix}\alpha$$
$$y = \begin{bmatrix}1 & 0 & 0\end{bmatrix}\begin{bmatrix}x\\ \dot{x}\\ \alpha\end{bmatrix} \tag{4}$$

where x is the position of the ball in one dimension, $\dot{x}$ is its time derivative, $\ddot{x}$ is its 2nd-time derivative, α is angle of the plate, y is the output of the system, K_s and T_s are identified constants of the state-space system.
Resulting identifications for different input angles showed a nonlinearity in the system, as shown in Fig. 4 which displays identification for the system described by (3). Both mathematical models were discretized with the period $T_0 = 0.05s$. Their discretization is necessary to design a discrete controller for future implementation of the real system.

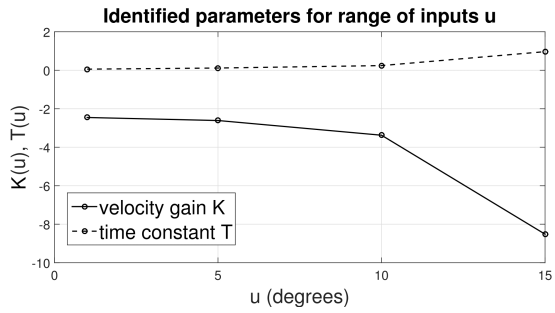

Fig. 4. Identified parameters

LQ POLYNOMIAL CONTROL

Discrete polynomial control is easy to implement because the discrete transfer function can be easily implemented in basic programming language and needs just previous values of input and output. Its disadvantage is a generally worse quality of control and limited possibilities compared to the state-space control, due to obvious reasons. The design of the 2DoF LQ digital controller is described in previous work of authors (Spacek et al. 2017).
The problem is solved by minimizing linear quadratic criterion in (5). This criterion can be solved in the polynomial form using spectral factorization (Bobal et al. 2005), instead of solving Riccati algebraic equation.

$$J = \sum_{k=0}^{\infty}\left\{\left[e(k)\right]^2 + q_u\left[u(k)\right]^2\right\} \tag{5}$$

where $e(k) = w(k) - y(k)$ is the error, $u(k)$ is controller output and q_u is a penalization constant, which

influences the controller output during the minimization process. Half of the solution (poles of characteristic polynomial in (6)) can be obtained from spectral factorization and other half is user-defined to adjust the behavior if needed (Spacek et al. 2017). This helps during the design of the controller, especially for the unstable system when poles of characteristic polynomial are harder to "guess".

$$D = AKP + BQ \tag{6}$$

where D is characteristic polynomial of a closed-loop system, B and A are numerator and denominator polynomials of the plant respectively, K is summation element, P and Q are denominator and numerator polynomials of the controller respectively. This equation is the same for continuous-time and discrete-time version, thus dependence notation of polynomials on complex variables s or z^{-1} is omitted.

The control law is designed for 2DoF controller structure (Fig. 5) because of its softer response to step changes. The 2DoF controller has 3 parts in this case: feedforward part C_f (7), feedback part C_b (8) and summation part $1/K(z^{-1}) = 1/(1 - z^{-1})$, which is extracted solely for practical purpose. Resulting implementation of the controller to programming environment is thus expressed in (9).

$$C_f(z^{-1}) = \frac{R}{P} = \frac{r_0}{1 + p_1 z^{-1} + p_2 z^{-2}} \tag{7}$$

$$C_b(z^{-1}) = \frac{Q}{P} = \frac{q_0 + q_1 z^{-1} + q_2 z^{-2} + q_3 z^{-3}}{1 + p_1 z^{-1} + p_2 z^{-2}} \tag{8}$$

$$\begin{aligned} u_k = (1 - p_1)u_{k-1} + (p_1 - p_2)u_{k-2} + p_2 u_{k-3} + \\ + r_0 w_k - q_0 y_k - q_1 y_{k-1} - q_2 y_{k-2} - q_3 y_{k-3} \end{aligned} \tag{9}$$

where q_i, p_i, and r_0 are controller parameters, u_{k-i} and y_{k-i} are outputs of the controller and plant respectively and w_k is desired (reference) value.

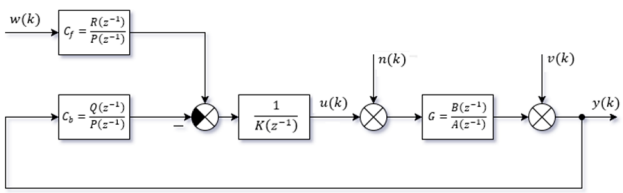

Fig. 5. Structure of 2DoF controller

LQR STATE-SPACE CONTROL

The state-space control is harder to implement, especially to the software of the robot because it not designed for such tasks in terms of speed and optimization. On the other hand, it has more options and may provide better results while using state observer, Kalman filter, MPC, and others. Because this paper deals with only simulation implementation it is not needed to use state observer

The state-space control (Fig. 6) is derived from the state-space description of the system either for continuous form in (10) or discrete form in (11).

