IMPROVED TPWL BASED NONLINEAR MOR FOR FAST SIMULATION OF LARGE CIRCUITS

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ABSTRACT

Trajectory Piecewise Linearisation(TPWL) is one of the nonlinear Model Order Reduction(MOR) techniques that involves multiple linearisations around suitably selected linearisation points(LPs). Selection of linearisation points play a major role in the accuracy of the reduced order model in TPWL technique. Standard linearisation point selection methods based on uniform distance criteria are found to be less efficient. This paper proposes a new adaptive scheme for the selection of LPs by taking into account of the 'sensitivity' of the nonlinear system towards the state. The proposed method has been implemented on two nonlinear benchmark problems, nonlinear transmission line circuit and nonlinear chain of inverter circuit and the simulation results are compared with the conventional schemes.

INTRODUCTION

Mathematical modelling of physical systems results in systems of very large order. The storage and simulation of these higher order systems is a challenge due to the huge requirement of memory and time[1]. This points towards the necessity of model order reduction(MOR) where the reduced order models with similiar input-output mappings are generated.

Common MOR methods for nonlinear systems are Proper Orthogonal Decomposition(POD)[2], DEIM(Dis crete Empirical Interpolation)[3], Linear or Polynomial expansion of the system nonlinearity[4], bi-linearisation [5], or Volterra series expansion [6] and Trajectory Piecewise Linearisation(TPWL).

The idea of TPWL was first introduced by Michal Rewienski and Jacob White[7] in which it was implemented on a nonlinear transmission line circuit model and micromachined switch[8][9]. Later this method was applied to diverse areas including fluid dynamics[15],nonlinear electronic circuit simulation[17][13][14],electromagnetics[16], etc.
Since this method involves considering multiple linearisations about suitably selected states of the system, judicious location of LPs play an important role in the accuracy of the generated reduced models. The LP selection methods proposed by Rewienski[7] takes distances from the previously selected LPs as the crieteria for location of LPs on either the exact or the approximate trajectory. This method was improved by S.A. Nahvi[18] giving the idea of FAS which includes a 'coarse run' over the whole trajectory, followed by selective refinement in areas where more LPs are required.

A new method has been proposed in this paper that adaptively decide the placement of the LPs along the trajectory. This is inspired by[11], where the authors have used adaptive sampling technique in parametric linear MOR with the example of a linear beam. In this paper, this adaptive sampling idea is incoorporated into TPWL method to locate the LPs. The regions where the nonlinear system is more sensitive to the state are identified based on the concept of distance between the subspaces or principal angles.The proposed method has been implemented on two nonlinear benchmark problems,nonlinear transmission line circuit and nonlinear chain of inverter circuit and the simulation results are compared with the conventional schemes.

The next two sections describe the basic TPWL process and the conventional methods for LP selection and their drawbacks respectively. The last section introduces the proposed adaptive scheme followed by its implementation on benchmark problems and error comparison.

TRAJECTORY PIECEWISE LINEARISATION

The TPWL process includes the following steps:
- The higher order system is simulated for a training input to obtain the nonlinear system trajectory.
- Location of suitable Linearisation Points along the exact or approximate trajectory[7] followed by the sub-

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sequent linearisations around these LPs.
• Finding out the dominant subspace and projecting the local-submodels into this dominant subspace.
• Combine the local reduced models using proper weight assignment stategies to form the final reduced model.

