

# USING THE CMA EVOLUTION STRATEGY FOR LOCATING SUBMARINE GROUNDWATER DISCHARGE

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## KEYWORDS

Covariance matrix adaptation evolution strategy, Autonomous underwater vehicles, Submarine groundwater discharge, Population-based search.

## ABSTRACT

For effective localisation of a search target by a swarm of Autonomous Underwater Vehicles (AUVs), a suitable cooperative search strategy should be utilised. Various aspects of the search task should be taken into account when selecting a search strategy. The nature of the search environment, the search target and the search agents should be considered. The Covariance Matrix Adaption Evolution Strategy (CMA-ES) is a well-known search strategy that proves its success in solving different continuous optimisation problems. This paper investigates utilising the CMA-ES to locate a Submarine Groundwater Discharge (SGD) using the temperature of water as a tracer. The impact of introducing some of the constraints, which are imposed by the search task, on the CMA-ES performance are studied. The influence of the number of the AUVs and their energy capacities on the search performance is investigated. The effect of the resolution of the temperature sensors together with the localisation and the navigation problems on the search behaviour are explored. The results show that these constraints have varying degrees of impact on the performance of the search strategy.

## INTRODUCTION

Technological advances in Autonomous Underwater Vehicles (AUVs) have opened the doors to explore and access areas previously considered inaccessible (Bhat & Stenius, 2018). Different types of AUVs have been developed and used in different applications (Paull, et al., 2014).

Searching is an important class of AUVs applications. AUVs can be used, for example, to detect mines, locate groundwater discharge sources, search for harmful dumped waste and lost ship containers (Zielinski, et al., 2009).

A swarm of AUVs can be used to explore a predefined search area to locate mobile or stationary targets (Nolle, 2015). A huge number of search algorithms has been successfully applied to solve real-world problems. However, selecting a suitable search algorithm to guide

an AUV towards a point of interest is not an easy task. Different aspects should be considered when selecting or developing a cooperative search strategy for AUVs. Sensors' quality, energy constraints, localisation errors, navigation capabilities and communication quality are among the factors that influence the performance of a swarm of AUVs (Tholen, et al., 2017).

A cooperative search strategy can be used to guide a group of AUVs, as search agents, towards the most promising region. This search strategy should have the capability to analyse the search information gathered by the AUVs to suggest the best path to the target. Different population-based search algorithms have the capability of efficient utilisation of search information to locate a global optimum. However, using AUVs as search agents can affect the behaviour of these algorithms.

The efficiency of a population-based search algorithm depends on its ability to utilise the shared search experience to capture a global view of the search problem. Building a global view of the search problem depends on the population size, which is determined by the number of the available AUVs. The captured global view also depends on the quality of the shared information. The shared information includes the location information and the target information (i.e. the tracer information of the target).

The quality of the search information depends on the quality of the sensors that collect the location information and the tracer information. The quality of the sensor information depends on the accuracy and the resolution of the sensor. It also depends on the sampling rate and the response time together with the speed of the AUV. To acquire search information with a specific quality, sensors can impose constraints on the acquiring rate of search information and on the speed of the AUVs.

AUVs, as real-time search agents, impose other constraints on the search algorithm. There are restrictions in terms of their physical movement and their search range. To evaluate a search decision for exploring a search region, an AUV should move to that region and collect the requested search information. Such an evaluation can take some time depending on the speed of the AUV and the distance to that region. In addition, the search information can be changed before collecting the requested information. This change can be due to the dynamic nature of both the search algorithm and the phenomenon. Furthermore, the decision for a detailed

exploration of the current search region should be taken immediately and should not be delayed for collecting more search information. This delay can waste energy of the AUVs by revisiting some locations more than once. It can further restrict the ability of the AUVs and the search algorithm to explore the whole search space due to the limited energy capacities of the AUVs.

Efficient utilisation of the energy of an AUV is essential for search algorithms to locate a target. The energy consumption can be minimised through avoiding exploring unpromising search regions. It can also be reduced by decreasing sharp changes in the AUVs directions. A search algorithm that creates smooth search paths for AUVs can help in extending the search range of the AUVs.

AUVs as search agents when navigate to explore the search space are prone to localisation and navigation errors (Paull, et al., 2014). There are different ways to alleviate localisation and navigation problems. However, an effective search strategy should consider these errors. Special attention should be paid for localisation errors. The localisation errors can influence the search behaviour through effecting the accuracy of the search information. In addition, the aim of the search process is to define the exact location of the global optimum with an acceptable accuracy.

