

# New fuzzy numbers comparison operators in energy effectiveness simulation and modeling systems

Wojciech T. Dobrosielski  
Casimir the Great University in Bydgoszcz  
Department of Computer Science  
ul. Chodkiewicza 30, 85-064 Bydgoszcz, Poland  
E-mail: wdobrosielski@ukw.edu.pl

Jacek M. Czerniak  
Casimir the Great University in Bydgoszcz  
Department of Computer Science  
ul. Chodkiewicza 30, 85-064 Bydgoszcz, Poland  
E-mail: jczerniak@ukw.edu.pl

Hubert Zarzycki  
University of Information Technology and Management  
Copernicus  
ul. Inowroclawska 56, 53-648 Wroclaw, Poland  
E-mail: hzarzycki@wsiz.wroc.pl

Janusz Szczepański  
Institute of Fundamental Technological Research  
Polish Academy of Sciences  
ul. Pawinskiego 5B, 02-106 Warsaw, Poland  
E-mail: jszczepa@ippt.pan.pl

## KEYWORDS

Fuzzy logic, comparison, OFN, GR, ML, TR

## ABSTRACT

Energy efficiency is often a key optimization problem. Many control systems use fuzzy logic and as a result applying compare operators to fuzzy numbers. The article deals with the issue of comparing fuzzy numbers. The similarity relation is most probably the most frequently used and the most difficult to precisely determine the convergence measure. Analysis of the similarity of two objects is a basic assessment tool and constitutes the basis for reasoning by analogy. It also directly affects the energy effectiveness of the universe that it controls. This article presents the methods for determining the similarity used in fuzzy logic. Many of these methods were dedicated only to fuzzy triangular or trapezoidal numbers (Dobrosielski et al. 2017, Chi-Tsuen Yeh 2017, Abbasbandy and Hajjar 2009). This was a computational inconvenience and posed a question about the axiological basis of this type of comparison. The authors proposed two new approaches for comparing fuzzy numbers using one of the known extensions of fuzzy numbers (Kacprzyk and Wilbik 2009, 2005). This allowed to simplify the operation and eliminate the duality (Zadrozny, 2004).

## INTRUDUCTION

In all fields of science for a long time it was necessary to compare certain objects. While some branches of science sought to answer the question about the nature of the similarities, others need precise, formal definition. Comparison of two objects or occurrences can be seen as an attempt to determine the relation between them (Piegat, 2015, Piegat et al., 2015). The most important and most frequently used relations between objects are similarity, difference and inclusion. In the literature, most attention is dedicated to the issue of the similarity of objects (Stachowiak and Dyczkowski 2013, Wenyi et

al 2016). In recent decades, the theory of fuzzy sets has been used in many areas of science and everyday life (Czerniak et al. 2017a, Zarzycki et al. 2017a, Dobrosielski et al. 2017, Ewald 2018a, Apiecionek et al. 2018, Marszałek 2014). The need to compare fuzzy sets emerged naturally from the very beginning of the theory. There are plenty of methods, often based on those used for conventional sets. Intensive development of fuzzy logic and its applications often need to identify new ways of comparing objects (Lebiediewa et al. 2016, Zarzycki et al. 2017b) or issues as Linguistic Summaries (Kacprzyk et al 2005, 2006, 2009, Zadrozny 2004). This issue is particularly important in the computer aided decision support, classification and processing of natural language. Although the issue of the comparison is crucial for many applications of fuzzy set theory, still we failed to clearly formalize the basic concepts such as similarity or inclusion (Adabitabar et al. 2012, Khorshidi and Nikfalayar 2017, Dabashree 2010). While some researchers concerned with fuzzy logic seeks to define precisely the concepts, others questioned this approach, saying that imposing rigid framework limits the practical applications (Czerniak et al., 2017). Through years of development of fuzzy logic, many researchers has been developing methods of comparing sets and fuzzy numbers. Among them, it is worth to recall Several fuzzy number comparison methods and indices have been researched since 1977 Zadeh, Yager and Kaufman, Chang, and Amado (Abbasbandy, 2009). Bortolan and De-gani and Dadgostar reviewed some of the methods for ranking fuzzy sets (Tran et al., 2002a, 2002b), including Yager's first, second and third indexes, Chang's algorithm, Adamo's method, Baas and Kwakernaak's method, Baldwin and Guild's method, Kerre's method (Wenyi et al, 2016), Jain's method and Dubois and Prade's four grades of dominance (PD, PSD, ND, NSD). Dadgostar and Kerr (Dabashree 2010) proposed a consistent method, called Partial Comparison Method (PCM) (Adabitabar et al. 2012). Wang and Kerre proposed several axioms as reasonable properties to determine the rationality of a fuzzy

ordering or ranking method and systematically compared a wide array of fuzzy ranking methods (Guixiang 2017).