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) + Du(t) \end{aligned} \tag{10}$$

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k \\ y_k &= Cx_k + Du_k \end{aligned} \tag{11}$$

where x, u and y are state, input and output vectors respectively, A, B, and C are system, input, and output matrices respectively and D is zero matrix for most real systems.

As mentioned above, the discrete controller is needed for future implementation, thus it will be designed for the discrete state-space model in (11). The goal is to obtain the optimal gain matrix K for state feedback control in (12) by minimization of the quadratic criterion in (13). The gain matrix K is then computed in MATLAB from (14).

$$u_k = -Kx_k \tag{12}$$

$$J = \sum_{k=1}^{\infty} \{x_k^T Q x_k + u_k^T R u_k\} \tag{13}$$

$$K = (B^T S B + R)^{-1} B^T S A \tag{14}$$

where Q and R are state (x_k) and input (u_k) weight matrices respectively, A and B are matrices described in (11) and S is the solution of discrete-time Riccati equation. N in Fig. 6 is precompensator equal to the 1st term of the gain matrix K (in this case). The precompensator is required because the reference value is not directly compared to the output, thus it needs a compensation.

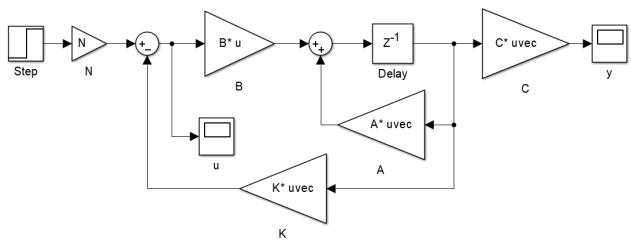

Fig. 6. State-space control in Simulink

As already mentioned, it is highly appropriate to use at least state observer to correctly estimate not measurable states, but in a simulation are all states "measurable", thus the more advanced control will be designed for the real system implementation only.

RESULTS

Designed controllers were implemented in MATLAB and Simulink to test and compare their results. Three types of controllers will be compared for this pilot study - adding a basic discrete PD controller to LQR state-space and LQ polynomial solutions. The PD controller

was designed by Naslin method (Balate 2003) and subsequently discretized, however its output was saturated to soften its output for large changes to interval $< -2; 2 >$. Results are shown for a step change (Fig. 7 and Fig. 8), sequence change (Fig. 9 and Fig. 10), ramp change (Fig. 11 and Fig. 12) and harmonic change (Fig. 13 and Fig. 14). Note that the plot is bounded for the step change, but the output of the PD controller reaches its saturation in $-2°$. The schematic of the Ball & Plate model virtualized in the RobotStudio for YuMi is shown in Fig. 15.

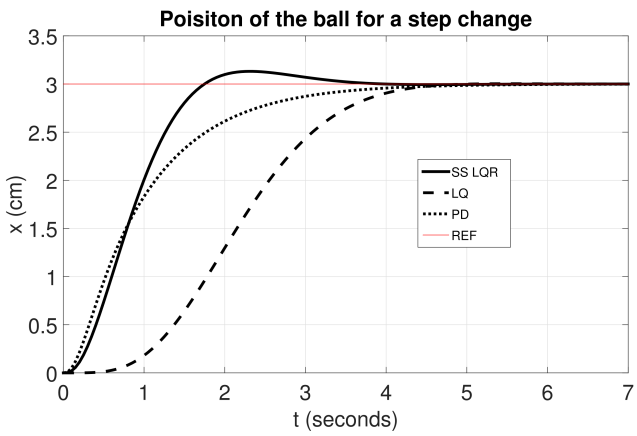

Fig. 7. Position of the ball - step change

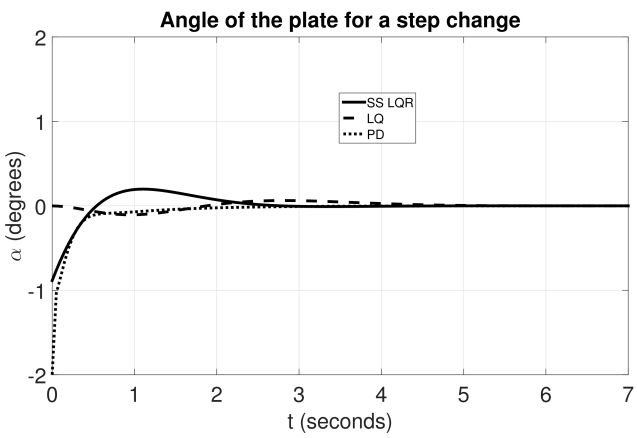

Fig. 8. Angle of the plate - step change

Quality criteria

The quality of control is shown in Tables 1-4. The criterion for the sum of squared errors is in (15) and for the sum of controller outputs in (16).