Evolution Strategies (ESs) (Schwefel, 1981) are black box population-based optimisation techniques. They have been studied for decades, leading to the many variants. The Matrix Adaption Evolution Strategy (CMA-ES) (Hansen & Ostermeier, 1996) is a well-known variant of ESs. It was originally designed for small population sizes and has been successfully applied to a considerable number of real world continuous domain problems (van Rijn, et al., 2017).

In this paper, the possibilities of utilising the CMA-ES as a search strategy for a swarm of AUVs to locate a Submarine Groundwater Discharge (SGD) using the temperature of water as a tracer is investigated. The robustness of the performance of the CMA-ES against some constrains of the task of locating SGDs is evaluated. These constrains include the number of AUVs and their energy capacity. They also include the resolution of the sensors. Furthermore, the impact of the localisation and the navigation problems on the algorithm behaviour is studied.

This paper is organised as follows. A very short introduction into SGDs is given in the second section. The CMA-ES algorithm, as described in (Hansen, 2006), is reviewed in the third section. The paper concludes by presenting and discussing the simulations' results.

## SUBMARINE GROUNDWATER DISCHARGE

Submarine Groundwater Discharge (SGD) is the flow of water across the sea floor. Groundwater discharge may be pure groundwater entering the sea from a coastal aquifer, or it may be recirculated seawater, or some combination of the two (Burnett, et al., 2006).

SGDs connect the land and ocean in the global water cycle (Taniguchi, et al., 2019). They can have a significant influence on the costal environment (Taniguchi, et al., 2019). They are important sources of nutrients, dissolved inorganic carbon or trace metals to coastal waters. This continuous loading of nutrients and trace metals alters the water quality and may lead to environmental degradation of coastal regions (Prakash, et al., 2018).

Due to their impact on coastal regions, locating SGDs and tracking their dispersal is an important as well as challenging task. Natural tracers can be used to locate and quantify SGDs. Natural tracers, other than temperature, include nutrients, radioisotopes, salinity, and trace elements such as silica, barium, methane and others (Ray & Dogan, 2016). By measuring the changes in the natural tracers, SGDs can be located and quantified.

The contrasts between groundwater and sea surface temperatures can be used to locate SGDs. Detecting such contrast in temperature can be done using simple temperature sensors. The temperature difference can also modify the colour and the transparency of seawater. These changes in colour and transparency enable identifying SGDs form aerial photographs or satellite image. Temperature as tracer can be used to identify shallow SGDs and SGDs with high flow rates. However, it might not be suitable for detecting deep SGDs or SGDs with low discharge flow due to the high heat capacity of the seawater (Kelly, et al., 2013).

Different search strategies have been used to guide a swarm of AUVs to locate an SGD using the temperature as a tracer (El-Mihoub, et al., 2019; Tholen, et al., 2018; Tholen, et al., 2017). The reported results of applying these strategies show that the search task's constrains influence their performances. To gain insight into the relations between these constrains and the search performance, an investigation in applying a variant of the CMA-ES algorithm as a search strategy for a swarm of AUVs is conducted.

## THE CMA-EVOLUTION STRATEGY

The CMA-ES algorithm optimises a fitness function  $f: x \in \mathbb{R}^n \rightarrow f(x) \in \mathbb{R}$  by sampling a population of  $\lambda$  solutions (individuals) from a multi-variate normal distribution. It selects the best  $\mu$  solutions (parents) out of the  $\lambda$  individuals to adaptively estimate the local covariance matrix of the objective function.

At generation  $g$ , the CMA-ES samples  $\lambda$  individuals according to

$$x_k^{g+1} \sim \mathcal{N}\left(m^{(g)}, (\sigma^{(g)})^2 C^{(g)}\right) \sim m^{(g)} + \sigma^{(g)} \mathcal{N}(0, C^{(g)}), \quad (1)$$

$$k = 1, \dots, \lambda$$

where

$\sim$  denotes the same distribution on the left and right side.

$\mathcal{N}(0, C^{(g)})$  is the multi-variate normal distribution with zero mean and a covariance of  $C^{(g)}$ .

$x_k^{g+1}$  is  $k$ -th offspring of generation  $g + 1$ .

$m^{(g)}$  is the mean value of the distribution at generation  $g$ .

$\sigma^{(g)}$  is the overall standard deviation, step size, at generation  $g$ .

$C^{(g)}$  is the covariance matrix at generation  $g$ .

$\lambda \geq 2$ , is the population size or the sample size.