### THREE NEW DEFUZZIFYCATORS

In this paragraph, the mathematical foundations of the three methods of defuzzifying will be presented. They are all sensitive to directing. This means that the methods applied to numbers in the OFN notation generally provide different defuzzifying results for numbers with opposite directing on the same coordinates (Ewald et al., 2018).

#### Golden Ratio (GR) defuzzification operator

Fibonacci series is based on the assumption that it starts with two ones, and each consecutive number is the sum of the previous two. The proposal for the Golden Ratio method of defuzzification is based on the proportion of the golden ratio (Dobrosielski et al., 2017). As a result of dividing each of the numbers by its predecessor, we always obtain quotients oscillating around the value of 1.618 the golden ratio number. The exact value of the limit is the golden number itself:

$$GR = \frac{\min(\text{supp}(A)) + |\text{supp}(A)|}{\Phi} \quad (1)$$

where  $\Phi = 1,618033998875\dots$

where: GR is the defuzzification operator,  $\text{supp}(A)$  is support for fuzzy set A in universe X.

#### Mandala Factor (ML) defuzzification operator

Buddhist monks can create amazing pictures of colored sand grains. Those pictures are called mandala. The Mandala Factor defuzzification operator is inspired by mandala. Calculation of the R value using the Mandala Factor for the rising edge, falling edge and core set function integral. Then the obtained value should be scaled from the center of the coordinate system by adding it to the start of the support value of the fuzzy number (Czerniak et al., 2018). When defuzzification is performed in the OFN arithmetic, then in the case of a positive order, one should proceed as described below, while in the case of a negative order, one should deduct the calculated value from the first coordinate of the OFN number corresponding to the outermost right side of the OFN support.

$$MF(A) = \begin{cases} c + r, & \text{if order } (A) \text{ is positive} \\ c - r, & \text{if order } (A) \text{ is negative} \end{cases} \quad (2)$$

where

$$r = \frac{1}{d-c} \int_c^d x dx - \frac{c}{d-c} \int_c^d dx + \frac{f}{f-e} \int_f^e dx - \frac{1}{f-e} \int_e^f x dx + \int_d^e dx \quad (3)$$

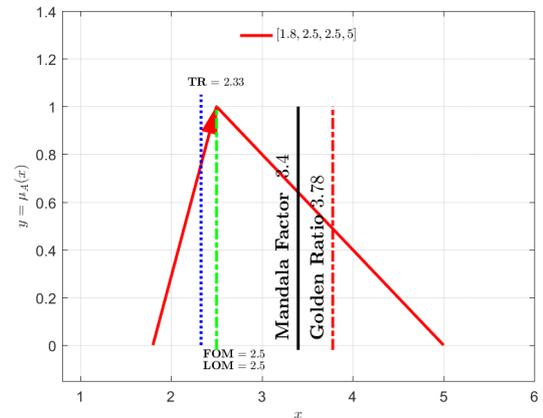
#### Triangular Expanding (TR) defuzzification operator

The above considerations include formal description of the proposed method in the OFN number defuzzification process that comes down to determining the equations for the intersection of two circles and the intersection of two linear functions (Dobrosielski et al., 2017).

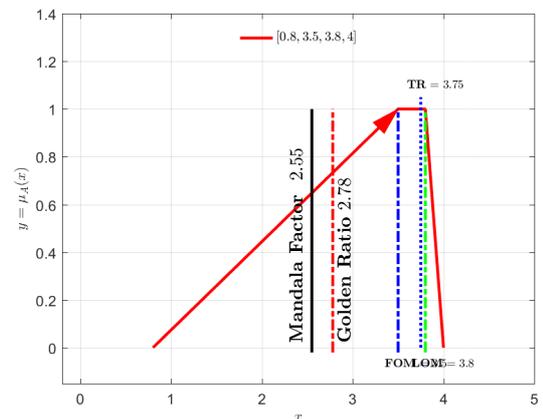
$$x_w = \frac{f(0)(f(1)s_1 - g(0)s_1 + g(0)) - g(0)x_1}{f(0) + f(1)s_1 - g(0)s_1 - x_1} \quad (4)$$