Mathematical Formulation

For the nonlinear dynamical system in the following state space form:

$$\frac{dx}{dt} = f(x) + Bu \quad (1)$$
$$y = Cx \quad (2)$$

Where $x \in R^n$ is a vector of system states, $f : R^n \rightarrow R^n$ is the nonlinear vector field, $B \in R^{n\times p}$ is the input matrix, $C \in R^{q\times n}$ is the output matrix, $u \in R^k$ is the input and $y \in R^q$ is the output. The Taylor series first-order approximation of $f(x)$ about an initial state x_0 is given by

$$\overline{f}(x) = f(x_0) + A_0(x - x_0) \quad (3)$$

Where A_0 is the Jacobian of $f(x)$ evaluated at x_0. Clearly, the error in this approximation is:

$$e(x) = f(x) - \overline{f}(x) \quad (4)$$

The linearised system at x_0 is hence

$$\frac{dx}{dt} = f(x_0) + A_0(x - x_0) + Bu \quad (5)$$
$$y = Cx \quad (6)$$

The dynamics are restricted to the dominant subspace by the state transformation $x = Vz$. $V \in R^{n\times r}$ is the projection matrix spanning the dominant subspace, $z \in R^r$, $V^TV = I$ and $r \ll n$. The reduced model at x_0 is:

$$\frac{dz}{dt} = A_{0r}z + V^T(f(x_0) - A_0x_0)) + B_ru \quad (7)$$
$$y = C_rz \quad (8)$$

where $A_{0r} = V^TA_0V, B_r = V^TB$, and $C_r = CV$. Assuming m similar linearised models generated along the training trajectory about the states $x_0, ..x_i, ..x_{m-1}$, the final TPWL model is expressed as the weighted sum:

$$\frac{dz}{dt} = \sum_{i=0}^{m-1} w_i(z)(A_{ir}z + V^T(f(x_i) - A_ix_i)) + B_ru \quad (9)$$
$$y = C_rz \quad (10)$$

where A_i is the Jacobian of $f(x)$ evaluated at x_i and $A_{ir} = V^TA_iV$.

CONVENTIONAL METHODS FOR LP SELECTION

Uniform Division of Trajectory

[10]The first proposed and most widely used methods for LP selection in TPWL based MOR are the ones which divide the system trajectory uniformly to create linearised systems. Two such methods, very similar to each other, but differing in whether the nonlinear system trajectory is exact or approximate, are given in [7]. The first method can be found in [7, p.45]. In it, given a training input $u(t)$ and an initial state x_0, the nonlinear system is simulated and a finite number of linear systems are generated on its trajectory. The linear systems are created by a uniform division of the trajectory at a constant, pre-selected euclidean distance δ. The second method given in [7, p.53], is similar to first, but it circumvents simulating the full-order nonlinear system. It is called Fast Approximate Simulation (FAS), and in it successively selected sub-models of the nonlinear system are simulated to get an approximate trajectory.

The uniform division of the trajectory suffers from two major drawbacks as pointed out in[12]. The first one is the heuristic nature of the δ being chosen. Selection of δ is mainly a game of 'hit and trial' and is unrelated with the nonlinearity involved. Moreover, these algorithms rely on the fact that the linearisation of a nonlinear function f at the state x_i is an accurate enough approximation at some other state x, provided that x is close enough to x_i, i.e., $||x - x_i|| < \delta$, or x lies within a ball of radius δ centered at x_i. The procedure contains an implicit assumption that dividing the trajectory uniformly would lead to acceptable sampling. In other words, it is assumed that δ is a constant throughout the trajectory, which is not necessarily true. There can be situations in which the trust regions don't overlap resulting in inadequate sampling and the trust region around one point covering the other trust region resulting in over-sampling as shown in Fig[1],[2].

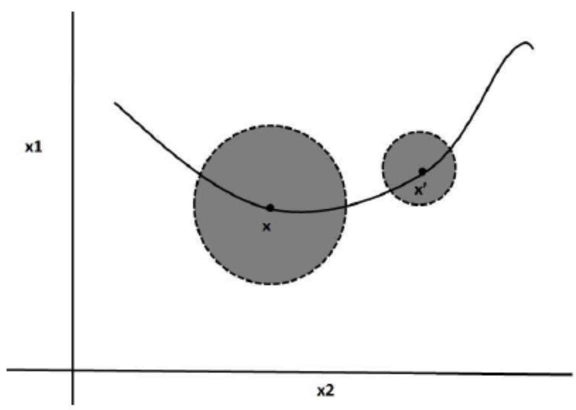

Fig. 1. The trajectory is inadequately sampled between the two trust regions

PROPOSED METHOD FOR ADAPTIVE SELECTION OF LPS

Uniform Division of the trajectory has proven to be a less efficient method for LP selection because of the above mentioned reasons. This section presents an algorithm to adaptively decide the number and placement of the linearisation points along the trajectory by