These  $\lambda$  individuals are evaluated and ranked. The mean  $m^{(g+1)}$  of the distribution is updated and set to the weighted sum of the best  $\mu$  individuals.

$$m^{(g+1)} = \sum_{i=1}^{\mu} w_i x_{i:\lambda}^{(g+1)} \quad (2)$$

where

$w_i > 0$  for  $i = 1, \dots, \mu$

$\sum_{i=1}^{\mu} w_i = 1$ ,

$x_{i:\lambda}^{(g+1)}$  is the  $i$ -th best ranked individual out of  $\lambda$  individuals of  $x^{(g+1)}$ .

The CMA-ES updates the covariance matrix through considering the evolution path,  $p_c$ . The evolution path is the search path the strategy takes over a number of generation steps. It can be expressed as a sum of consecutive steps of the mean. The evolution path can be constructed through exponential smoothing. Starting with  $p_c^{(0)} = 0$ ,  $p_c^{(g+1)}$  can be calculated.

$$p_c^{(g+1)} = (1 - c_c)p_c^{(g)} + \sqrt{c_c(2 - c_c)\mu_{eff}} \frac{m^{(g+1)} - m^{(g)}}{\sigma^{(g)}} \quad (3)$$

where

$p_c^{(g)}$  is the evolution path at generation  $g$ .

$c_c \leq 1$ , is the learning rate for cumulation update of the covariance matrix.

$\mu_{eff} = \frac{1}{\sum_{i=1}^{\mu} w_i^2}$ , is the variance effective selection mass.

The evolution path is used for updating the covariance matrix. The rank-one and rank- $\mu$  updates of the CMA can be combined in a single formula with  $\mu_{cov}$  to determine their relative weighting.

$$C^{(g+1)} = (1 - c_{cov})C^{(g)} + \frac{c_{cov}}{\mu_{cov}} U_{rank-one} + c_{cov} \left(1 - \frac{1}{\mu_{cov}}\right) U_{rank-\mu} \quad (4)$$

where

$U_{rank-one} = p_c^{(g+1)} p_c^{(g+1)T}$ , is the rank-one update of the covariance matrix

$U_{rank-\mu} = \sum_{i=1}^{\mu} w_i \left(\frac{x_{i:\lambda}^{(g+1)} - m^{(g)}}{\sigma^{(g)}}\right) \left(\frac{x_{i:\lambda}^{(g+1)} - m^{(g)}}{\sigma^{(g)}}\right)^T$ , is the rank- $\mu$  update of the covariance matrix.

$\mu_{cov} \geq 1$ , is a parameter for weighting between rank-one and rank- $\mu$  update.

$c_{cov} \approx \min(\mu_{cov}, \mu_{eff}, n^2)/n^2$ , is the learning rate for the covariance matrix update.

The CMA-ES uses a step size control for better estimation of the step size,  $\sigma^g$ . The cumulative path length control adapts the step size utilising the concept of evolution path. It compares the length of the evolution path with the expected length under random selection, i.e.  $E \|\mathcal{N}(0, I)\|$ . The cumulative path length control increases the step size if the evolution path is longer than expected, which indicates single steps are pointing to similar directions. On the other hand, the step size is

decreased, if the evolution path is shorter than expected, which indicates single steps cancel each other.

To make the length of evolution path independent of its direction for estimating the step size a conjugate evolution path  $p_\sigma$  can be calculated.

$$p_\sigma^{(g+1)} = (1 - c_\sigma)p_\sigma^{(g)} + \sqrt{c_\sigma(2 - c_\sigma)\mu_{eff}} C^{(g)-\frac{1}{2}} \frac{m^{(g+1)} - m^{(g)}}{\sigma^{(g)}} \quad (5)$$

where

$c_\sigma$  is the learning rate for the step size update.

The conjugate evolution path is used to update the step size,  $\sigma^{g+1}$  based on the current step size.

$$\sigma^{g+1} = \sigma^g \times \exp\left(\frac{c_\sigma}{d_\sigma} \left(\frac{\|p_\sigma\|}{E \|\mathcal{N}(0, I)\|} - 1\right)\right). \quad (6)$$

where

$d_\sigma$  is a damping parameter, which scales the change magnitude of the step size.

In the CMA-ES algorithm, there is a number of strategy parameters that control, separately, change rates of the mean  $m^g$ , the covariance matrix,  $C^g$ , and the step size,  $\sigma^g$ . However, default values for these strategy parameters have been defined and are applicable to a range of optimisation problems (Hansen, 2006).

The success of CMA-ES algorithms is due to estimating the parameters of the mutation distribution based on a set of selected steps not on a set of points (Hansen & Ostermeier, 1996). The use of cumulative path length control also enables adapting nearly optimal step sizes (Hansen, 2006). This improves convergence speed and global search capabilities at the same time.

