SIMULATION OF LARGE-SCALE QUEUEING SYSTEMS

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KEYWORDS

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ABSTRACT

In this paper we consider the dynamics of large-scale queueing systems with an infinite number of servers. We assume that a Poisson input flow of requests with intensity $N\lambda$. We suppose that each incoming request selects two any servers randomly and a next step of an algorithm includes sending this request to the server with the shorter queue instantly. A share $u_k(t)$ of the servers that have the queues lengths with not less than k can be described using an system of ordinary differential equations of infinite order. We investigate this system of ordinary differential equations of infinite order with a small real parameter. A small real parameter allows us to describe the processes of rapid changes in large-scale queueing systems. We use the simulation methods for this large-scale queueing systems analysis.

INTRODUCTION

The recent research of large-scale queueing systems with complex routing discipline in [16], [25], [26], [27], transport networks [1],[7], [8] and the asymptotic behavior of Jackson networks [21] faced with the problem of proving the global convergence of the solutions of certain infinite queueing systems of ordinary differential equations to a time-independent solution. Scattered results of these studies, however, allow a common approach to their justification. In work [17] the countable systems of differential equations with bounded Jacobi operators were studied and the sufficient conditions of global stability and global asymptotic stability were obtained. In [15] it was considered finite closed Jackson networks with N first come, first serve nodes and M customers. In the limit $M \to \infty$, $N \to \infty$, $M/N \to \lambda > 0$, it was got conditions when mean queue lengths are uniformly bounded and when there exists a node where the mean queue length tends to ∞ under the above limit (condensation phenomena, traffic jams), in terms of the limit distribution of the relative utilizations of the nodes. It was deriven asymptotics of the partition function and of correlation functions. In papers [5], [11], [20] the authors built various models of large-scale queueing systems and considered their dynamics.

Cauchy problems for the systems of ordinary differential equations of infinite order was investigated A.N.Tihonov [22], K.P.Persidsky [18], O.A.Zhautykov [28], [29], Ju.Korobeinik [10], A.M.Samoilenko, Yu.V.Teplinskii [19] other researchers. For example, Markus Kreer, Aye Kzlers and Anthony W. Thomas [13] investigated fractional Poisson processes, a rapidly growing area of non-Markovian stochastic processes, that are useful in statistics to describe data from counting processes when waiting times are not exponentially distributed. They showed that the fractional KolmogorovFeller equations for the probabilities at time t could be represented by an infinite linear system of ordinary differential equations of first order in a transformed time variable. These new equations resemble a linear version of the discrete coagulationfragmentation equations, well-known from the non-equilibrium theory of gelation, cluster-dynamics and phase transitions in physics and chemistry. The singular perturbed systems of ordinary differential equations with a small parameter was studied by A.N. Tihonov [23], A.B.Vasil'eva [24], S.A. Lomov [14] other researchers.

In paper [2] we investigated the singular perturbed systems of ordinary differential equations of infinite order of Tikhonov-type $\epsilon\dot{x} = F(x(t, g_x), y(t, g_y), t)$, $\dot{y} = f(x(t, g_x), y(t, g_y), t)$ with the initial conditions $x(t_0) = g_x$, $y(t_0) = g_y$, where x, $g_x \in X$, $X \subset l_1$ and y, $g_y \in Y$, $Y \in \mathbf{R^n}$, $t \in [t_0, t_1]$ $(t_0 < t_1)$, $t_0, t_1 \in T$, $T \in \mathbf{R}$, g_x and g_y are given vectors, $\epsilon > 0$ is a small real parameter.

In this paper we apply Dobrushin approaches from [26]. We consider the dynamics of large-scale queueing systems that consists of infinite number of servers with a Poisson input flow of requests of intensity $N\lambda$. We can use an algorithm that selects two any servers for each incoming request and sent it to the one of a server with the shorter queue instantly. We suppose that service time has mean $1/\mu$ with exponential distribution. In this case a share $u_k(t)$ of the servers that have the queues lengths with not less than k can be described using an system of ordinary differential equations of infinite order. We investigate this system of ordinary differential equations of infinite order with a small real parameter. A small real parameter allows us to describe the processes of rapid changes in large-scale queueing

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systems. Tikhonov type Cauchy problem for this system with small parameter ϵ and initial conditions is investigated.