where:  $x_1, s_1$  are the determined coordinates of the point crisp value

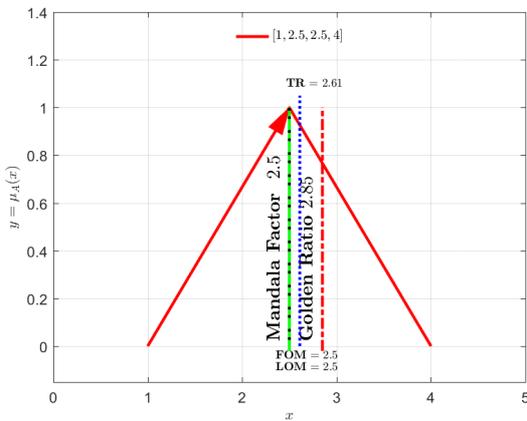
The following set of drawings presents the visualization of the seven fuzzy numbers. We are going to compare these numbers, which is not a trivial task in the case of fuzzy logic.



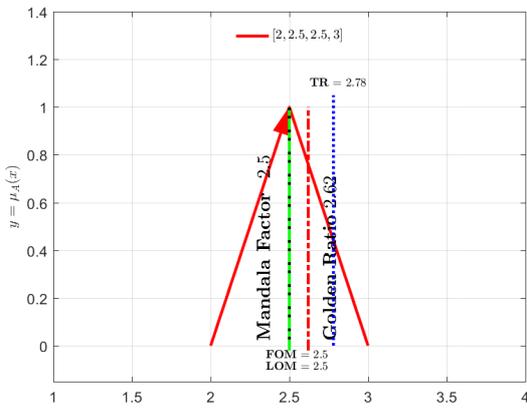
Figures 1: Crisp value of bA1 by GR,ML and TR



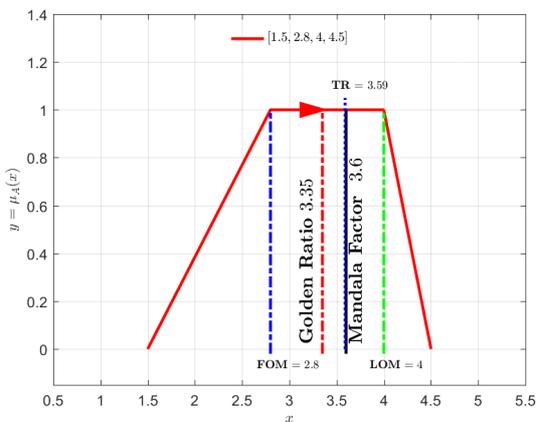
Figures 2: Crisp value of bA2 by GR,ML and TR



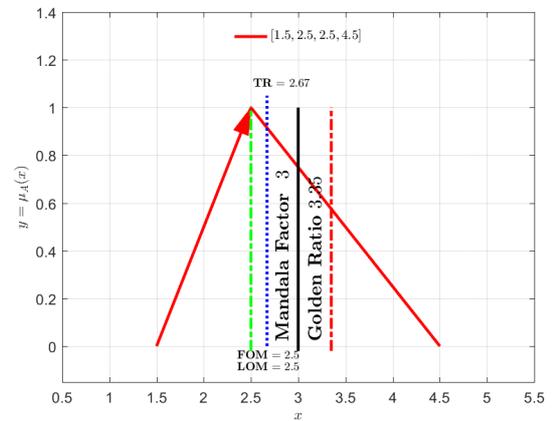
Figures 3: Crisp value of dA1 by GR,ML and TR



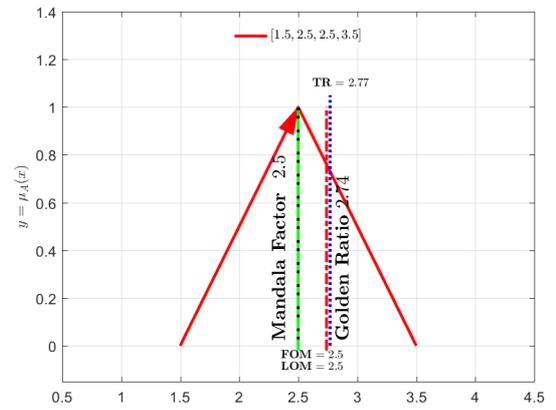
Figures 4: Crisp value of dA2 by GR,ML and TR



