DELIVERY RISK IN A SUPPLY CHAIN WITH A DOMINATING MEMBER: MODELING THE EFFECT OF THE INVENTORY POLICY

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ABSTRACT  
This paper simulates a three-member supply chain with delivery risk by a stochastic model based on the Prékopa-Ziermann model. The research aims to identify cases where the influencing power of a dominating supply chain member is needed to keep inventory at its optimum for the whole supply chain to maximize competitiveness. We set up a supply chain in which the production phases take place at locations geographically remote from each other. Thus, it is not only the timing of deliveries but also the duration of the distribution that needs to be managed as a random process. Introducing cost factors into the model allows us to get a more comprehensive picture of the profitability and apply our results for other economic and financial problems. We found that it is not only the behaviour of other chain members but also the level of margin provided that drives the inventory behaviour of the member firms. The overuse bargaining power of the dominating player to push down margins of other supply chain members might lead to increased risk and reduced competitiveness of the whole chain.

INTRODUCTION  
Nowadays we do not see firms competing with each other rather those different supply chains that fight for the customer. Compared to a single firm model supply chains have additional risk emerging (Bredell – Walters, 2017) one of which is that of delivery. Should any earlier chain member be late in delivery, the final seller may lose some profit due to not being able to serve the customer. Thus, the competitiveness of the supply chain depends also on the punctuality of deliveries between the members. To reduce this risk both emerging from the (partial) failure of delivery and unexpected delay of transportation, firms stick up some raw material inventory. Once the profit loss due to our own delivery failure and the cost of holding inventory is given, a profit maximising company can determine the optimum level of inventory. But what is optimal for a single firm may not be that for the whole supply chain. If there were a dominating member, it could push other members away from their individual optimum to enhance the competitiveness of the whole chain. This paper investigates when this would be necessary and what margin rates the chain has to provide so that the given member chooses the chain optimum as its inventory level.

LITERATURE REVIEW  
Inventory risk is covered from various aspects of the literature. Hsien-Jen Lin (2017) examined long-term cost when a connection between the inventory level and the production facility exists. He quantified the optimal level of the control parameters like the level of the inventory (m, M) to minimize the long-run total expected cost of production. Wang and Mersereau (2017) examined the effects of global news, the local and the internal events on demand forecasting capability and management efficiency.

Brányi et al. (2015) have surveyed the effects of competitiveness of the dominant company in the supply chain among Hungarian firms. They showed that the competitiveness of a company depends heavily on its size and turnover. The dominant, powerful firm in the supply chain (SC) is usually more competitive than its partners. A prominent company influences its partner's results in a given sector. This influence might be both positive and negative, but the SC only works successfully in the case of a positive impact. Enhancing SC competitiveness requires closer cooperation of the partners to promote the transmission of competitiveness across SC members. Sanchez-Rodrigues et al. (2010) showed that the main factors jeopardising the sustainability of transportations within SCs are: delivery constraints, delays, poor information, and insufficient SC integration. Our model assumes perfect information and focuses on the remaining three issues.

Zhao et al. (2013) underline that the main driver of supply delivery risk is the lack of SC integration. Internal cooperation could significantly reduce this uncertainty. An excellent review of earlier supply chain disruption models is offered by Bugert and Lasch (2018). They propose that new papers should put more focus on resilience and risk control gave us a special motivation.
THE PRÉKOPA-ZIEMANN MODEL

Our model is based on the Prékopa-Ziemann model (PZ-model) (Prékopa [2006]), which was designed for the ordering and delivery habits of the 1950s and 60s in Hungary: when the ordered quantity arrived at the customer in smaller units almost randomly in time (s), but the total order was delivered until a final date.