$$S_e = \frac{1}{N}\sum_{k=1}^{N} e^2(k) \tag{15}$$

$$S_u = \frac{1}{N}\sum_{k=1}^{N} u^2(k) \tag{16}$$

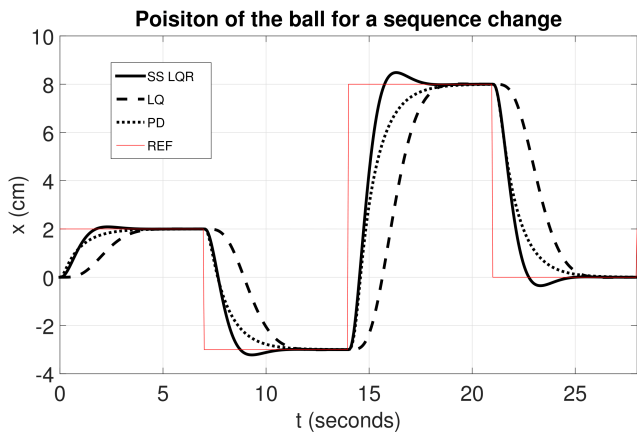

Fig. 9. Position of the ball - sequence change

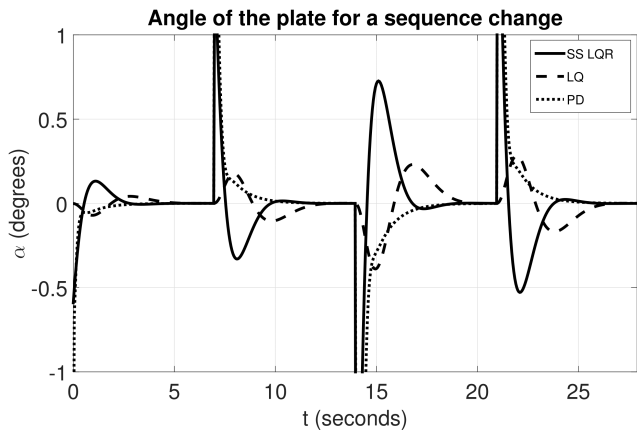

Fig. 10. Angle of the plate - sequence change

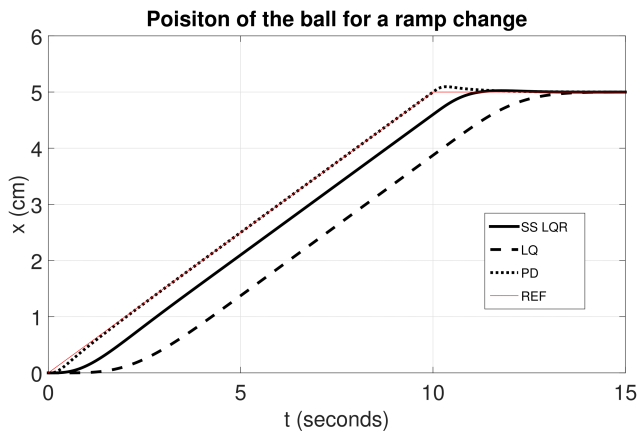

Fig. 11. Position of the ball - ramp change

Tab. 1. Quality control for the step change

	LQR	LQ	PD
S_e	0.0080	0.0229	0.0081
S_u	2.4203	0.2963	5.4281

Tab. 2. Quality control for the sequence change

	LQR	LQ	PD
S_e	0.0475	0.1348	0.0548
S_u	13.9880	1.4078	16.9862

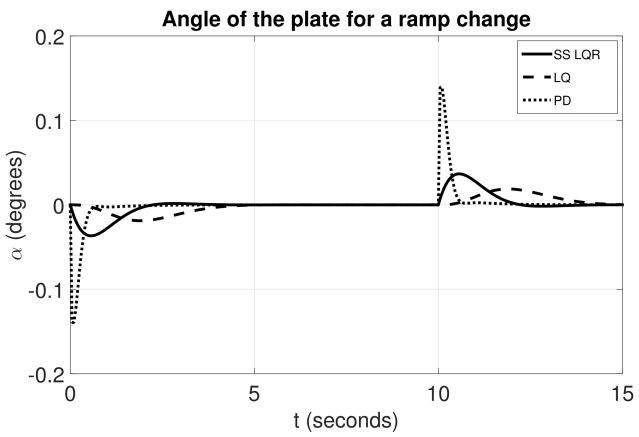

Fig. 12. Angle of the plate - ramp change

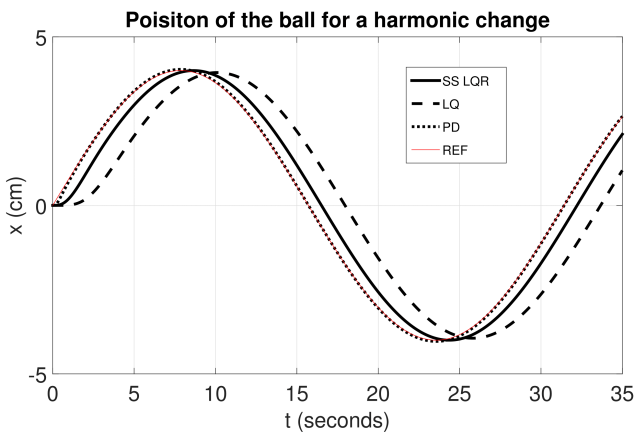

Fig. 13. Position of the ball - harmonic change

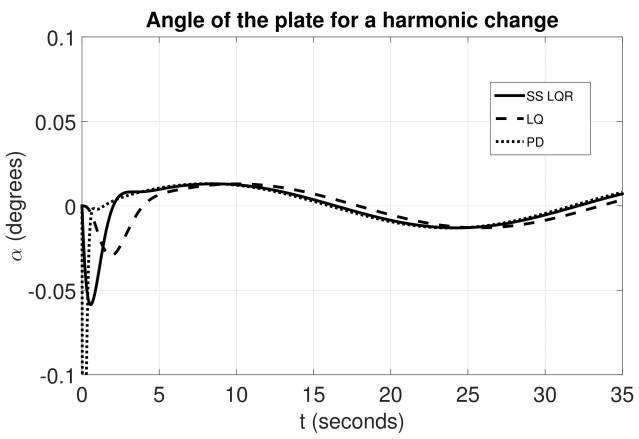

Fig. 14. Angle of the plate - harmonic change

Tab. 3. Quality control for the ramp change

	LQR	LQ	PD
S_e	0.0010	0.0076	~ 0
S_u	0.1292	0.1248	0.1637

Tab. 4. Quality control for the harmonic change

	LQR	LQ	PD
S_e	0.0022	0.0156	~ 0
S_u	0.0450	0.0490	0.0625

CONCLUSION

It is clearly visible from graphs that the PD controller shows a very good quality of control. However, from previous experiences with Ball & Plate system can be stated, that this type of controller would not work in the real system, because it has fast behavior and generates large changes in its output. Not to mention the noise from the environment and disturbances. It also nicely managed to follow a linear and sinusoidal trajectory. Two other controllers can be of course designed to be more competitive, but the goal was to show their true nature. For