Figures 5: Crisp value of eA1 by GR,ML and TR



Figures 6: Crisp value of eA2 by GR,ML and TR

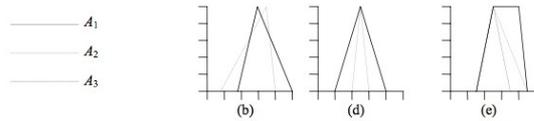


Figures 7: Crisp value of eA3 by GR,ML and TR

### EXAMPLES OF FUZZY NUMBER COMPARISON

This paragraph presents the results obtained using the ML, GR and TR methods for the following set of compared numbers.

Table 1: Fuzzy number comparison methods



Methods		$A_1$	$A_2$	$A_1$	$A_2$	$A_1$	$A_2$	$A_3$
Yager	F1	0.61	0.53	0.50	0.50	0.62	0.56	0.50
	F2	0.66	0.69	0.61	0.54	0.81	0.64	0.58
	F3	0.58	0.56	0.50	0.50	0.62	0.54	0.50
Chang		0.40	0.34	0.29	0.10	0.56	0.33	0.20
Adamo	0.9M	0.55	0.66	0.53	0.51	0.81	0.54	0.52
	0.9m	0.55	0.66	0.53	0.51	0.81	0.54	0.52
	0.5	0.75	0.72	0.65	0.55	0.85	0.70	0.60
Baas-Kwakernaak		0.84	1	1	1	1	1	1
Baldwin-Guild	lap.	0.42	0.33	0.27	0.28	0.45	0.37	0.27
	g.	0.44	0.37	0.30	0.24	0.53	0.40	0.28
	r.a.	0.34	0.24	0.20	0.23	0.31	0.28	0.21
Kerre		0.96	0.89	0.91	0.91	1	0.85	0.75
Own results	GR	3.78	2.78	2.85	2.62	3.35	3.35	2.74
	ML	3.4	2.55	2.5	2.5	3.6	3	2.5
	TR	2.33	3.75	2.61	2.78	3.59	2.67	2.77
Dubois-Prade	PD	0.84	1	1	1	1	1	1
	PSD	0.54	0.46	0.73	0.24	0.80	0.20	0
	ND	0.54	0.46	0.27	0.76	0.50	0.50	0.50
	NSD	0	0.16	0	0	0	0	0
Lee-Li	U.m.	0.61	0.53	0.50	0.50	0.62	0.56	0.50
	U.g.	-	-	0.12	0.04	-	-	-
	P.m.	0.53	0.58	0.50	0.50	0.63	0.55	0.50
	P.g.	-	-	0.09	0.03	-	-	-
Dadgostar-Kerr	PCM	0.57	0.43	0.50	0.50	0.63	0.37	0.25
Dorohonceanu-Marin	B2	0.59	0.41	0.50	0.50	0.65	0.60	0.40
	B2 x	0.59	0.41	0.50	0.50	0.68	0.61	0.39

As can be seen from the table above, the use of defuzzification operators GR, ML and TR allowed to reduce the uncertainty of fuzzy numbers and allowed to compare them. Thus, in the first group (b) where the  $A_1$  and  $A_2$  numbers are compared, individual defuzzifiers allow to detect the following relations: GR:  $A_1 > A_2$ , ML:  $A_1 > A_2$ , TR:  $A_1 < A_2$ .

In group (d) defuzzification allowed detection of the following relations:

GR:  $A_1 > A_2$ , ML:  $A_1 = A_2$ , TR:  $A_1 < A_2$ .

The third group (e) of fuzzy numbers, the elements of which have been defuzzified, will present the following set of relations:

GR:  $A_1 = A_2 > A_3$ , ML:  $A_1 > A_2 > A_3$ , TR:  $A_1 > A_3 > A_2$

## CONCLUSION

The article presents three defuzzifying methods that can be applied both to classic fuzzy numbers and to numbers in OFN notation. This time, the discussed defuzzifiers were used, ie GR, ML and TR as operators preparing data for comparing fuzzy numbers.