The initial assumptions of the PZ-model are as follows: r represents the amount of raw material used for production during one unit of time. Thus \( r^*T = Q \), where Q is the amount of raw material required to maintain production. The material arrives in \( n \) parts during the examined period \([0; T]\), but the timing of deliveries is random, chosen from the interval \([0, T]\) according to a uniform distribution independently. The model quantifies the optimum amount of initial stock, \( M(n, \varepsilon) \) by which a shortage of raw material can be avoided with probability \((1 - \varepsilon)\). Using \( Y_t \) for the amount of the delivered stock until the date of \( t \) and \( X_t \) for the actual inventory at the time \( t \) implicates the following formula:

\[
X_t = M + Y_t - r^*t
\]

where \( 0 \leq Y_t \) and \( Q = r^*T = Y_T \).

Ensuring the continuous consumption to be not disrupted with probability \((1 - \varepsilon)\) they minimized \( M \geq 0 \)

subject to \( \Pr(X_t \geq 0) = 1 - \varepsilon \)

\( \varepsilon \) is a continuous jump process which contains exactly \( n \) jumps in the interval \([0, T]\) at random times. The jumps are independent of each other, and their time distribution is evenly distributed over the simulation time horizon. The size of each jump can have a minimum: \( \alpha Q / n \), \( 0 \leq \alpha \leq 1 \).

If \( \alpha = 1 \), the magnitude of each \( Y_t \) is exactly \( Q/n \). In this case, the initial safety stock is

\[
M(n, \varepsilon) = r^*T \sqrt{\frac{\ln n}{2n}}
\]

which means it has to be \( \sqrt{\frac{\ln n}{2n}} \)-th of the consumption of the whole period \([0; T]\).

SUPPLY CHAIN WITH A DOMINATING MEMBER

In our modified simulation, we examined a SC as a critical supplier line of a multinational manufacturing company. There is one kind of raw material at every stage of the process, and the continuity of production depends on this crucial input. The figure below schematically shows the production process simulated by us.

![Figure 1. Structure of the assumed supply chain](Image)

The SC in our model has three levels: \( P_3 \) sells to final customer (market), \( P_2 \) is the leading supplier of \( P_3 \) and the main buyer of \( P_1 \). Each \( P_i \) has two warehouses: one for the input stock and one for the output stock. In this model, there is not any storage capacity limit, but the cost of storage will be considered.

The timing of deliveries was set based on a normal distribution around the predetermined delivery dates evenly distributed over the simulation time horizon. Compared to the PZ model, due to less random deliveries, significantly lower inventory levels can ensure continuous production. Given the characteristics of the normal distribution, a supply chain member (SCM) can most likely (99.73%) avoid the scarcity of the input by having stock enough for the delivery period plus three times the deviation of the delivery (\( S^* \)). We assume that this is the target level of availability and any inventory level below that increases the operational risk and decreases the competitiveness of the whole SC as the final customer may not be well served.

Compared to the Prékopa-Ziermann model, the lead time was integrated into the model for transportation between the individual production phases. For the sake of simplicity, we set its expected value to one day by a deviation of one. (The actual time need for delivery was the whole number nearest to the received random number.)

We also introduced the total cost of production (\( c_i \)) and warehousing (\( w_i \)), and sales price (\( p_i \)). All of them depend on the quantity of the total produced output (\( c_i \) and \( p_i \)) and of the average input (\( w_i \)) where \( i \) stands for the identifier of the firm. For to keep the model simple, \( w_i \) covers the warehousing cost of one unit input for the whole \([0; T]\) period. In this model, we ignored the economies of scale effect of the fixed costs, so the profit of a firm can be calculated by the formula:

\[
\Pi_i = (p_i - c_i)q_i - w_i^*m_i^*T
\]

where \( q_i \) is the total amount of output in \([0; T]\), and \( m \) is the average input stock for a day of the period, i.e. the average of the stock levels of \( t_j \), where \( j = 1, ..., n \) and \( 0 < t_1; t_2; ...; t_n \leq T \).

Depending on the input and output ratio, i.e. how much of the critical input is required for the production of one unit of the output, the profit might change. This ratio can be interpreted as well as the quantity of a single critical input, or the amount of the essential inputs. We assumed that \( w_i \) is equal for all input of a firm: the physical size and the storage conditions can be the same.

