POSITIVITY AND STABILITY OF DESCRIPTOR CONTINUOUS-TIME LINEAR SYSTEMS WITH INTERVAL STATE MATRICES

Tadeusz Kaczorek
Białystok University of Technology
Faculty of Electrical Engineering
Wiejska 45D, 15-351 Białystok, Poland
e-mail: kaczorek@ee.pw.edu.pl

ABSTRACT

Necessary and sufficient conditions for the positivity and stability of descriptor continuous-time linear systems with interval state matrices are established. The convex linear combination of the Hurwitz polynomials of positive descriptor linear systems is analyzed. It is shown that the convex linear combination of the Hurwitz polynomials of positive linear systems is also the Hurwitz polynomial. The Kharitonov theorem is extended to the positive descriptor linear systems with interval state matrices.

KEY WORDS

interval, positive, descriptor, linear, continuous-time, system, stability, Kharitonov theorem.

INTRODUCTION

A dynamical system is called positive if its state variables take nonnegative values for all nonnegative inputs and nonnegative initial conditions. The positive linear systems have been investigated in (Berman and Plemmons 1994, Farina and Rinaldi 2000, Kaczorek 2002 and 2008) and positive nonlinear systems in (Kaczorek 2014, 2015a, 2015b, 2015c and 2016).
Examples of positive systems are industrial processes involving chemical reactors, heat exchangers and distillation columns, storage systems, compartmental systems, water and atmospheric pollution models. A variety of models having positive linear behavior can be found in engineering, management science, economics, social sciences, biology and medicine, etc.
Positive linear systems with different fractional orders have been addressed in (Busłowicz 2012, Kaczorek 2010 and 2011). Descriptor (singular) linear systems have been analyzed in (Kaczorek 1993, 1997, 2012, 2014, 2018a) and the stability of a class of nonlinear fractional-order systems in (Kaczorek 2015c and 2016, Xiang-Jun et al. 2008). Application of Drazin inverse to analysis of descriptor fractional discrete-time linear systems has been presented in (Kaczorek 2013) and stability of discrete-time switched systems with unstable subsystems in (Zhang et al. 2014a). The robust stabilization of discrete-time positive switched systems with uncertainties has been addressed in (Zhang et al. 2014b). Comparison of three method of analysis of the descriptor fractional systems has been presented in (Sajewski 2016a). Stability of linear fractional order systems with delays has been analyzed in (Busłowicz 2008) and simple conditions for practical stability of positive fractional systems have been proposed in [4]. The stability of interval positive continuous-time linear systems has been addressed in (Kaczorek 2018b).
In this paper the positivity and the asymptotic stability of descriptor continuous-time linear systems with interval state matrices will be investigated.
The paper is organized as follows. In section 2 some basic definitions and theorems concerning descriptor linear systems and elementary row and column operations are recalled. Necessary and sufficient conditions for the positivity of the descriptor linear systems are established in section 3. The convex linear combination of Hurwitz polynomials of positive linear systems and an extension of the Kharitonov theorem are given in section 4. The stability of positive descriptor linear systems with interval state matrices is addressed in section 5. Concluding remarks are given in section 6.
The following notations will be used: $\Re$ - the set of real numbers, $\Re^{n\times m}$ - the set of $n\times m$ real matrices, $\Re_+^{n\times m}$ - the set of $n\times m$ real matrices with nonnegative entries and $\Re_+^n=\Re_+^{n\times 1}$, M_n - the set of $n\times n$ Metzler matrices (real matrices with nonnegative off-diagonal entries), I_n - the $n\times n$ identity matrix.

PRELIMINARIES

Consider the autonomous descriptor continuous-time linear system
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$$E\dot{x}=Ax, \tag{1}$$

where $x=x(t)\in\Re^n$ is the state vector and $E,A\in\Re^{n\times n}$.
It is assumed that

$$\det[Es-A]\neq 0 \text{ for some } s\in\mathbf{C}, \tag{2}$$

where **C** is the field of complex numbers and the system (1) has unique solution for admissible initial conditions $x_0=x(0)\in\Re^n$.
It is well-known (Kaczorek 1993) that if (2) holds then there exists a pair of nonsingular matrices $P,Q\in\Re^{n\times n}$ such that

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$$P[Es-A]Q=\begin{bmatrix} I_{n_1}s-A_1 & 0 \\ 0 & Ns-I_{n_2} \end{bmatrix},$$
$$A_1\in\Re^{n_1\times n_1},\ N\in\Re^{n_2\times n_2}, \tag{3}$$

where $n_1=\deg\{\det[Es-A]\}$ and N is the nilpotent matrix, i.e. $N^{\mu}=0$, $N^{\mu-1}\neq 0$ (μ is the nilpotency index).

