QUALITY EVALUATION OF MODELS AND POLYMODEL COMPLEXES:
SUBJECT-OBJECT APPROACH

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progress, qualimetry, complex modeling and proactive
control.

ABSTRACT
New approach of models and multiple-models quality
evaluation is proposed. This approach is based on
twofold ideas. First, when selecting an object for
modeling it is reasonable to select not a really existing
(designed or abstract) object but a situation in progress,
that includes the objects and the subjects of the
modeling (people responsible for making decisions
(DM), people responsible for the substantiation of a
decision (solution), experts, and people responsible for
the implementation of solutions). Second, the process
of modeling is here assumed as a control process of
developing situation under uncertain conditions, caused
by absence of information needed for forming the
substantiated decisions. The descriptive and formal
statement of quality control of models and multiple-
model complexes are interpreted as problems of
structural-functional synthesis of a model (a multiple-
model complex) and also, selection of optimal programs
of control and regulation of structural dynamics of a
situation in progress (quality control of models and
multiple-model complexes). The example of solving the
task of poly-model finding distances from a vertex to all
other vertices of a graph is proposed.

INTRODUCTION
In the modern world mathematical modeling can be
considered a universal tool for study, research and
design of objects in various fields of practice. By now
the theory, methods and technologies for creation and
application of mathematical models have developed to a
quite high level. However, unfortunately, the problems
of multi-criteria quality evaluation of mathematical
models, analysis and classification of different types of
models and reasonable selection of models for solving
certain practical tasks have not been studied well enough
yet. The listed problems are the main objects of study in
this paper, which introduces the results of research of a
new applied theory being developed by the authors – the
qualimetry of mathematical models and multiple-model
complexes describing various kinds of complex objects
(CO) (Ceany and Raiffa 1981; Ivanov, D.A. et al. 2010;
Krishans 2011; Mikoni 2015; Okhtilev 2006).
The paper provides methodological basis for the
proposed theory of models’ quality evaluation which
includes concepts, principles and approaches to solving
its main problems. There is also a general formal
description and dynamic interpretation of models of a
situation in progress, the participants of which are the
subjects and objects of modeling, as well as the models
used. This description allows to develop the most
generalized approach to solving the tasks of models’
qualimetry based on fundamental and applied results
achieved in the modern theory of CO control. There is
an example illustrating the achieved results (Azgal'dov
1982; Val'kman 1996).

**GENERALIZED DESCRIPTION OF THE
PROBLEMS OF SUBJECT-OBJECT MODELING
OF A SITUATION IN PROGRESS AND THEIR
CONTROL INTERPRETATION**

There is a big number of definitions of the word *model*,
which is known for its polysemy – the phenomenon of
having different meanings depending on a context.
Currently there are a few hundred definitions of the
concepts of a model and modeling (Aframchuk et al.
1998; Merkuryeva et al. 2011; Mesarovich and
Takahara 1978; Rostovtsev and Yusupov 1991). One
element of them is the following - a *model* is a
multiplace reflection of an original object that, together
with its absolutely true content, contains conditionally
true and false content, which reveals itself in the process
of the object’s creation and practical use (Rostovtsev
1991); *modeling* is one of the stages of
cognitive activity of a subject that includes the
development (selection) of a model, using it for
research, obtaining and analyzing the results, developing
recommendations on the further activity of the subject
and the estimation of the quality of the model itself
considering the solved problem and its specific
conditions.

The analysis of the mentioned definitions leads to the
conclusion that every model that was designed correctly
contains the objective truth (that is, correctly reflects the
original object in a certain way) (Okhtilev et al. 2006).
In addition to this, due to a limited number of elements
and relations in designed (applied) models, which
describe objects of the unlimitedly diverse reality, and
limited resources available for modeling (time, finance
and materials), a model always reflects an original
object in a simplified and approximate way. However,
human practical experience gives enough evidence for
these specific features of a model to be quite acceptable
for the solution of problems that subjects have to deal
with. Applying the principle of accuracies balance, it is
always possible to reach a compromise between how
detailed the description of an original object is and the
pragmatic value of a designed model (Peschel 1981;
Sethi and Thompson 2006; Sokolov and Yusupov
2004; Sokolov and Yusupov 2002).