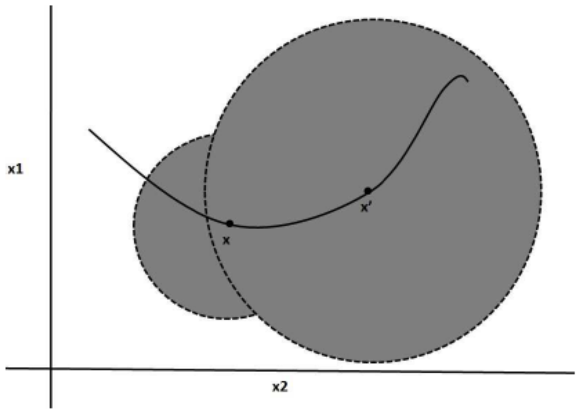

Fig. 2. The trust region around x covers x^1

placing more LPs in highly sensitive zones and lightly gridding the less sensitive zones. The high and low 'sensitive' regions are identified based on the concept of distance between the subspaces or principal angles.

Principal Angle Concept

Let V_1 and V_2 be the orthonormal projection matrices at the linearization points x_1 and x_2. The largest subspace angle across these two points is computed as

$$\theta_{12} = arcsin(\sqrt{(1-\sigma_r^2)}) = arccos(\sigma_r)$$

σ_r is the smallest singular value of $V_1^T V_2$.[11]

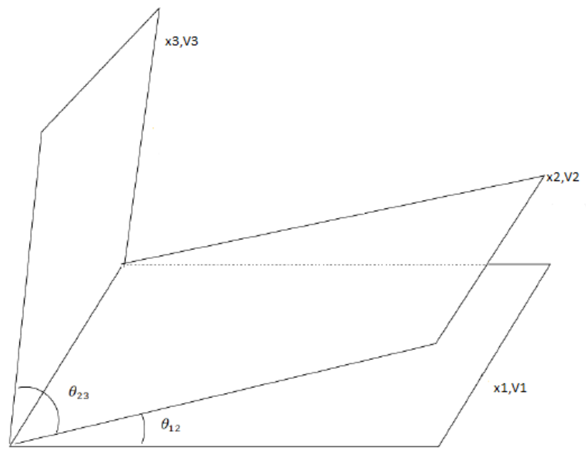

Fig. 3. Principal angles between V_1, V_2, V_3

Higher the angle between the subspaces, higher the sensitivity of the ROM and hence more LPs should be considered. Hence we have exploited the concept of subspace angles as a crieteria for deciding the selection the linearization points.This method hence assures more judicious placement of the LPs when compared with the uniform trajectory division.The new scheme for adaptive selection of LPs first perform a rough uniform sampling by selectiong a very high value of δ followed by the refinement in those regions where the system is more sensitive to state. In other words, the method adds more LPs to those regions where the subspace angle is more than θ_{max}, the maximum tolerable subspace angles. The proposed adaptive sampling algorithm is given in the following section.

Automatic Adaptive Sampling Algorithm

- Input θ_{max}, maximum error between the subspace angles that can be tolerated.
- Select linearization points $x_1, x_2, ...x_k$ by initial rough sampling by selecting a very high value for δ.
- While all $l_{i,i+1} > 1$

a) Calculate the projection matrices $V_1, V_2...V_k$ corresponding to each of these values $x_1, x_2, ...x_k$.
b) Compute subspace angles $V_{12}, V_{23}, ...V_{k-1,k}$ between these V_is, each taken pairwise.
c) Calculate $l_{12} = \frac{\theta_{12}}{\theta_{max}}, l_{23} = \frac{\theta_{23}}{\theta_{max}},.......$
d) Divide the interval between x_1 and x_2 into l_{12} further intervals. Likewise do the same for all the other intervals.