To simplify the considerations it is assumed that the matrix N has only one block. The nonsingular matrices P and Q can be found for example by the use of elementary row and column operations [20]:

1) Multiplication of any i-th row (column) by the number $c\neq 0$. This operation will be denoted by $L[i\times c]$ ($R[i\times c]$).
2) Addition to any i-th row (column) of the j-th row (column) multiplied by any number $c\neq 0$. This operation will be denoted by $L[i+j\times c]$ ($R[i+j\times c]$).
3) Interchange of any two rows (columns). This operation will be denoted by $L[i,j]$ ($R[i,j]$).

POSITIVITY OF DESCRIPTOR LINEAR SYSTEMS

Definition 1. The descriptor system (1) is called (internally) positive if $x(t)\in\Re_+^n$, $t\geq 0$ for all admissible nonnegative initial conditions $x(0)\in\Re_+^n$.

Definition 2. A real matrix $A=[a_{ij}]\in\Re^{n\times n}$ is called Metzler matrix if its off-diagonal entries are nonnegative, i.e. $a_{ij}\geq 0$ for $i\neq j$. The set of $n\times n$ Metzler matrices will be denoted by M_n.

Theorem 1. The descriptor system (1) is positive if and only if the matrix E has only linearly independent columns and the matrix $A_1\in M_{n_1}$.

Proof. Knowing $n_1=\deg\{\det[Es-A]\}$ and $\operatorname{rank}E$ we may find the nilpotency index $\mu=\operatorname{rank}E-n_1+1$ of the matrix N. Using column permutation of E we choose its n_1 linearly independent columns as its first columns. Next using elementary row operations we transform the matrix E to the form $\begin{bmatrix} I_{n_1} & 0 \\ 0 & N \end{bmatrix}$ and the matrix A to the form $\begin{bmatrix} A_1 & 0 \\ 0 & I_{n_2} \end{bmatrix}$.

From (3) it follows that the system (1) has been decomposed into two independent subsystems

$$\dot{x}_1=A_1x_1,\ x_1\in\Re^{n_1} \tag{4}$$

and

$$N\dot{x}_2=x_2,\ x_2\in\Re^{n_2}, \tag{5}$$

where

$$Q^{-1}x=\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{6}$$

and Q and Q^{-1} are permutation matrices.

It is well-known (Farina and Rinaldi 2000, Kaczorek 2002) that the solution $x_1=e^{A_1t}x_1(0)$ of (4) is not negative if and only if $A_1\in M_{n_1}$ and the solution x_2 of (5) is zero for $t>0$. □

Definition 3. (Farina and Rinaldi 2000, Kaczorek 2002) The positive system (4) is called asymptotically stable if

$$\lim_{t\to\infty} x_1(t)=0 \text{ for all admissible } x_1(0)\in\Re_+^{n_1}. \tag{7}$$

Theorem 2. (Farina and Rinaldi 2000, Kaczorek 2002 and 2018a) The positive system (4) is asymptotically stable if and only if one of the equivalent conditions is satisfied:

1) All coefficients of the polynomial

$$\det[I_{n_1}s-A_1]=s^{n_1}+a_{n_1-1}s^{n_1-1}+...+a_1s+a_0 \tag{8}$$

are positive, i.e. $a_k>0$ for $k=0,1,...,n_1-1$.

2) All principal minors $\overline{M}_i$, $i=1,...,n_1$ of the matrix $-A_1$ are positive, i.e.

$$\overline{M}_1=|-a_{11}|>0,\ \overline{M}_2=\begin{vmatrix} -a_{11} & -a_{12} \\ -a_{21} & -a_{22} \end{vmatrix}>0,\ ...,\ \overline{M}_{n_1}=\det[-A_1]>0 \tag{9}$$

3) There exists a strictly positive vector $\lambda=[\lambda_1 \ \cdots \ \lambda_{n_1}]^T$, $\lambda_k>0$, $k=1,...,n_1$ such that

$$A_1\lambda<0 \text{ or } A_1^T\lambda<0. \tag{10}$$

If $\det A\neq 0$ then we may choose $\lambda=-A_1^{-1}c$, where $c\in\Re^{n_1}$ is any strictly positive vector.