We investigate the truncation system of this ordinary differential equations of infinite order with a small real parameter order N. Tikhonov type Cauchy problem for this truncation system with small parameter ϵ and initial conditions is used for the simulation of behavior solutions and for analysis of large-scale queueing systems with taking into account parameters λ, μ, ϵ.

LARGE-SCALE QUEUEING SYSTEMS MODEL

The basic model considered there is a queueing system S_N, with N identical infinite-buffer FCFS (First-Come, First-Served) single-servers, with a Poisson arrival flow of rate $N\lambda$ and with i.i.d. exponential service times of mean $1/\mu$, where $0 < \lambda < \mu$. Upon its arrival each task chooses m servers at random (i.e., independently of the pre-history of the queueing system (QS) and with probability $1/(N^m)$) and then selects, among the chosen ones, the server with the lowest queue-size, i.e., the lowest number of tasks in the buffer (including the task in service). If there happen to be more than one server with lowest queue-size, the task selects one of them randomly.

One is interested in the 'typical' behavior of a server in S_N, as $N \to \infty$. Formally, it means that $\forall t \geq 0$ and $k = 0, 1, \ldots$, we consider the fraction $q_k(t) = M_k(t)/N$ where $M_k(t)$ is the (random) number of servers with the queue-size k at time t. Clearly, $0 \leq q_k(t) \leq 1$, $\sum_k q_k(t) = 1$; and $Q(t) = (q_k(t))$, $t \geq 0$, forms a Markov process (MP). Technically, it is more convenient to pass to the tail probabilities $r_k(t) = \sum_{j \geq k} Q_k(t)$; the state space of the corresponding MP $U_N(t) = (f_k(t)), t \geq 0$, is the set $\mathbf{U}_N$ of non-increasing non-negative sequences $\mathbf{u} = (u_k, k = 0, 1, \ldots)$ with $u_0 = 1$, $\sum_{k>1} u_k < \infty$ and with the u_k's multiple of $1/N$, which implies that $u_k = 0$ for all k large enough. It is convenient to prolong the sequences $\mathbf{u} \in \mathbf{U}_N$ to the negative k's by the value 1.

The generator of $\{U_N(t)\}$ is an operator $\mathbf{A}$ acting on functions $f : \mathbf{U}_N \to C^1$ and given by

$$\mathbf{A}_N f(\mathbf{u}) = N \sum_{k>0} (u_k - u_{k+1}) \left[f\left(\mathbf{u} - \frac{\mathbf{e}_k}{N} - f(\mathbf{u}) \right) \right] + \\ +\lambda N \sum_{k>0} \left((u_{k-1})^2 - (u_k)^2 \right) \left[f\left(\mathbf{u} + \frac{\mathbf{e}_k}{N} - f(\mathbf{u}) \right) \right]. \tag{1}$$

Here, $\mathbf{e}_k$ stands for the sequence with the k-th entry 1 and all others 0, the addition of the sequences is componentwise. Process $\{U_N(t)\}$ is positive-recurrent and thus possess a unique invariant distribution, π_N; given any initial distribution ϖ, the distribution of $U_N(t)$ approaches π_N as $t \to \infty$. The main result of [25] is that, as $N \to \infty$, the expected value $\mathbf{E}_{\pi_N} r_k(t)$ converges to the value $\{a_k\}$, where

$$a_k = \left(\frac{\lambda}{\mu} \right)^{(m^k - 1)/(m-1)}, \ k \geq 0. \tag{2}$$

Pictorially speaking, it means that, as $N \to \infty$, an 'average' server in the QS will have k or more tasks in the buffer with probability a_k.