example, 2DoF LQ polynomial controller was quite slow and although it has similar settling time, it looks lazy. This is actually the point of using the 2DoF controller, which has a very subtle behavior well suited for the Ball & Plate model. State-space LQR controller could be also modified for better control, but its behavior is sufficient enough for this paper.

Tables clearly show that the LQ polynomial control excels at the quality of controller effort. This is caused by its 2DoF structure and desired, especially when the settling time is almost as small as for two other designed controllers. PD controller shows very good results for ramp and harmonic change because of their relatively slow rise time.

Results presented in this paper are helpful for future research on the topic and authors are keen to continue further. Next steps are to add state observer or Kalman filter to the state-space control, implement designed algorithms to the programming language of the robot and run tests in RobotStudio simulation environment. RobotStudio virtualizes the robot quite precisely, thus it is a good intermediate step in the implementation. The hardest part will be to implement state-space control in the programming language of the robot because it does not have appropriate tools and it is not optimized for this type of application. Using an external control unit is considered as the best option, which can simply send angles directly to the robot and do the "heavy" work.

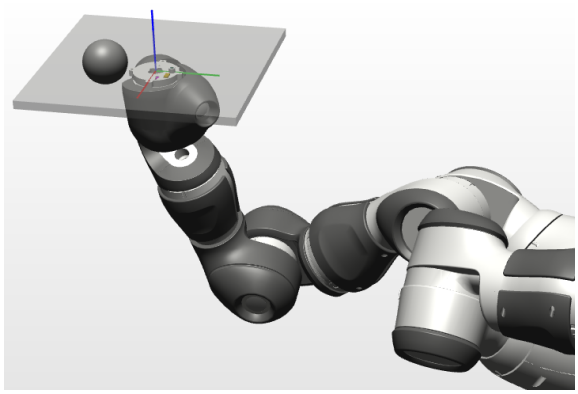

Fig. 15. Ball & Plate modeled for the YuMi in the RobotStudio

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