Example 1. Consider the descriptor system (1) with the matrices

$$E=\begin{bmatrix} 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & -2 \\ 1 & -2 & 0 & 0 \\ 0 & 0 & 0 & -2 \end{bmatrix},\ A=\begin{bmatrix} 0 & 1 & 0 & -4 \\ 1 & -4 & 0 & 4 \\ 0 & 6 & 1 & 0 \\ 1 & -1 & 0 & 4 \end{bmatrix}. \tag{11}$$

The condition (2) for (11) is satisfied since

$$\det[Es-A]=\begin{vmatrix} 0 & -1 & 0 & 2s+4 \\ -1 & s+4 & 0 & -2s-4 \\ s & -2s-6 & -1 & 0 \\ -1 & 1 & 0 & -2s-4 \end{vmatrix} =-2s^2-10s-12 \tag{12}$$

and $n_1=2$. In this case $\operatorname{rank}E=3$ and $\mu=\operatorname{rank}E-n_1+1=2$.

To transform the matrix $Es-A$ with (11) to the desired form

$$\begin{bmatrix} I_2 s - A_1 & 0 \\ 0 & Ns - I_2 \end{bmatrix}$$

$$\text{with } A_1 = \begin{bmatrix} -2 & 1 \\ 0 & -3 \end{bmatrix},\ N = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}. \tag{13}$$

The following elementary column operations $R[4\times\frac{1}{2}]$, $R[4,1]$ and elementary row operations $L[2+4\times(-1)]$, $L[4+1\times1]$, $L[3+2\times2]$ have been performed.

In this case the matrices Q and P have the form

$$Q = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \frac{1}{2} & 0 & 0 & 0 \end{bmatrix},\ P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 2 & 1 & -2 \\ 1 & 0 & 0 & 1 \end{bmatrix}. \tag{14}$$

Note that the matrix A_1 defined by (13) is the stable Metzler matrix and the descriptor system with (11) is positive and asymptotically stable.

CONVEX LINEAR COMBINATION OF HURWITZ POLYNOMIALS AND EXTENSION OF KHARITONOV THEOREM

Consider the set (family) of the n-degree polynomials

$$p_n(s) := a_n s^n + a_{n-1} s^{n-1} + \ldots + a_1 s + a_0 \tag{15a}$$

with the interval coefficients

$$\underline{a_i} \le a_i \le \overline{a_i},\ i = 0,1,\ldots,n. \tag{15b}$$

Using (15a) we define the following four polynomials

$$\begin{aligned} p_{1n}(s) &:= \underline{a_0} + \underline{a_1} s + \overline{a_2} s^2 + \overline{a_3} s^3 + \underline{a_4} s^4 + \underline{a_5} s^5 + \ldots, \\ p_{2n}(s) &:= \underline{a_0} + \overline{a_1} s + \overline{a_2} s^2 + \underline{a_3} s^3 + \underline{a_4} s^4 + \overline{a_5} s^5 + \ldots, \\ p_{3n}(s) &:= \overline{a_0} + \underline{a_1} s + \underline{a_2} s^2 + \overline{a_3} s^3 + \overline{a_4} s^4 + \underline{a_5} s^5 + \ldots, \\ p_{4n}(s) &:= \overline{a_0} + \overline{a_1} s + \underline{a_2} s^2 + \underline{a_3} s^3 + \overline{a_4} s^4 + \overline{a_5} s^5 + \ldots. \end{aligned} \tag{16}$$

Theorem 3. (Kharitonov Theorem) The set of polynomials (15) is asymptotically stable if and only if the four polynomials (16) are asymptotically stable.

Proof. The proof is given in (Kharitonov 1987, Kaczorek 1993).

The polynomial

$$p(s) := s^n + \overline{a_{n-1}} s^{n-1} + \ldots + \overline{a_1} s + \overline{a_0} \tag{17}$$

is called Hurwitz if its roots s_i, $i = 1,\ldots,n$ satisfy the condition $\operatorname{Re} s < 0$ for $i = 1,\ldots,n$.