It is interesting to compare $\mathbf{S}_N$ with another queueing system $\mathbf{L}$, where the arriving task chooses the server completely randomly (i.e., independently of the pre-history and with probability $1/N$). Clearly, $\mathbf{L}$ is equivalent to an isolated $M/M/\infty$ queue with the arrival and service rates λ and μ, respectively, which justifies omitting subscript N in this notation. More precisely, the average server in $\mathbf{L}$ will have k or more tasks in the buffer with the geometrical probability

$$a_k^0 = \left(\frac{\lambda}{\mu} \right)^k \ k \geq 1, \tag{3}$$

(independently of N), which is much larger than a_k.

In fact, as was shown in [25], the whole process $\{U_N(t)\}$ is asymptotically deterministic as $N \to \infty$. More precisely, let $\mathbf{U}$ denote the set of the non-increasing non-negative sequences $\mathbf{u} = (u_k, \ k \in \mathbf{Z})$ with $u_k = 1$ for $k \leq 0$ and $\sum_{k \leq 0} u_k < \infty$.Then, if the distribution ϖ of initial state $U_N(0)$ approaches a Dirac delta-measure concentrated at a point $\mathbf{g} = \{g_k\} \in \mathbf{U}$, the distribution of $\{U_N(t)\}$ is concentrated in the limit at the 'trajectory' $\mathbf{u}(t) = u_k(t)$, $t \geq 0$, giving the solution to the following system of differential equations

$$\dot{u}_k(t) = \mu \left(u_{k+1}(t) - u_k(t) \right) + \\ +\lambda \left((u_{k-1}(t))^2 - (u_k(t))^2 \right), \\ u_0(t) = 0, u_k(0) = g_k \geq 0, \ k = 1, 2, \ldots, \ t \geq 0, \tag{4}$$

where $g = \{g_k\}_{k=1}^{\infty}$ is a numerical sequence ($1 = g_1$, $g_k \geq g_{k+1}$, $k = 1, 2, \ldots$) [25]. Point $\mathbf{a} = (a_k)$ (see (2)) is a (unique) fixed point for system (4) in $\mathbf{U}$.

These results illustrate the essence of the mean-field approximation for QS S_N. Equations (4) describe a 'self-compatible' evolution of vector $\mathbf{u}(t)$, or, equivalently, of the probability distribution $\mathbf{q}(t) = \{q_k(t)\}$ defined by $q_k(t) = u_k(t) - u_{k+1}(t)$, $t \geq 0$, $k = 0, 1, \ldots$ As before, $\mathbf{u}(t)$ is simply the sequence of the tail probabilities for $\mathbf{q}(t)$.

We can compare system (4) with the linear system

$$\dot{y}_k(t) = \mu \left(y_{k+1}(t) - y_k(t) \right) + \lambda \left(y_{k-1}(t) - y_k(t) \right), \tag{5}$$

(where $k \geq 1$) describing the evolution of the probability distribution $\mathbf{q}^0(t) = (q_k^0(t), \, q_k^0(t) = y_k(t) - y_{k+1}(t))$ in a standard $M/M/1/\infty$ queue with the arrival and service rates λ and μ, respectively. The μ-terms in (4) and (5) are the same; they correspond with the departure of the tasks and 'push' the probability mass in $\mathbf{q}(t)$ and $\mathbf{q}^{(0)}(t)$ towards $k = 0$. On the other hand, the λ-terms (different in both SQ) correspond with the arrival of the tasks; these terms shift the probability mass to larger k's. The λ-term in (4) is smaller than the one in (5) when $u_k(t)$ is small; pictorially speaking, system (4) provides (for the same values of λ and μ) more 'protection', for large k, against the shift to the right, which may lead to an 'explosion', when the relation $\sum_{k>1} u_k(t) < \infty$ or $\sum_{k>1} y_k(t) < \infty$ may fail as

$t \to \infty$. Because of this, the entries a_k of sequence $\mathbf{a}$ (see (2)) giving the fixed point of (4) decrease 'super-exponentially', in contrast with the exponential decay of the tail probabilities in the fixed point $\mathbf{a}^0 = (a_k^0)$ of (5).