The analysis of definitions given above shows that for
modeling of different types of CO, both natural and
artificial, it is reasonable to define the following basic
elements and relations that characterize a certain
process: firstly, a subject or subjects ($S_{<m>}$), an original
object ($Ob_{<m>}^{op}$), model-object ($Ob_{<m>}^{m}$), the environment
in which the modeling is performed ($CP_{<m>}^{m}$); and,
secondly, binary relations between the listed elements
$R_{<1>}(Ob_{<m>}^{op},S_{<m>})$, $R_{<2>}(S_{<m>},Ob_{<m>})$,
$R_{<3>}(Ob_{<m>}^{op},Ob_{<m>})$, $R_{<4>}(CP_{<m>},Ob_{<m>})$,
$R_{<5>}(CP_{<m>},Ob_{<m>})$, and $R_{<6>}(CP_{<m>},S_{<m>})$. The
subscripts "$<m>" used here stand for personal names of
objects (subjects) and relations (Okhtilev 2001; Okhtilev
et al. 2006; Sethi and Thompson 2006; Sokolov and
Yusupov 2004). It is important to mention that by
subjects of modeling we understand the following
classes of social subjects – decision-makers (DM); the
people who substantiate decisions (PSD); experts; the
people who utilize models; the people who design
models. Figure 1 represents possible interrelation
between the listed elements and relations between them
(Okhtilev et al. 2006; Rostotsev and Yusupov 1991).

**Figures 1: All possible interrelations of objects and
subjects of modeling of a situation in progress**

From the analysis of this figure it can be inferred that the
process of modeling is based on the processes of
interaction between subjects ($S_{<m>}$), an original object
($Ob_{<m>}^{op}$), a model object ($Ob_{<m>}$) and an environment
($CP_{<m>}$), which are set by binary relations between the
listed elements $R_{<1>}(Ob_{<m>}^{op},S_{<m>})$; $R_{<2>}(S_{<m>},Ob_{<m>})$;
$R_{<3>}(Ob_{<m>}^{op},Ob_{<m>})$; $R_{<4>}(CP_{<m>},Ob_{<m>})$;
$R_{<5>}(CP_{<m>},Ob_{<m>})$; $R_{<6>}(CP_{<m>},S_{<m>})$. It is important
to note that all listed elements and relations constantly
change with time due to objective-subjective and
external-internal reasons. Based on that, we will call any
structure condition of these four elements in a certain
moment a *situation*, and their change with time – a
situation in progress (SP). With such description the
process of subject-object modeling of CO can be
interpreted as a controlled process (as control of a
situation in progress).

The purpose of such process will be the constant
minimization of a discrepancy between an original
object and a model at all stages of their life cycle by
constant adaptation of the model to the changes
occurring to $Ob_{<m>}^{op}$, as well as to $CP_{<m>}$, $S_{<m>}$ (for
example, if a subject changes the purposes of
functioning and modeling of $Ob_{<m>}^{op}$).
The given interpretation of the process of object’s model development in case of a situation in progress is a very perspective one. Such approach allows to use a quite well-developed set of tools for analysis and synthesis of complex technical systems and their control systems and apply it to such objects of control as models and multiple-model complexes, as well as to a situation in progress as a whole (Ivanov et al. 2010; Krishans et al. 2011; Merkuryeva et al. 2011; Okhtilev et al. 2006; Rostovtsev and Yusupov 1991; Sokolov and Yusupov 2004).

So far, a lot of constructive approaches have been developed allowing to describe different kinds of models in general terms, which is necessary for their evaluation and comparative analysis (Laue and Müller 2016; Merkuryeva et al. 2011; Mesarovich and Takahara 1978; Okhtilev et al. 2006; Sokolov and Yusupov 2004; Steinberg et al. 1998; Trotsky and Gorodetsky 2009). Now it is necessary to describe possible technologies of subject-object modeling, also in general terms, thoroughly and formally (in general case — subject-object system modeling), applying the proposed control interpretation.

On a descriptive level the problem of modeling of a situation in progress at different stages of its life cycle is reduced to a solution of the following three main types of tasks (Okhtilev et al 2006):
- the task of analysis of structural dynamics of a situation in progress;
- the task of estimation (monitoring) of structural conditions and structural dynamics of a situation in progress;
- the task of structural-functional synthesis of a model (a multiple-model complex) and selection of optimal programs of control and regulation of structural dynamics of a situation in progress (quality control of models and multiple-model complexes) in various environmental conditions.

Let us give an example of a descriptive and formal statement of the task of structural-functional synthesis of a model (multiple-model complex) and also, selection of optimal programs of control and regulation of structural dynamics of a situation in progress (quality control of models and multiple-model complexes) in various environmental conditions.

The descriptive statement of the task of control of structural dynamics of a situation in progress is reduced to the following: we know the initial structural state of a situation in progress, we know the elements, possible variants of a structure of a situation in progress, we know the space-time, technical and technological constraints of a situation in progress, we know the time interval during which the control over the situation in progress takes place and a certain system of quality indicators for the given control.