Definition 4. The polynomial

$$p(s) := (1-k)p_1(s) + kp_2(s) \text{ for } k \in [0,1] \tag{18}$$

is called convex linear combination of the polynomials

$$\begin{aligned} p_1(s) &= s^n + a_{n-1}s^{n-1} + \ldots + a_1 s + a_0, \\ p_2(s) &= s^n + b_{n-1}s^{n-1} + \ldots + b_1 s + b_0. \end{aligned} \tag{19}$$

Theorem 4. The convex linear combination (18) of the Hurwitz polynomials (19) of the positive linear system is also a Hurwitz polynomial.

Proof. By Theorem 2 the polynomials (19) are Hurwitz if and only if

$$a_i > 0 \text{ and } b_i > 0 \text{ for } i = 0,1,\ldots,n-1. \tag{20}$$

The convex linear combination (18) of the Hurwitz polynomials (19) is a Hurwitz polynomial if and only if

$$(1-k)a_i + kb_i > 0 \text{ for } k \in [0,1] \text{ and } i = 0,1,\ldots,n-1. \tag{21}$$

Note that the conditions (20) are always satisfied if (21) holds.

Therefore, the convex linear combination (18) of the Hurwitz polynomials (19) of the positive linear system is always the Hurwitz polynomial. □

Example 1. Consider the convex linear combination (18) of the Hurwitz polynomials

$$\begin{aligned} p_1(s) &= s^2 + 5s + 2, \\ p_2(s) &= s^2 + 3s + 4. \end{aligned} \tag{22}$$

The convex linear combination (18) of the polynomials (22) is a Hurwitz polynomial since

$$\begin{aligned} (1-k)5 + 3k = 5 - 2k > 0 \text{ and} \\ (1-k)2 + k4 = 2 + 2k > 0 \text{ for } k \in [0,1]. \end{aligned} \tag{22}$$

The above considerations for two polynomials (19) of the same order n can be extended to two polynomials of different orders (Kaczorek 2018b).

Consider the set of positive interval linear continuous-time systems with the characteristic polynomials

$$p(s) = p_n s^n + p_{n-1}s^{n-1} + \ldots + p_1 s + p_0, \tag{23a}$$

where

$$0 < \underline{p_i} \le p_i \le \overline{p_i},\ i = 0,1,\ldots,n. \tag{23b}$$

Theorem 5. The positive interval linear system with the characteristic polynomial (23a) is asymptotically stable if and only if $\underline{p_i} > 0$ for $i = 0,1,\ldots,n.$.

Proof. By Kharitonov Theorem the set of polynomials (23) is asymptotically stable if and only if the polynomials (16) are asymptotically stable. Note that the coefficients of polynomials (16) are positive if $\underline{p_i} > 0$ for $i = 0,1,\ldots,n$. Therefore, by Theorem 2 the positive interval linear system with the characteristic polynomials (23a) is asymptotically stable if and only if $\underline{p_i} > 0$ for $i = 0,1,\ldots,n.$ □

Example 2. Consider the positive linear system with the characteristic polynomial

$$p(s) = a_3 s^3 + a_2 s^2 a_1 s + a_0 \tag{24a}$$

with the interval coefficients

$$\begin{aligned} &0.5 \le a_3 \le 2,\ 1 \le a_2 \le 3, \\ &0.4 \le a_1 \le 1.5,\ 0.3 \le a_0 \le 4. \end{aligned} \tag{24b}$$

By Theorem 5 the interval positive linear system with (24) is asymptotically stable since the coefficients a_k, $k = 0,1,2,3$ of the polynomial (24a) are positive, i.e. the lower and upper bounds are positive.

STABILITY OF DESCRIPTOR POSITIVE LINEAR SYSTEMS WITH INTERVAL STATE MATRICES

Consider the autonomous descriptor positive linear system

$$E\dot{x} = Ax, \tag{25}$$

where $x = x(t) \in \Re^n$ is the state vector, $E \in \Re^{n\times n}$ is constant (exactly known) and $A \in \Re^{n\times n}$ is an interval matrix defined by

$$\underline{A} \le A \le \overline{A} \text{ or equivalently } A \in [\underline{A}, \overline{A}]. \tag{26}$$