The novelty that the article brings is, in addition to the development of three proprietary defuzzification methods, applying them to the fuzzy numbers mentioned in the introduction, commonly used as peculiar benchmarks of fuzzy logic. As a result, a new application of a novel defuzzification operators is shown.

The GR and ML defuzzification operators showed in group (b) the same results as most compared operators.

In group (d), only the ML operator indicated the same type of relationship as the majority of known operators, and GR and TR were among the minority compared operators. However, in the third group (e) the ML operator indicated the relation that the majority

indicated and the GR and TR operators signaled another type of relation not mentioned in the group.

## REFERENCES

- Adabitarbar Firozja M.; G.H. Fath-Tabar; Z. Eslampia. 2012. "The similarity measure of generalized fuzzy numbers based on interval distance", Applied Mathematics Letters, Volume 25, Issue 10, pp.1528-1534, Elsevier.
- Apiecionek Ł.; H. Zarzycki; J.M. Czerniak; W.T. Dobrosielski; D. Ewald. 2018. "The Cellular Automata Theory with Fuzzy Numbers in Simulation of Real Fires in Buildings". In: Atanassov K. et al. (eds) Uncertainty and Imprecision in Decision Making and Decision Support: Cross-Fertilization, New Models and Applications. IWIFSGN 2016. Advances in Intelligent Systems and Computing, vol 559. Springer, Cham.
- Chi-Tsuen Yeh. 2017. "Existence of interval, triangular, and trapezoidal approximations of fuzzy numbers under a general condition". Fuzzy Sets and Systems, Volume 310, pp. 1-13, Elsevier
- Czerniak J.M.; Zarzycki H. 2017. "Artificial Acari Optimization as a new strategy for global optimization of multimodal functions". Journal of Computational Science, Volume 22f, pp. 209-227, Elsevier.
- Czerniak J.M.; Zarzycki H.; Apiecionek Ł.; Palczewski W.; Kardasz P. 2018. "A Cellular Automata-Based Simulation Tool for Real Fire Accident Prevention" Mathematical Problems in Engineering 2018, Article ID 3058241, 12 pages, Hindawi.
- Dobrosielski W.; J. Szczepański.; H. Zarzycki. 2016. A Proposal for a Method of Defuzzification based on the Golden Ratio - GR, In: Atanassov K. et al. (eds) Novel Developments in Uncertainty Representation and Processing. Advances in Intelligent Systems and Computing, Volume 401, pp 75-84, Springer, Cham.
- Dobrosielski W.; J. M. Czerniak; J. Szczepański; and H. Zarzycki. 2017. "Triangular expanding, a new defuzzification method on ordered fuzzy numbers," Advances in Intelligent Systems and Computing, Volume 642, pp. 605-619, Springer.
- Debashree Guha; Debjani Chakraborty. 2010. "A new approach to fuzzy distance measure and similarity measure between two generalized fuzzy numbers", Applied Soft Computing, Volume 10, Issue 1, pp. 90-99, Elsevier.
- Dyczkowski K. 2007. "A Less Cumulative Algorithm of Mining Linguistic Browsing Patterns in the World Wide Web". EUSFLAT Conf., Volume 2, pp.129-135.
- Ewald D.; J.M. Czerniak; H. Zarzycki. 2018. "OFN Bee Method Used for Solving a Set of Benchmarks". In: Kacprzyk J., Szmidt E., Zadrożny S., Atanassov K., Krawczak M. (eds) Advances in Fuzzy Logic and Technology 2017. IWIFSGN 2017, EUSFLAT 2017. Advances in Intelligent Systems and Computing, vol 642. Springer, Cham.
- Guixiang Wang; Jing Li. 2017. "Approximations of fuzzy numbers by step type fuzzy numbers", Fuzzy Sets and Systems, Volume 310, pp. 47-59, Elsevier.
- Abbasbandy S.; T. Hajjari. 2009. "A new approach for ranking of trapezoidal fuzzy numbers".
- Jiang Wen; Xin Fan; Dejie Duanmu; Deng Yong. 2011. "A modified similarity measure of generalized fuzzy numbers", Procedia Engineering, Volume 15, pp.2773-2777, Elsevier.
- Kacprzyk J.; Wilbik A. 2009 "Using Fuzzy Linguistic Summaries for the comparison of time series: an