LARGE-SCALE QUEUEING SYSTEMS MODEL WITH A SMALL PARAMETER

Let's consider a system that consists of N servers with a Poisson input flow of requests of intensity $N\lambda$. Each request arriving to the system randomly selects two servers and is instantly sent to the one with the shorter queue. The service time is distributed exponentially with mean $\bar{t} = 1/\mu$. Let $u_k(t)$ be a share servers that have the queues lengths with not less than k. It is possible to investigate the asymptotic distribution of the queue lengths as $N \to \infty$ and $\lambda < 1$ [25]. The considered system of the servers is described by ergodic Markov chain. There is a stationary probability distribution for the states of the system and if $N \to \infty$ the evolution of the values $u_k(t)$ becomes deterministic and the Markov chain asymptotically converges to a dynamic system the evolution of which is described by system of ordinary differential equations of infinite order

$$\dot{u}_k(t) = \mu\left(u_{k+1}(t) - u_k(t)\right) + \tag{6}$$
$$+\lambda\left((u_{k-1}(t))^2 - (u_k(t))^2\right).$$

For this system of ordinary differential equations of infinite order we can formulate Cauchy problem in the form

$$\dot{u}_k(t) = \mu\left(u_{k+1}(t) - u_k(t)\right) + \tag{7}$$
$$+\lambda\left((u_{k-1}(t))^2 - (u_k(t))^2\right),$$
$$u_0(t) = 0,\ u_k(0) = g_k \geq 0,\ k = 1, 2, \ldots,\ t \geq 0,$$

where $g = \{g_k\}_{k=1}^{\infty}$ is a numerical sequence ($1 = g_1$, $g_k \geq g_{k+1}$, $k = 1, 2, \ldots$) [25].

We can investigate Cauchy problem for system of ordinary differential equations of infinite order with small parameter such form

$$\dot{u}_k(t) = \mu\left(u_{k+1}(t) - u_k(t)\right) + \lambda\left((u_{k-1}(t))^2 -,\right.$$
$$-(u_k(t))^2,\ k = 0, 1, \ldots, n-1, \tag{8}$$
$$\dot{u}_n(t) = \mu\left(U_{n+1}(t) - u_n(t)\right) + \lambda\left((u_{n-1}(t))^2 - (u_n(t))^2\right),$$
$$\epsilon\dot{U}_k(t) = \mu\left(U_{k+1}(t) - U_k(t)\right) + \lambda\left((U_{k-1}(t))^2 -\right.$$
$$-(U_k(t))^2,\ k = n+1, n+1, \ldots,$$
$$u_k(0) = g_k \geq 0,\ k = 0, 1, 2, \ldots, n,$$
$$U_k(0) = g_k \geq 0,\ k = n+1, \ldots,$$

where $\epsilon > 0$ is a small parameter that bring a singular perturbation to the system (7), which allows us to describe the processes of rapid change of the systems.

Using (8) we can write Tikhonov problems for systems of ordinary differential equations of infinite order with a small parameter ϵ and initial conditions

$$\dot{u} = f(u(t, \mu, \lambda, g_u), U(t, \mu, \lambda, g_U), t),$$
$$\epsilon\dot{U} = F(U(t, \mu, \lambda, g_U), t); \tag{9}$$
$$u(0, \mu, \lambda, g_u) = g_u,\ U(0, \mu, \lambda, g_U) = g_U,$$

where $u, f \in X$, $X \in \mathbf{R^n}$ are n-dimensional functions; $U, F \in Y$, $Y \subset l_1$ are infinite-dimensional functions and $t \in [0, T_0]$ $(0 < T_0 \leq \infty)$, $t \in T$, $T \in \mathbf{R}$; $g_u \in X$ and $g_U \in Y$ are given vectors ($g_u = \{g_k\}_{k=0}^{n}, g_U = \{g_k\}_{k=n+1}^{\infty}$), $\epsilon > 0$ is a small real parameter; $u(t, g_u) = g_u = \{u_k\}_{k=0}^{n}$ and $U(t, g_U) = \{u_k\}_{k=n+1}^{\infty}$ are solutions of (9). Given functions $f(u(t, \mu, \lambda, g_u), U(t, \mu, \lambda, g_U), t)$ and $F(U(t, \mu, \lambda, g_U), t)$ are continuous functions for all variables