IMPLEMENTATION ON NONLINEAR BENCHMARK PROBLEMS

The adaptive method for LP selection is implemented on the two nonlinear bench mark problems ie, Nonlinear Transmission Line and Inverter Chain Circuit. The results are compared with the LP selection method using uniform division of trajectory.

Example-1 : Nonlinear Transmission Line

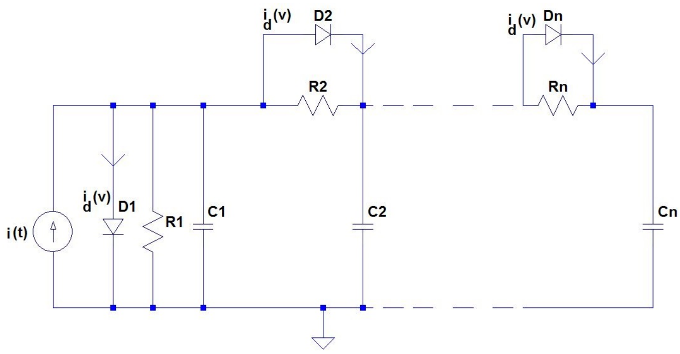

Fig. 4. Nonlinear transmission line circuit

The first example considered is a nonlinear transmission line circuit model shown in Figure 4. The circuit consists of resistors, capacitors, and diodes with a constitutive equation $i_d(v) = exp(40v) - 1$, where v is the voltage between diodes terminals. For simplicity we assume that all the resistors and capacitors have unit resistance and capacitance, respectively ($R_1 = R_2 = ... = R_n = 1, C_1 = C_2 = ... = C_n = 1$). The input is the current source entering at node 1: $u(t) = i(t)$, and the output is chosen to be the voltage at node 1: $y(t) = v_1(t)$. Using constitutive relations for capacitors, resistors, and diodes, as well as kirchhoffs current law, we obtain an input affine nonlinear dynamical system given by equations 1 and 2. This higher order nonlinear circuit is reduced using TPWL method with adaptive selection of LPs and the results are compared with the uniform trajectory division case. In both the cases the reduced order and the number of LPs are kept same. In this case, the value of δ is 1 for uniform trajectory division and while using adaptive case the trajectory is first roughly sampled with δ=3 and then used the adaptive algorithm keeping maximum error tolerance angle, θ_{max} as 5°. As a result, the number of LPs came out to be 5 in both the cases. The results show that the proposed method gives only 4.2% error at the output voltage compared to 6.74% error in the conventional case.

Error Analysis

The error in the output voltage between Full Order Model(FOM) and Reduced Order Model(ROM)

$$e(t) = (y_r(t) - y(t)) \tag{11}$$

where y_r and y are the output voltages of ROM and FOM respectively. The absolute error for the entire time span can be calculated as

$$E = \int_0^T |e(t)|dt \tag{12}$$

Using the above equations, the absolute error for uniform trajectory division, $E_u = 0.6233$ V while in the adaptive case, the error is $E_a = 0.3129$ V, clearly showing $E_a < E_u$.
The output voltages for both the cases are plotted in Fig 5 and 6 along with the FOM response. It is clearly visible from the graphs that the error peak during time 3.5 sec to 4.5 sec is reduced drastically in adaptive sampling case.