During the Monte Carlo simulation, we tested how the initial inventory level impacts the profit of each firm over the 3-year time horizon. We looked at how price over cost ratio affects inventory management, and whether the optimal inventory level of the given SCM would be different to that minimizing delivery risk across the whole SC. Should a deviation emerge, the dominating company might be forced to motivate the firm to change its inventory policy to maintain the competitiveness of the chain? Beside of using pure bargaining power, increasing the realised margin of the given SCM could also offer a solution.

Our assumptions for production in the simulation were the following. At \( P_1 \), three units of critical input (\( A \)) is
needed to produce one unit of B (the standard daily output was 8 units). P₂ requires two units of critical input (B) to produce one piece of product C (the standard daily output was 4 units), while P₃ needs two units of input (C) to create a product D (the standard daily output was 2). We set the expected period between deliveries to 30 days at each of the firms. (The standard deviation of this period length was either 5 or 10 days.)

Based on the results of Brányi (2006) we determined the prices and costs and the potential output so that the dominant firm has the highest turnover. We supposed that the dominant firm always has a starting inventory of S* to secure its production. To guarantee the total final SC output, it prefers all other SCMs to do the same, i.e. keeping an initial input inventory equal to their S*. (Note that the absolute amount of S* may be different for the individual SCMs due to the different conversion ratios of their manufacturing process. It is the level of operational security that S* provides that is the same.) The prices of A, B, C, D are 5, 50, 500, and 4000 respectively, the warehousing cost is 10 at each firm in our simulation. The production of B, C, D needs respectively 3 of A, 2 of B, 2 of C respectively as input. Having the highest turnover, the firms produce the following amount of output in each case (i.e. if P₁ or P₂ is dominant, it must have customers outside of the supply chain as well to get the highest turnover).

<table>
<thead>
<tr>
<th>Firm</th>
<th>P₁ prod of B</th>
<th>P₂ prod of C</th>
<th>P₃ prod of D</th>
</tr>
</thead>
<tbody>
<tr>
<td>P₁</td>
<td>20</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>P₂</td>
<td>40</td>
<td>20</td>
<td>2</td>
</tr>
<tr>
<td>P₃</td>
<td>8</td>
<td>4</td>
<td>2</td>
</tr>
</tbody>
</table>

Figure 1: The production of the firms

In our examination, we simulated 1000 times the results of the production of the three firms. In each case in each Monte Carlo simulation, independent of the initial inventory stock the average, the volatility and the relative volatility reaches a stable level after about 500 simulations. Only the cases of higher production (i.e. 20 and 40) needed the simulation of 1000 to stabilize the average and the relative volatility.

The dominant firm at the end of SC

First, we assumed that it is the firm at the end of the SC that is dominant as being the exclusive marketer of the others just like it is typical in case of car manufacturing. However, its production security depends on the inventory level of the other SCMs. Obviously when all SCMs have an initial input inventory of S*, the dominant firm at the end of the supply chain can reach its maximum profit. However, how motivated are the suppliers to keep an inventory like that?

We considered four cases:

- C₁: every firm has S* input stock
- C₂: P₂, P₃ have S* input stock
- C₃: P₁, P₃ have S* input stock
- C₄: only P₁ has S* input stock.

The other firms which do not set the input stock at S*, have the inventory level enough for the delivery period of 30 days (S). We compared the two different level of the input inventory stock (S and S*) with several aspects. Moreover, we also examined the effect of the profit-margin (defined by equation 5) on the willingness to increase the input inventory by different deviations.

\[ m_i = \frac{(P_i - c_i)}{c_i} = \frac{E_i}{c_i} - 1 \quad i = B, C, D \] (5)

In the first case, the standard deviation of the delivery date reached only one-sixth (5 days) of the average lead time, while in the second case it was one-third (10 days) of it. After the Monte Carlo simulation of 1000 thousand cases, we found the following:

- if only the dominant P₁ has S* initial inventory (C₄), is worth also for P₁ to have its starting stock to push up to S* above a margin level of 10.47%. If also P₂ has an inventory level of S* at initiation (C₂), the minimum P₁ margin to increase the storage voluntarily to S* is 26.66%.
- In Case 4, it pays off for P₂ to increase the inventory to S* at a margin level higher than 15.98%, otherwise, if all other SCMs keep S* (C₃), even a margin level of 6.67% is enough for it.