$$f_k(u(t, \mu, \lambda, g_u), t) = \mu\left(u_{k+1}(t) - u_k(t)\right) +$$
$$+\lambda\left((u_{k-1}(t))^2 - (u_k(t))^2\right),\ k = 0, 1, \ldots, n-1,$$
$$f_n(u(t, \mu, \lambda, g_u), U(t, \mu, \lambda, g_U), t) =$$
$$\epsilon\left(U_{n+1}(t) - u_n(t)\right) + \lambda\left((u_{n-1}(t))^2 - (u_n(t))^2\right), \tag{10}$$
$$F_k(U(t, \mu, \lambda, g_U), t) = \epsilon\left(U_{k+1}(t) - U_k(t)\right) +$$
$$+\lambda\left((U_{k-1}(t))^2 - (U_k(t))^2\right),\ k = n+1, n+2, \ldots$$

Let S is an integral manifold of the system (9) in $X \times Y \times T$. If any point $t^* \in [0, T_0]$ $(u(t^*), U(t^*), t^*) \in S$ of trajectory of this system has at least one common point on S this trajectory $(u(t, G), U(t, g), t) \in S$ belongs the integral manifold S totally.

If we assume in (9) that $\epsilon = 0$ than we have a degenerate system of the ordinary differential equations and a problem of singular perturbations

$$\dot{u} = f(u(t, \mu, \lambda, g_u), U(t), t),$$
$$0 = F(u(t, \mu, \lambda, g_u), U(t, \mu, \lambda), t); \tag{11}$$
$$u(0, \mu, \lambda, g_u) = g_u,$$

where the dimension of this system is less than the dimension of the system (9), since the relations $F(u(t, \mu, \lambda), U(t, \mu, \lambda), \lambda, t) = 0$ in the system (11) are the algebraic equations (not differential equations). Thus for the system (10) we can use limited number of the initial conditions then for system (9). Most natural for this case we can use the initial conditions $u(0, \mu, \lambda, g_u) = g_u$ for the system (11) and the initial conditions $U(0, \mu, \lambda, U_y) = g_U$ disregard otherwise we get the overdefined system. We can solve the system (11) if the equation $F(u(t, \mu, \lambda), U(t, \mu, \lambda), \mu, \lambda, t) = 0$ has roots. If it is possible to solve we can find a finite set or countable set of the roots $U_q(t, \mu, \lambda, g_u) = u_q(u(t, \mu, \lambda, g_u), t)$ where $q \in \mathbf{N}$. If the implicit function $F(u(t, \mu, \lambda), U(t, \mu, \lambda), \mu, \lambda, t) = 0$ has not simple structure we must investigate the question about the choice of roots. Hence we can use the roots $U_q(t, \mu, \lambda, g_u) = u_q(u(t, \mu, \lambda, g_u), t)$ $(q \in \mathbf{N})$ in (11) and solve the degenerate system

$$\dot{u_d} = f(u_d(t, \mu, \lambda, g_u), u_q(u_d(t, \mu, \lambda, g_u), t), \lambda, t); \tag{12}$$
$$U_d(0, \mu, \lambda, g_u) = g_u.$$

Since it is not assumed that the roots $U_q(t, \mu, \lambda, g_u) = u_q(u(t, \mu, \lambda, g_u), \mu, \lambda, t)$ satisfy the initial conditions of the Cauchy problem (9) ($U_q(0) \neq g_u$, $q \in \mathbf{N}$), the solutions $U(t, \mu, \lambda, g_U)$ (9) and $U_q(t, \mu, \lambda, g_u)$ do not close to each other at the initial moments of time $t > 0$. Also there is a very interesting question about behaviors of the solutions $u(t, \mu, \lambda, g_u)$ of the singular perturbed problem (9) and the solutions $u_d(t, \mu, \lambda, g_u)$ of the degenerate problem (11). When $t = 0$ we have $u(0, \mu, \lambda, g_u) = u_d(0, \mu, \lambda, g_u)$. Do these solutions close to each other when $t \in (0, T_0]$? The answer to this question depends on using roots $U_q(t, \mu, \lambda, g_u) = u_q(u(t, \mu, \lambda, g_u), t)$ and the initial conditions, which we apply for the systems (9) and (12).