It is required to perform multicriteria dynamic structural-functional synthesis of both multiple-model complex itself (based on the purposes of modeling set by a subject and indicators of quality of modeling that are used) and a corresponding technology of system modeling of a situation in progress, so that for each given scenario of changing disturbing actions on a situation in progress the most preferable transition from its current to a required structural state is reached.

Let us provide a formalization of these tasks with the use of the theory of control of structural dynamics of CO, which is being developed by the authors (Ivanov and Sokolov 2010; Ivanov et al. 2010; Okhtilev et al. 2006; Sokolov and Yusupov 2002). For a constructive description of relations between above listed subjects and objects which are basic components of a situation in progress we will introduce a dynamic system alternative multigraph (DSAM) with transformable structures that looks the following way:

\[ G^a_X := <X^a_X, F^a_X, Z^a_X >, \]

where \( \chi \) - index characterizing basic components of a situation in progress, \( \chi \in NS = \{1,2,3,4\} \) – the set of indices corresponding to elements \( Ob^{m}\_>, Ob^{m}\_<>, S^m, CP^m, \), \( t \in T \) – the set of time moments; \( X^a_X = \{x^a| l \in L^a\} \) – the set of basic components existing in the structure \( G^a_X \) (the set of DSAM nodes) in the moment of time \( t \); \( F^a_X = \{f^a| l \in L^a\} \) – the set of DSAM edges of the type \( G^a_X \) reflecting the relations between its basic components in the moment of time \( t \); \( Z^a_X = \{z^a| l \in L^a\} \) – the set of values of parameters that provide quantitative characteristic of the relation between corresponding basic components of DSAM (for example, the parameters of material, energy and information flows that circulate between basic components of a situation in progress).

**Graphic control interpretation** of the studied tasks of system subject-object modeling of a situation in progress (control of its structural dynamics) in this case is reduced to the search of such structural state \( S^a \in \{S_1, S_2, ..., S_{K_A}\} \) and such sequence (composition) of performing operations of mapping in time \( \Pi_{\Delta t} \_\delta_i, \delta_j \_\delta_k \_\delta_m \_\delta_n \_\delta_x \_\delta_y \_\delta_z \) which enable multicriteria dynamic structural-functional synthesis of both multiple-model complex itself (based on the purposes of modeling set by a subject and indicators of quality of modeling that are used) and a corresponding technology of system modeling of a situation in progress, so that the following conditions are met:

\[ <U\_a, S^a >, \]
Let the model to be nonnegative edge weights. Let us consider the problem of finding distances from a vertex to all other vertices of a graph with unbounded edge weights. The analysis of a formal statement of the studied problem shows that it refers to the class of problems of multlcriteria selection. We will clarify what it means by giving an example.

THE EXAMPLE OF SOLVING THE TASK OF POLY-MODEL FINDING DISTANCES FROM A VERTEX TO ALL OTHER VERTICES OF A GRAPH

Below is the illustration of the main ideas of the proposed approach to model quality evaluation. Let us consider the problem of finding distances from a vertex to all other vertices of a graph with unbounded nonnegative edge weights. Let the model to be considered be a matrix $D$ of edge weights. An example of the matrix $D$ is given below:

$$
D_i = \begin{bmatrix}
0 & 9 & 6 & 3 \\
9 & 0 & 8 & \infty \\
\infty & 2 & 4 & 0 \\
\infty & 4 & \infty & \infty
\end{bmatrix}
$$

In practice three methods (algorithms) of finding distances from a vertex to all other vertices of a graph are used:

1. Multiplying $i$-th row of the distance matrix by $D$ according to the formula:

$$
d_{ik}^s = \min \left\{ d_{ik}^{s-1}, \min_j \left( d_{ij}^{s-1} + d_{jk} \right) \right\}, i, j = 1, \ldots, n. \quad (4)
$$

2. Dijkstra's algorithm of dynamic programming with stepwise reduction of the shortest route between vertices $v_i$ and $v_k$:

$$
d_{ik}^s = \min \left\{ d_{ik}^{s-1}, \min_j \left( d_{ij}^{s-1} + d_{jk} \right) \right\};
$$

$$
t = \arg \min_j \left( d_{ij}^{s-1} \right). \quad (5)
$$

3. Finding all paths from the vertex to all other vertices with the use of the modified adjacency matrix and the following calculation of the minimal distances (Pavlovsky 2000).