In the case of higher deviation (one-third of the lead time), we found fewer differences among the willingness of increasing the inventory level:

- for P₁, it paid off to increase the inventory in C₂ even at a margin of 39.03% (low std. dev.: 26.66%). However, in the previously favourable case (C₄), the breakeven level increased from 10.47% to 49.93%.
- in the case of P₂, the change of the required margin was bigger: from 15.98% down to 1.07% for C₄ and from 6.67% up to 9.83% for C₂. Note, that the direction of the change was the opposite of what we saw in case of P₁.

![Graph: P₁ and P₂ break even margins to set inventory at the security level if the dominant firm at the end of SC](image)

Practically, if the uncertainty of the delivery decreases then other influencing tools might be required for getting P₁ (P₂) to raise the input inventory to the level of S* in the case of C₄ (C₃).

The dominant firm in the middle of SC

Next, we examine the case when P₂ is the dominant firm. Because P₂ is both a buyer and a supplier in the SC, it should also have customers not belonging to the given SC to have the highest turnover as required by the definition of the dominant firm in this model. In our simulation, P₁...
produces only the amount needed for the production of $P_2$, but $P_2$ produces five times more than the usage of product $C$ at $P_1$.

In case $p/c=2$, the highest average profit can be reached at each firm if their inventory level at time 0 is enough for the periods until next delivery plus three times the deviation of the delivery date ($S^*$). Theoretically, high warehousing costs may refute this statement by leading to higher profit with no extra inventory for possible delays, but that calls for a cost structure that seems extreme.

In the simulation, the considered cases were as follows:

- **CC$_1$**: each firm has an input inventory of $S^*$
- **CC$_2$**: $P_1$ and $P_2$ have $S^*$ input inventory
- **CC$_3$**: $P_1$ and $P_3$ have $S^*$ input inventory
- **CC$_4$**: only $P_2$ has $S^*$ input inventory.

In each case, the firms not having the level $S^*$ have inventory level ($S$) enough only for the period until the next delivery. Based on our results, both $P_1$ and $P_2$ have a positive profit if their margin is above 15.75% and 3.13%, respectively. Performing the analysis in the previous sub-section, we have received the following results in the case of the lower deviation (one-sixth of the lead time):

- For $P_1$, it is already worth to increase initial inventory at a margin level higher than 29.23% if only the dominant $P_2$ has $S^*$ ($CC_2$). If also $P_1$ has an inventory level of $S^*$ ($CC_3$), $P_1$ is best off when also having an inventory of $S^*$ at a margin of 15.93% or above.

- For $P_3$, it is only worth to keep an initial inventory of $S^*$ in case of $CC_4$ if the margin reaches than 1.04%. For the case $CC_2$, a margin level of 0.73% is needed. Practically, at any margin level, it is worth for $P_3$ to set the beginning inventory level at $S^*$. For example, in our model, setting the margin level between 1.55% and 3.13% $P_3$ will realize a negative profit in some cases, but all of them become profitable above that level.

In the case of a higher deviation (one-third of the lead time), contrary to our findings with a dominant $P_3$, $P_1$ is in a more difficult situation. It must have a higher margin to benefit from the higher inventory. After our calculation, $P_1$ would have $S^*$ inventory (moving from $CC_2$ to $CC_4$) if the margin on B is more than 37%. Similarly, $P_1$ prefers $CC_1$ for $CC_3$ if the margin ($p/c$) is at almost 32%.