TRUNCATION LARGE-SCALE QUEUEING MODEL AND NUMERICAL ANALYSIS

Using (8) we can rewrite system of differential equations order N in the form

$$\dot{u}_k(t) = \mu\left(u_{k+1}(t) - u_k(t)\right) + \lambda\left((u_{k-1}(t))^2 -,\right.$$
$$\left. -(u_k(t))^2,\ k = 0, 1, \ldots, n-1, \right. \tag{13}$$
$$\dot{u}_n(t) = \mu\left(U_{n+1}(t) - u_n(t)\right) + \lambda\left((u_{n-1}(t))^2 - (u_n(t))^2\right),$$
$$\epsilon\dot{U}_k(t) = \mu\left(U_{k+1}(t) - U_k(t)\right) + \lambda\left((U_{k-1}(t))^2 -\right.$$
$$\left. -(U_k(t))^2,\ k = n+1, n+1, \ldots, N. \right.$$

For this truncation system of ordinary differential equations of order N we can formulate Cauchy problem in the form

$$\dot{u}_k(t) = \mu\left(u_{k+1}(t) - u_k(t)\right) + \lambda\left((u_{k-1}(t))^2 -,\right.$$
$$\left. -(u_k(t))^2,\ k = 0, 1, \ldots, n-1, \right. \tag{14}$$
$$\dot{u}_n(t) = \mu\left(U_{n+1}(t) - u_n(t)\right) + \lambda\left((u_{n-1}(t))^2 - (u_n(t))^2\right),$$
$$\epsilon\dot{U}_k(t) = \mu\left(U_{k+1}(t) - U_k(t)\right) + \lambda\left((U_{k-1}(t))^2 -\right.$$
$$\left. -(U_k(t))^2,\ k = n+1, n+1, \ldots, N, \right.$$
$$u_k(0) = g_k \geq 0,\ k = 0, 1, 2, \ldots, n,$$
$$U_k(0) = g_k \geq 0,\ k = n+1, \ldots, N.$$

The numerical analysis was carried out using the adaptive step Runge-Kutta integration method, which is one of the most commonly used methods for the numerical solution of the singularly perturbed system of differential equations.

The numerical example is presented in the figure (see Fig. 1, 2) where $n = 7$, $N = 10$, $\lambda = 0.5$, $\mu = 1.1$, $g_0 = 1$ $g_k = 1 - 0.1k$, $k = \overline{0,9}$ and a small parameter $\epsilon = 0.1$ (Fig. 1) , $\epsilon = 0.01$ (Fig. 2), $\epsilon = 0.001$ (Fig. 3). In these numerical examples we can see the existence of steady state conditions for evolutions $u_i(t)$, $i = \overline{0,5}$ and quasi-periodic conditions with boundary layers for evolutions $u_i(t)$, $i = \overline{6,10}$.

The numerical simulation show that the solution of the singularly perturbed systems of differential equations have an area of rapid change of the function, which is usually located in the initial point of the problem. This area of rapid function change is called the area of the mathematical boundary layer. The thickness of the boundary layer depends on the value of a small parameter, and when the small parameter decreases, the thickness of the boundary layer decreases. The integration area is divided into external (outside the boundary layer) and internal (inside the boundary layer). The solution of the singularly perturbed equation is sought in the form of a solution suitable for the outer domain, which is then refined in the vicinity of the boundary point where the boundary layer is located. The numerical examples are shown the existence of steady state conditions for evolutions $u_i(t)$ and quasi-periodic conditions with boundary layers for evolutions $u_i(t)$.