It is assumed that

$$\det[Es - \underline{A}] \ne 0 \text{ and } \det[Es - \overline{A}] \ne 0 \tag{27}$$

and the matrix E has only linearly independent columns. If these assumptions are satisfied then there exist two pairs of nonsingular matrices (P_1, Q_1), (P_2, Q_2) such that

$$P_1[Es - \underline{A}]Q_1 = \begin{bmatrix} I_{\underline{n}_1} s - \underline{A}_1 & 0 \\ 0 & \underline{N}s - I_{\underline{n}_2} \end{bmatrix},$$

$$\underline{A}_1 \in \Re^{\underline{n}_1 \times \underline{n}_1},\ \underline{N} \in \Re^{\underline{n}_2 \times \underline{n}_2},\ \underline{n}_1 + \underline{n}_2 = n, \tag{28a}$$

and

$$P_2[Es - \overline{A}]Q_2 = \begin{bmatrix} I_{\overline{n}_1} s - \overline{A}_1 & 0 \\ 0 & \overline{N}s - I_{\overline{n}_2} \end{bmatrix},$$

$$\overline{A}_1 \in \Re^{\overline{n}_1 \times \overline{n}_1},\ \overline{N} \in \Re^{\overline{n}_2 \times \overline{n}_2},\ \overline{n}_1 + \overline{n}_2 = n, \tag{28b}$$

where $\underline{n}_1 = \deg\{\det[Es - \underline{A}]\}$ and $\overline{n}_1 = \deg\{\det[Es - \overline{A}]\}$.

Theorem 6. If the assumptions are satisfied then the interval descriptor system (25) is positive if and only if

$$\underline{A}_1 \in M_{\underline{n}_1} \text{ and } \overline{A}_1 \in M_{\overline{n}_1}. \tag{29}$$

Proof. The proof is similar to the proof of Theorem 1.

Definition 5. The descriptor interval positive system (25) is called asymptotically stable (Hurwitz) if the system is asymptotically stable for all matrices E, A, $A \in [\underline{A}, \overline{A}]$.

Theorem 7. If the matrices $\underline{A}$ and $\overline{A}$ of the positive system (25) are asymptotically stable then their convex linear combination

$$A = (1-k)\underline{A} + k\overline{A} \text{ for } 0 \le k \le 1 \tag{30}$$

is also asymptotically stable.

Proof. By condition (10) of Theorem 2 if the positive systems are asymptotically stable then there exists strictly positive vector $\lambda \in \Re_+^n$ such that

$$\underline{A}\lambda < 0 \text{ and } \overline{A}\lambda < 0. \tag{31}$$

Using (30) and (31) we obtain

$$A\lambda = [(1-k)\underline{A} + k\overline{A}]\lambda = (1-k)\underline{A}\lambda + k\overline{A}\lambda < 0 \quad \text{for } 0 \le k \le 1. \tag{32}$$

Therefore, if the matrices $\underline{A}$ and $\overline{A}$ are asymptotically stable and (31) hold then the convex linear combination is also asymptotically stable. □

Theorem 7. The interval descriptor positive system (25) with (26) and matrix E with only linearly independent columns is asymptotically stable if and only if there exists a strictly positive vector $\lambda \in \Re_+^n$ such that

$$P_n \underline{A}\lambda < 0 \text{ and } P_n \overline{A}\lambda < 0, \tag{33}$$

where P_n is the submatrix of P consisting of its first n rows.

Proof. If by assumption the matrix E has only linearly independent columns then $\lambda = Q\lambda_q \in \Re_+^n$ with all positive components for any $\lambda_q \in \Re_+^n$ with all positive components. By condition (10) of Theorem 2 and Theorem 6 the interval descriptor positive system (25) with (26) is asymptotically stable if and only if the conditions (32) are satisfied. □

Example 3. (Continuation of Example 1) Consider the descriptor positive system (25) with the matrix E of the form (11) and the interval matrix A with

$$\underline{A} = \begin{bmatrix} 0 & -1 & 0 & -2 \\ 1 & -3 & 0 & 2 \\ 0 & 4 & 1 & 0 \\ 1 & -1 & 0 & 2 \end{bmatrix},\ \overline{A} = \begin{bmatrix} 0 & -1 & 0 & -6 \\ 1 & -5 & 0 & 6 \\ 0 & 8 & 1 & 0 \\ 1 & -1 & 0 & 6 \end{bmatrix}. \tag{34}$$

The matrices $(E, \underline{A})$ and $(E, \overline{A})$ satisfy the assumptions (27) and the matrix E given by (11) has only linearly independent columns.