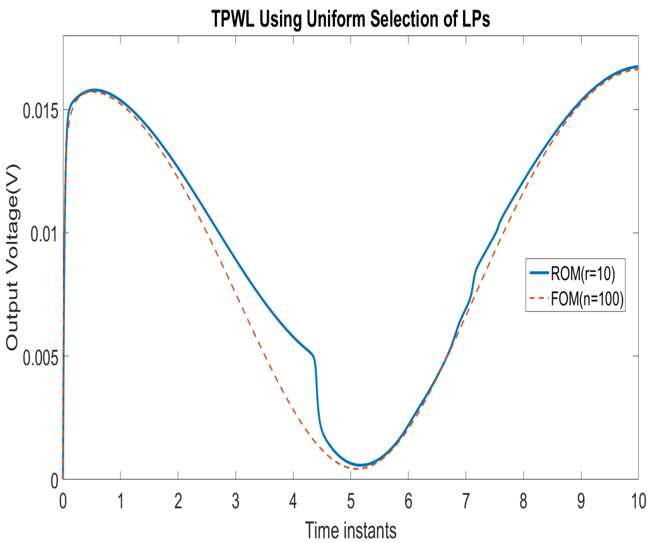

Fig. 5. TPWL Using Uniform Division of Trajectory for Nonlinear Transmission Line Model

Example-2 : Inverter Chain Circuit

The inverter circuit shown in Fig.7 is a nonlinear circuit consisting of an input voltage source, capacitors, resistors, and inverters whose input-output relation is

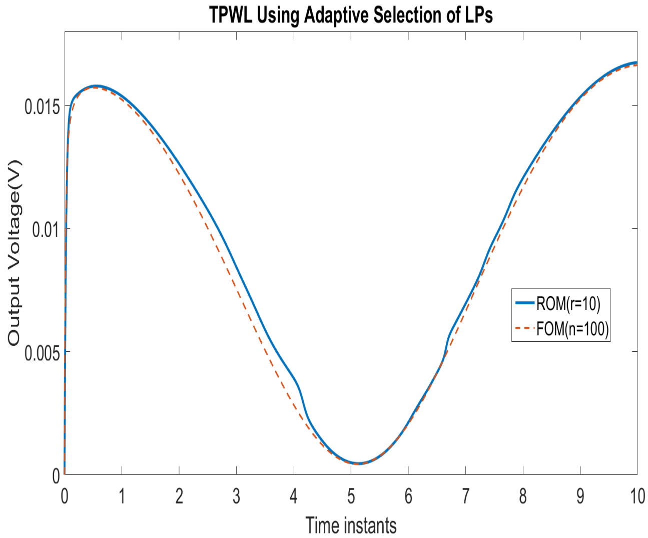

Fig. 6. TPWL Using Adaptive Selection of LP for Nonlinear Transmission Line Model

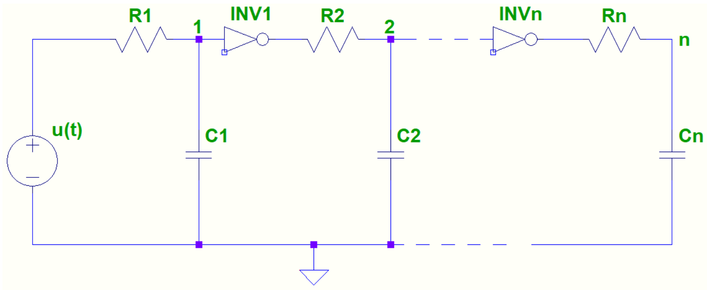

Fig. 7. Inverter Chain Circuit

$$V_{out} = f(V_{in}) = V_{dd}tanh(AV_{in}) \tag{13}$$

where V_{dd} is the supply voltage and A is a parameter. We choose $V_{dd} = 1$ and $A = 5$. Input is the voltage source $u(t)$ and output is the voltage at node 1, $v_1(t)$. The states are the corresponding voltage at each node. The parameter values are selected as $R_1 = R_2 = ... = R_n = 1\,\Omega, C_1 = C_2 = ... = C_n = 1\,$F. With the above given values the circuit has input affine form as given by equations 1 and 2. This higher order nonlinear circuit is reduced using TPWL method with adaptive selection of LPs and the results are compared with the uniform trajectory division case. In both the cases the reduced order and the number of LPs are kept same. In this case, the value of δ is 6 for uniform trajectory division and while using adaptive case the trajectory is first roughly sampled with δ=9 and then used the adaptive algorithm keeping maximum error tolerance angle, θ_{max} as 10°. As a result, the number of LPs came out to be 4 in both the cases. By using the proposed method we are able to reduce the percentage error at the output from 11.5% to 9% without increasing the number of LPs. Hence we can conclude that, in this case also the adaptive sampling with $E_a = 36.2522$ V gives a less erroneous ROM as compared to uniform division with $E_u = 54.8893$ V.