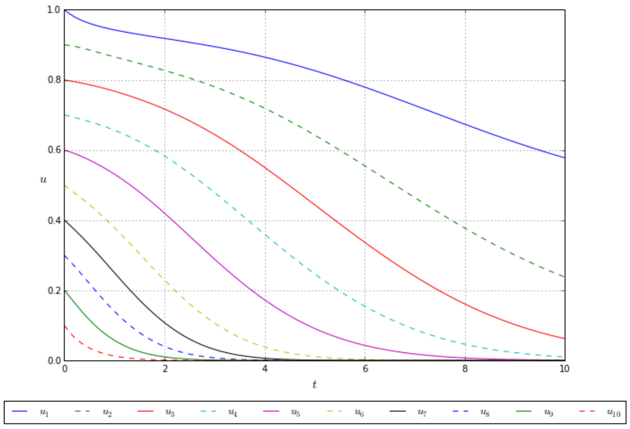

Fig. 1. Evolution analysis of u_k ($\epsilon = 0.1$).

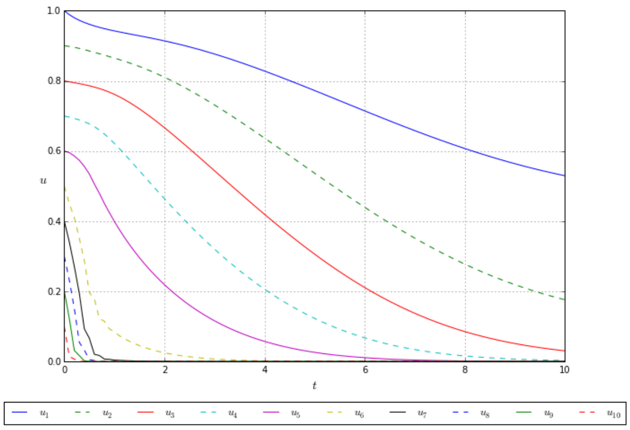

Fig. 2. Evolution analysis of u_k ($\epsilon = 0.01$).

CONCLUSIONS

We investigate the dynamics of large-scale queueing systems that consists of infinite number of servers with a Poisson input flow of requests of intensity $N\lambda$. Each request arriving to the system randomly selects two servers and this request is instantly sent to the one with the shorter queue. We suppose that service time has mean $1/\mu$ with exponential distribution. In this case a share $u_k(t)$ of the servers that have the queues lengths with not less than k can be described using an system of differential equations of infinite order. Tikhonov type Cauchy problem for this system with small parameter ϵ. Tikhonov type Cauchy problem for this system with small parameter ϵ and initial conditions is inves-

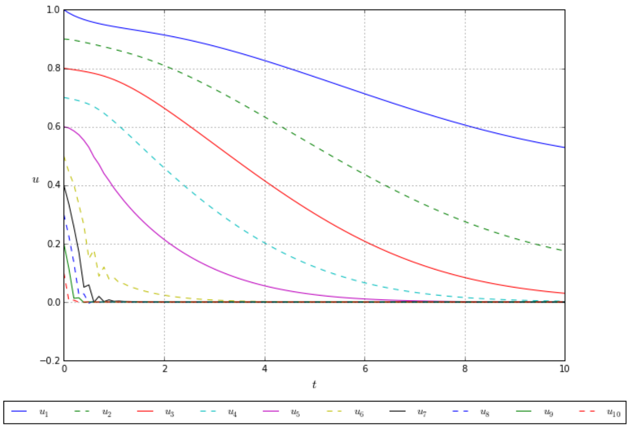

Fig. 3. Evolution analysis of u_k ($\epsilon = 0.001$).

tigated. We use the simulation methods for behavior solutions analysis with taking into account parameters λ, μ, ϵ. The numerical examples are shown the existence of steady state conditions for evolutions $u_i(t)$ and quasi-periodic conditions with boundary layers for evolutions $u_i(t)$.

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