There is only a slight difference between the required $P_1$ margin rates between low and high deviation level cases. If the deliveries become more uncertain, $P_1$ benefits from increasing the starting inventory in the case of $CC_2$ and $CC_4$ if the margin level is higher than 0.35% and 0.43% respectively. However, these are such low margin limits, that it can be stated that the firm at the end of the SC would always benefit from holding $S^*$ of input inventory.

In our model, for some cases with a deviation of 10 days, the profit of $P_3$ was negative under a margin of 3.49% (BreakEven levels are the following: $CC_2$ 3.49%; $CC_3$ 3.37%; $CC_4$ 1.55%; $CC_1$ 1.06%).

**The dominant firm at the beginning of the SC**

Being a dominant firm, $P_1$ has to face a huge demand to serve the SC and fulfill the turnover condition. In this case, we see the results being quite similar to the case when the dominant firm is at the end of the SC.

In this simulation, the considered cases were as follows (using the markers of previous subchapters):

- **C$_1$**: every firm has $S^*$ input stock
- **C$_2$**: $P_1$, $P_2$ have $S^*$ input stock
- **C$_3$**: $P_1$, $P_2$ have $S^*$ input inventory
- **A**: only $P_1$ has $S^*$ input inventory.

In our simulation, $P_2$ and $P_3$ have a positive profit if their margin is above 15.01% and 3.16%, respectively. Scanning their willingness to hold inventory of $S^*$ in the case of the lower deviation (dev=5 days) we found the follows:

- If all of other SCMs hold an inventory of $S^*$ then $P_2$ would have it as well at all the acceptable margin levels.
- If only $P_1$ has an inventory of $S^*$, $P_2$ increases its storage only if its margin would rise above 30.19%.
- $P_1$ has in any case higher profit when maintaining an initial inventory level of $S^*$.

Higher insecurity in the transportation eventuates the lowest profit at the firms which means that higher markup is required for not operating at a loss. $P_2$ needs a margin of almost 26%, while for $P_1$ a margin of 3.52% would be enough. $P_1$ still prefers to adopt the $S^*$ as the desired inventory level at time 0. As for $P_2$, the interval of the margin narrows, a margin below 7.28% results in maintaining the lowest inventory, but a margin above 23.39% forces the firm to set the level at $S^*$.

Figure 3 shows the narrowing of the margin gap. In Figure 3, we can ascertain that the increase of the deviation of the delivery decreases the resistance of the $P_2$ in raising the level of input storage when only the dominant firm has $S^*$ inventory initially. But if both firms ($P_1$ and $P_2$) have an input inventory of $S^*$ higher deviation stops being a motivator as the added security of $P_1$ increased inventory removes a part of the risk $P_2$ faced originally.
Figure 4: P2 and P3 breakeven margins to set inventory at the security level if the dominant firm at the beginning of SC

CONCLUSIONS

We have examined three different cases depending on the position of the dominant SCM. For each of them, we clarified which situations would automatically lead to the maximisation of availability of the SC and where is some kind of additional motivation needed so that the strategy of minimising delivery risk and that of maximising profit does not diverge. The results of our simulations are summarized in Table 1.