Estimation of asymptotic complexity of the methods of solution

1. Matrix multiplication according to (4)

Number of operations: Multiplication of a row by a column: $n$; Determination of the minimal sum: $n-1$; Comparing with the last shortest distance: 1; Total number of row-by-column multiplication operations: $n(n-1) = 2n$; Multiplication of a row by $n$ columns: $2n(n-1)$; Total number of row-by-row multiplications for achieving all vertices: $n-1$; The total number of operations is equal to $2n^2(n-1)$.

2. Dijkstra's algorithm of dynamic programming

Number of operations: Determining the vertex nearest to the previous one: $n-1$; Adding the distance between these vertices to the whole distance from the initial vertex: 1; Removal of the labelled vertex from the list of vertices: 1; Total number of searches for the nearest vertex: $n+1$; Removal of labelled vertices reduces the number of searching operations to $n+1 - i$; $i = 1, n-1$.

The total number of operation for $n$ iterations is equal to

$$
\sum_{i=1}^{n-1} \left( n+1-i - n(n-1) \right).
$$

3. Finding distances through all paths with the use of the modified adjacency matrix

Number of operations: Multiplication of a row by a column: $n$; Determining and removal of cyclic routes: 1; Total number of row-by-column multiplication operations: $n+1$; Multiplication of a row by $n$ columns: $(n+1)n$; Total number of row-by-row multiplications for achieving all vertices: $n-1$; The total number of
operations for the first phase is equal to \((n+1)n(n-1)=n(n^2-1).\) Calculation of the length for an arbitrary route: \(n(n-1)/2;\) The total number of operations for two phases is equal to \(n(n^2-1)+n(n-1)/2.\)

The first two methods have a common model and focus only on the calculation of distances. Therefore, they are compared only by complexity. Unlike the method of finding distances by multiplying row by matrix realizing a parallel search procedure, the dynamic programming algorithm implements a reduced sequential search having the complexity \(2n\) times less. Consequently, while giving the same results, it is preferable for solving the problem under consideration. The third method uses a more complex model (edge weights and adjacency matrices), which allows to determine not only the shortest distance, but also all distances from a vertex to all the other vertices. So, it is more universal compared to the first two methods, however it is also more complex than these methods. For the comparison of these methods a two-criteria evaluation of models must be applied (see Table 3).

The last row of Table 3 shows the preferences on the set of parameters. For these preferences Pareto’s set is formed in the second and third algorithm. To select one of them, one should aggregate numerical estimates, bringing them to a common scale. The estimates of the third algorithm, that exceed the estimates of other algorithms, are used as normalizing values. The two-criteria estimates of the algorithms depend on the significance of parameters defined by a client (an expert).

Table 1: Capitalize Caption with No-Period

<table>
<thead>
<tr>
<th>No.</th>
<th>The name of the method (algorithm)</th>
<th>Versatility (number of routes)</th>
<th>Complexity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Multiplying by the edge weights matrix</td>
<td>((n-1))</td>
<td>(2n^2(n-1))</td>
</tr>
<tr>
<td>2</td>
<td>Dijkstra’s algorithm</td>
<td>((n-1))</td>
<td>(n(n-1))</td>
</tr>
<tr>
<td>3</td>
<td>The use of the modified adjacency matrix</td>
<td>((n-1)(n-1))</td>
<td>(n(n^2-1)+n(n-1)/2)</td>
</tr>
<tr>
<td>Preferences</td>
<td>max</td>
<td>min</td>
<td></td>
</tr>
</tbody>
</table>

In conference presentation we will propose example of practical implementation of an interdisciplinary approach that uses broadly the Earth’s remote sensing data, service architecture-based forecasting systems, and an intelligent interface to select the type and adjust the parameters of hydrological models, providing the interpretation, user-friendly representation, and accessibility of operational river-flood forecast results as web services.

CONCLUSION

As the result of the conducted research on complex objects’ models quality evaluation several conclusions have been made. First, when selecting an object for modeling it is reasonable to select not a really existing (designed or abstract) object but a situation in progress, that includes the objects and the subjects of the modeling (people responsible for making decisions (DM), people responsible for the substantiation of a decision (solution), experts, and people responsible for the implementation of solutions. The main feature of a situation in progress is that the set of states for all its participants varies in time due to different kinds of reasons (objective, subjective, internal, external, etc.). Secondly, modeling of objects is interpreted here as a process of control of structural dynamics of a situation in progress which takes place in uncertain conditions caused by lack of information necessary for subjects to form substantiated decisions. The discussed specifics of the conceptual and formal description of models and multiple-model complexes allow to apply mathematical structures that are being developed in the modern theory of control and engineering knowledge for the purpose of formal representation and study of models and multiple-model complexes.

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