In this case

$$n = \deg\{\det[Es - \underline{A}]\} = \begin{vmatrix} 0 & -1 & 0 & 2s+2 \\ -1 & s+3 & 0 & -2s-2 \\ s & -2s-4 & -1 & 0 \\ -1 & 1 & 0 & -2s-2 \end{vmatrix} \quad (35a)$$

$$= \deg(-2s^2 - 6s - 4) = 2,$$

$$n = \deg\{\det[Es - \overline{A}]\} = \begin{vmatrix} 0 & -1 & 0 & 2s+6 \\ -1 & s+5 & 0 & -2s-6 \\ s & -2s-8 & -1 & 0 \\ -1 & 1 & 0 & -2s-6 \end{vmatrix} \quad (35b)$$

$$= \deg(-2s^2 - 14s - 24) = 2$$

and from (14) we have

$$P_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix}. \quad (35)$$

Using (29) and (35) for $\lambda = [1 \quad 1 \quad 1 \quad 1]^T$ we obtain

$$P_2 \underline{A}\lambda = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 & -2 \\ 1 & -3 & 0 & 2 \\ 0 & 4 & 1 & 0 \\ 1 & -1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \quad (36a)$$

$$= \begin{bmatrix} -3 \\ -2 \end{bmatrix} < \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

and

$$P_2 \overline{A}\lambda = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 & -6 \\ 1 & -5 & 0 & 6 \\ 0 & 8 & 1 & 0 \\ 1 & -1 & 0 & 6 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \quad (36b)$$

$$= \begin{bmatrix} -7 \\ -4 \end{bmatrix} < \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

Therefore, by Theorem 7 the interval positive descriptor system is asymptotically stable.

CONCLUDING REMARKS

The positivity and asymptotic stability of descriptor linear continuous-time systems with interval state matrices have been investigated. It has been shown that the descriptor system is positive if and only if the matrix E has only linearly independent columns and the matrix A_1 is a Metzler matrix (Theorem 1) and the convex linear combination of the Hurwitz polynomials of positive linear systems is also the Hurwitz polynomial (Theorem 3). The Kharitonov theorem has been extended to positive descriptor linear systems with interval state matrices (Theorem 5). Necessary and sufficient conditions for the asymptotic stability of descriptor positive linear systems has been also established (Theorem 7). The considerations have been illustrated by numerical examples.

The above considerations can be extended to positive linear discrete-time systems and to fractional linear systems. An open problem is an extension of these considerations to standard (non-positive) descriptor linear systems.

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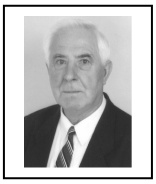

TADEUSZ KACZOREK, born 27.04.1932 in Elzbiecin (Poland), received the MSc., PhD and DSc degrees from Electrical Engineering of Warsaw University of Technology in 1956, 1962 and 1964, respectively. In the period 1968 - 69 he was the dean of Electrical Engineering Faculty and in the period 1970 - 73 he was the prorector of Warsaw University of Technology. Since 1971 he has been professor and since 1974 full professor at Warsaw University of Technology. In 1986 he was elected a corresp. member and in 1996 full member of Polish Academy of Sciences. In the period 1988 - 1991 he was the director of the Research Centre of Polish Academy of Sciences in Rome. In June 1999 he was elected the full member of the Academy of Engineering in Poland. In May 2004 he was elected the honorary member of the Hungarian Academy of Sciences. He was awarded by the title doctor honoris causa by 13 Universities.

His research interests cover the theory of systems and the automatic control systems theory, specially, singular multidimensional systems, positive multidimensional systems and singular positive and positive 1D and 2D systems. He has initiated the research in the field of singular 2D and positive 2D linear systems and in the fractional positive linear and nonlinear systems. He has published 28 books (7 in English) and over 1100 scientific. He has given invited lectures in more than 50 universities in USA, Canada, UK, Germany, Italy, France, Japan, Greece etc. He has been a member of many international committees and programme committees.

He supervised 69 Ph.D. theses. More than 20 of this PhD students became professors in USA, UK and Japan. He is editor-in-chief of Bulletin of the Polish Academy of Sciences, Techn. Sciences and editorial member of about ten international journals.