- application to the analysis of investment fund quotations". IFSA/EUSFLAT Conf., pp.1321-1326, 2009.
- Kacprzyk J., Wilbik A., Zadrożny S., On some types of linguistic summaries of time series, in: Proceedings of 3rd International IEEE Conference Intelligent Systems, London, UK, Sept. 4-6, 2006, IEEE Press, pp.373-378
- Kacprzyk J.; Zadrożny S. 2005. "Fuzzy Linguistic Summaries in Text Categorization for Human-Consistent Document-Driven Decision Support Systems". In: Reusch B. (eds) Computational Intelligence, Theory and Applications. Advances in Soft Computing, volume 33, pp. 1373-1380, Springer.
- Khorshidi Hadi Akbarzade; Sanaz Nikfalazar. 2017. "An improved similarity measure for generalized fuzzy numbers and its application to fuzzy risk analysis", Applied Soft Computing, Volume 52, pp. 478-486, Elsevier.
- Lebiediewa S.; H. Zarzycki; W.T. Dobrosielski. 2016. "A New Approach to the Equivalence of Relational and Object-Oriented Databases". In: Atanassov K. et al. (eds) Novel Developments in Uncertainty Representation and Processing. Advances in Intelligent Systems and Computing, vol 401. Springer, Cham.
- Macko M.; J. Flizikowski. 2010. "The method of the selection of comminution design for non-brittle materials". AICHE Annual Meeting, Conference Proceedings 2010.
- Marszałek A.; T. Burczyński. 2014. "Modeling and forecasting financial time series with ordered fuzzy candlesticks". Information Science, Volume 273, pp.144-155, Elsevier.
- Mikołajewska E.; D. Mikołajewski. 2011. "Exoskeletons in Neurological Diseases - Current and Potential Future Applications". Advances in Clinical and Experimental Medicine, Volume 20, pp.227-233, 2011.
- Piegat A. 2005. "A new definition of the fuzzy set", Appl. Math. Comput. Sci. Volume 15, No. 1, pp. 125-140.
- Piegat A.; M. Pluciński. 2015. "Computing with words with the use of inverse RDM models of membership functions". International Journal of Applied Mathematics and Computer Science, 25(3), pp. 675-688, De Gruyter
- Rojek I. 2016. "Technological process planning by the use of neural networks", Artificial Intelligence for Engineering Design, Analysis and Manufacturing, Volume 31, No. 1, pp. 1-15, Cambridge University Press.
- Stachowiak A.; Dyczkowski K. 2013. "A similarity measure with uncertainty for incompletely known fuzzy sets". 2013 Joint IFSA World Congress and NAFIPS Annual Meeting (IFSA/NAFIPS), pp. 390-394.
- Szmidt E.; Janusz Kacprzyk. 2000. "Distances between intuitionistic fuzzy sets, Fuzzy Sets and Systems", Volume 114, Issue 3, pp.505-518, Elsevier.
- Śmigielski G.; W. Toczek; R. Dygdała. 2016. "Metrological Analysis of Precision of the System of Delivering a Water Capsule for Explosive Production of Water Aerosol". Metrology and Measurement Systems, Volume 23, No. 1, pp. 47-58, De Gruyter.
- Śmigielski G., R. Dygdała, H. Zarzycki, and D. Lewandowski, "Real-time system of delivering water-capsule for firefighting," Advances in Intelligent Systems and Computing, vol. 534, pp. 102–111, Springer.
- Tofigh Allahviranloo, M. Adabitabar Firozja, Ranking of fuzzy numbers by a new metric, Soft Computing, 2010, Volume 14, Number 7, Page 773-782
- Tran L., L. Duckstein, Multiobjective fuzzy regression with central tendency and possibilistic properties, Fuzzy Sets and Systems 130 (2002) pp.21–31.
- Tran L., L. Duckstein, Comparison of fuzzy numbers using a fuzzy distance measure, Fuzzy Sets and Systems 130 (2002) pp.331–341
- Wenyi Zeng; Deqing Li; Qian Yin. 2016. "Distance and similarity measures between hesitant fuzzy sets and their application in pattern recognition", Pattern Recognition Letters, pp.267-271, Volume 84, Elsevier.
- Zarzycki H.; Czerniak J.M.; and Dobrosielski W.T. 2017. "Detecting Nasdaq Composite Index Trends with OFNs". In: Prokopowicz P., Czerniak J., Mikołajewski D., Apiecionek Ł., Slezak D. (eds) Theory and Applications of Ordered Fuzzy Numbers. Studies in Fuzziness and Soft Computing, vol 356. Springer, Cham.
- Zarzycki H.; J.M. Czerniak; D. Lakomski, P. Kardasz. 2017. Performance Comparison of CRM Systems Dedicated to Reporting Failures to IT Department. In: Madeyski L., Śmiałek M., Hnatkowska B., Huzar Z. (eds) Software Engineering: Challenges and Solutions. Advances in Intelligent Systems and Computing, vol 504. Springer, Cham.
- Zadrożny S., Kacprzyk J., On the use of linguistic summaries for text categorization, in: Proceedings of IPMU'2004 – International Conference on Information Processing and Management of Uncertainty in Knowledge-based Systems, 2004, vol. 2, pp.1373-1380