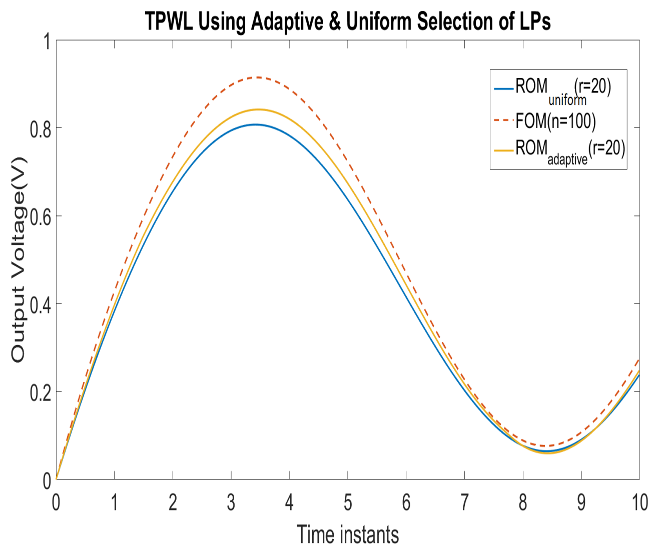

Fig. 8. TPWL using Adaptive Selection of LPs

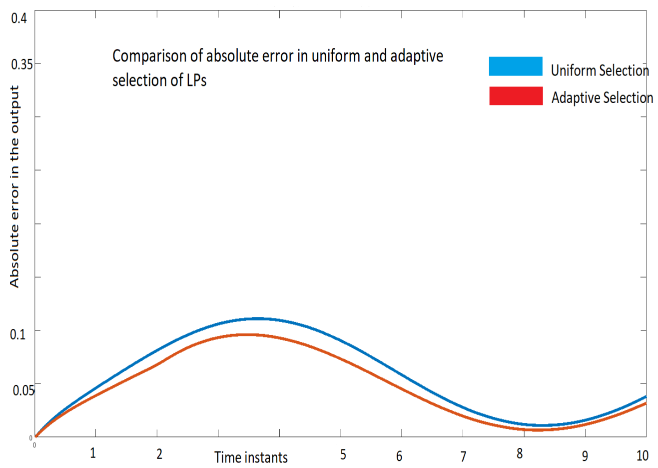

Fig. 9. Absolute error in the output voltage between the FOM and ROM for adaptive LP selection method and uniform trajectory division method

	NL transmission line		Chain of inverters	
	Adaptive	Uniform	Adaptive	Uniform
Full Order	100	100	100	100
Reduced Order	10	10	20	20
% Error in Output	4.2	6.74	9	11.5
No of LPs	5	5	4	4
Online Simulation Time	0.1s	0.1s	0.1s	0.1s

Table 1: Comparison of proposed method with uniform selection of LPs for nonlinear transmission line and chain of inverter circuit.

CONCLUSION

This paper presents an improved TPWL method for nonlinear MOR in which the LPs are placed more accurately. The adaptive LP selection algorithm takes into account of the 'sensitivity' of the nonlinear system towards state removing the uncertainities associated with uniform division methods. Hence it is able to reduce the percentage error in the output between FOM and ROM compared to unform trajectory division by judiciously locating the LPs on the trajectory.The new method is thus able to develop more efficient reduced order models without increasing the number of linearisation points as seen from the simulation results of the two nonlinear benchmark problems. The proposed method can be applied to more nonlinear higher order systems and hence fast simulation of such systems can be achieved more efficiently.

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