Table 1: The minimum margins to hold $S^*$

<table>
<thead>
<tr>
<th>Analysed firm</th>
<th>Dominant</th>
<th>All other SCMs</th>
<th>Dominant</th>
<th>All other SCMs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Firms with $S^*$ inventory</td>
<td>P1</td>
<td>P2</td>
<td>P1</td>
<td>P2</td>
</tr>
<tr>
<td>Minimum margin to hold $S^*$ st. dev. = 5 days</td>
<td>Dominant only</td>
<td>10.47%</td>
<td>26.66%</td>
<td>15.98%</td>
</tr>
<tr>
<td>Minimum margin to hold $S^*$ st. dev. = 10 days</td>
<td>Dominant only</td>
<td>49.93%</td>
<td>39.03%</td>
<td>1.07%</td>
</tr>
<tr>
<td>Firms with $S^*$ inventory</td>
<td>Dominant</td>
<td>All other SCMs</td>
<td>Dominant</td>
<td>All other SCMs</td>
</tr>
<tr>
<td>Minimum margin to hold $S^*$ st. dev. = 5 days</td>
<td>Dominant only</td>
<td>29.23%</td>
<td>15.93%</td>
<td>1.04%</td>
</tr>
<tr>
<td>Minimum margin to hold $S^*$ st. dev. = 10 days</td>
<td>Dominant only</td>
<td>37.06%</td>
<td>31.91%</td>
<td>0.35%</td>
</tr>
<tr>
<td>Firms with $S^*$ inventory</td>
<td>Dominant</td>
<td>All other SCMs</td>
<td>Dominant</td>
<td>All other SCMs</td>
</tr>
<tr>
<td>Minimum margin to hold $S^*$ st. dev. = 5 days</td>
<td>Dominant only</td>
<td>30.19%</td>
<td>5.88%</td>
<td>all</td>
</tr>
<tr>
<td>Minimum margin to hold $S^*$ st. dev. = 10 days</td>
<td>Dominant only</td>
<td>17.28%</td>
<td>23.39%</td>
<td>all</td>
</tr>
</tbody>
</table>

We found that if the dominant firm is at the end of the chain, the effect of the uncertainty of delivery depends on the position of the SCM the chain. If the SCM is next to the dominant player, then it inclines to increase the inventory level if both of its neighbours in the SC have the optimal $S^*$ storage. If the firm is the supplier of the whole SC (P1), then it prefers to set the inventory level at $S^*$ if only the dominant firm has the optimal $S^*$ initial stock. So, we can set a potential process of increasing the inventory stock from level S to level $S^*$ by the less deviation case: P3, P1, P2. The minimum margins in this process are 10.47% for P1 and 6.67% for P2 after P2 set its stock level at $S^*$. However, the sequence to increase the stock would be P3, P2, P1; the margins are 15.98% for P2 and 26.66% for P1. Higher required margin levels can make increasing stock levels more uncertain. If deliveries are more uncertain (st. dev. =10), then the firms are forced to increase the stocks, and it is more likely to set the stock at level $S^*$ in the order of P3, P2, P1.

However, if the dominant firm stands in the middle of the supply chain, only the subsequent firms benefit from higher stock if the delivery becomes more uncertain, but for the preceding firms, the favourable scale of inventory build-up narrows. Thus, the dominant firm might expect that P3 will increase the stock to strengthen the progress of the SC. In contrast, in the relationship with its supplier, the dominant firm has to intervene as P1 is always better off when keeping an initial inventory less than $S^*$. Thus, P1 may need compensation if pushed to hold $S^*$ amount of inventory.

In the case of the cooperation of the three firms (i.e. negotiation of increasing the inventory stock), they can afford lower margins compared to their own decisions. If all of them increase the stock at the same time that the minimum margins are 15.93% (P1) and 0.73% (P3), however in a consecutive decision series the margins needed to increase the stocks are 15.93% and 1.04% respectively or 29.23% and 0.73% respectively depending on which one increase the inventory first. Almost the same is true in the case of more uncertain deliveries, only at P3 is a slight difference in the minimum margins but the margin to get any profit is higher of them. If the dominant firm is the first member of the SC, the profit maximising SCM at the end of the chain sets its inventory automatically for the optimal $S^*$ level. In this case, the firm in the middle benefits from doing that as well only if its margin level is higher than 30.19%. This means that not providing enough margin to given SCM may lead to an increased delivery risk in the SC and it could jeopardise the competitiveness of the whole chain. If they cooperate in the case of the lower deviation than much smaller margin is needed. Perhaps if P2 is aware of the increase done by P3 after that P2 increase its stock as well. In the case of more uncertain deliveries there is a little conflict of interest between P1 and P3; therefore perhaps P3 is interested in forcing P2 to increase its input stock.
REFERENCES


AUTHOR BIOGRAPHIES

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