## AUTHOR BIOGRAPHIES



**JACEK M. CZERNIAK** received Ph.D. degrees in Computer Science in 2005 from Technical University of Szczecin in Poland. In 2000, Dr. Czerniak received a M.Eng. degree in Computer Science from the Technical University of Szczecin. Dr. Czerniak is currently employed as an Assistant Professor in the Computer Science Department at Casimir the Great University in Bydgoszcz. Dr. Czerniak is a founding director the AIRlab Artificial Intelligence and Robotics Laboratory. He is also founding Editor-in-Chief journal of Studies and Materials in Applied Computer Science. Dr. Czerniak has been awarded twice by the President of Casimir the Great University in Bydgoszcz. First in october 2009, awarded Outstanding Teacher Award, and the second time in november 2015 and third in 2017 awarded Outstanding Researcher Award. His e-mail address is : JCzerniak@ukw.edu.pl and his Web-page can be found at [http://www. JCzerniak.ukw.edu.pl](http://www.JCzerniak.ukw.edu.pl)



**HUBERT ZARZYCKI** servers as an assistan proffesor at University of Information Technology and Management Copernicus in Wroclaw. He received his

MSc and PhD (2006) degree in the discipline of Computer Science from Technical University of Szczecin. He also holds a master's degree in the field of Finance and Banking. His research interests developed while working in different companies and research centers in Poland, the United States, Germany and Italy. He collaborates with renowned institutions, domestic and foreign. He has many years of experience in the information technology industry. His research interests relate to artificial intelligence, fuzzy numbers, computer system architecture, databases and programming. His e-mail address is : [hzarzycki@wsiz.wroc.pl](mailto:hzarzycki@wsiz.wroc.pl).



**WOJCIECH T. DOBROSIELSKI** graduated from the Faculty of Computer Science at Technical University of Szczecin in specialty programming techniques and information systems. He worked as a research assistant at the Casimir the Great University in Bydgoszcz and Systems Research Institute of Polish Academy of Sciences. His research interests are related to Artificial Intelligence, with the main focus is associated with fuzzy logic. More specifically his scientific work often discusses applications of ordered fuzzy numbers. Related research interest are: control systems, semantic networks, swarm intelligence - ACO, robotics, parallel programming, cryptography, software engineering and electronics. His e-mail address is : [wdobrosielski@ukw.edu.pl](mailto:wdobrosielski@ukw.edu.pl) and his Web-page can be found at <http://www.it.ukw.edu.pl>

**JANUSZ SZCZEPAŃSKI** received the M.Sc. degree in mathematics (1979) at the Warsaw University, the Ph.D. degree in applied mathematics (1985) and habilitation degree in computer sciences (2007) in Polish Academy of Sciences. Currently, he is a professor



at the Institute of Fundamental Technological Research, PAS and deputy-chairmen of the Scientific Council. In 2001–2004 he was a Consultant on Cryptography with the Polish Certification Authority “Centrast” Co. For Public Key Infrastructure. In 2000 and 2003 he was a Visiting Scientist at the Miguel Hernandez University, Elche, Spain. In 2004 he was a Visiting Scholar at the University of California, San Diego. His research interests include information theory, neuroscience, application of dynamical systems and stochastic processes to biological systems and cryptography. Prof. Szczepanski received the PAS Award in 1989.

His e-mail address is : [jszczepa@ippt.pan.pl](mailto:jszczepa@ippt.pan.pl) and his Web-page can be found at [http://www.ippt.pan.pl/staff/jszczepa](http://http://www.ippt.pan.pl/staff/jszczepa)