## SIMULATION AND RESULTS

A set of experiments was conducted using CMA-ES to locate the SGD, which has the highest discharge rate in an intermedia sized area with two SGDs. The aim of these experiments is to investigate the effect of some constrains, which are imposed by the search task, on the CM-ES behaviour. The strategy parameters of the CMA-ES algorithm were set to the default values as defined in (Hansen, 2006). The experiments were conducted with the assumption that AUVs can travel to any point in the search space in zero time and without any communication problems.

The problem of locating SGDs in a marine environment with the dimensions of 400m x 400m is simulated by a two-dimensional search space. In this space, two SGDs are located randomly. A Gaussian shape with maximum temperature at its centre is used to represent an SGD.

The average temperature of the water is set to 30 °C. The temperature of the centre of the SGD with the highest rate is set to 24 °C and the temperature of the centre of the second SGD was set to 26 °C (Akawwi, 2006). The radiuses of the basin of SGDs are selected randomly in the range from 10 to 20 m. The plume areas with these radiuses can be produced by SGDs with flow rates in the range from 0.00433 m<sup>3</sup>/s to 0.01524 m<sup>3</sup>/s (Kelly, et al., 2013).

The performance criterion in the simulations is the accuracy of locating the global SGD. The SGD with the

centre of highest temperature is the global SGD. The distance between the best-found location by an algorithm and the exact location of the global SGD's centre was defined as the error of that algorithm.

Each experiment was repeated for 100 times using the mentioned above search environment. The experiments' results were used to extract the cumulative distribution of the errors of each algorithm in each experiment. This cumulative distribution is used to estimate the probability of an algorithm to locate the global optimum with less than or equal to a specific error value.

Each experiment was conducted with a different number of AUVs, to study the combined effect of each constrain and the population size on the performance. The number of AUVs was set to  $\lambda_{min}$ ,  $2.5\lambda_{min}$ ,  $5\lambda_{min}$  and  $10\lambda_{min}$ .  $\lambda_{min}$  is the default value for the population size strategy parameter.  $\lambda_{min} = 4 + \lceil 3 \ln(n) \rceil$ , where  $n$  is the dimension of the optimisation problem (Hansen, 2006).

### Energy Capacity

The aim of the first set of experiments is to study the influence of the energy capacity of the AUV on the performance of the CMA-ES.

In literature, the number of function evaluations is usually used as a termination criterion when evaluating the performance of optimisation algorithms. However, for the task of locating SGDs using AUVs, energy consumption is a more realistic termination criterion. The cost of function evaluations, which is sensing the water's temperature, can be ignored. The goal of the swarm of AUVs is to locate the target before consuming their energy. With the goal of studying the effect of the number of AUVs and their energy capacity on the performance of the CMA-ES algorithm, the stopping criteria for search is consuming the energy stored on the AUVs.

The energy capacity of the AUVs can be translated in terms of meters travelled by the AUV. The energy

capacity of the AUV used in this research is sufficient for travelling a distance of 4,500 meter with an average speed of 1 m/s (Tholen, et al., 2018). This capacity was used as a reference in these experiments. The experiments were conducted for AUVs with 2250 meter, 4500 meter, 9000 meter, and unlimited energy capacities. The cost of changing the direction by an angle of  $180^\circ$  was assumed to be equivalent to travelling 4 meters (El-Mihoub, et al., 2019).

The experiments were conducted with the assumption that the AUVs estimate their location without errors, navigate with zero navigation error, and are using ideal sensors.

Figure 1 shows the performance of the CMA-ES algorithm with different number of AUVs and with different energy capacities. The figure shows that even with a population size of 60 AUVs and without any constrains, the CMA-ES is only able to find the global optimum with an accuracy of less than 5 meter in about 70% of the experiments. It also demonstrates as expected that the decrease in the energy capacities of the AUVs degrade the algorithm performance. It also shows that as the capacity decreases, the difference in the performance between the algorithms with 60, 30 and 15 AUVs decreases. In other words, there is a minimum of energy requirements for effective cooperation regardless of the number of cooperating AUVs. The figure shows that increasing the energy capacity and increasing the number of AUVs does not guarantee locating the global optimum even with an off the shelf state-of-the-art search algorithm. Another set of experiments has been done using the genetic algorithm optimisation tool of MATLAB to solve this problem. The results, which are not shown here, demonstrate that the probability of locating the global optimum with an accuracy of less than 5 meter is less than 0.6.

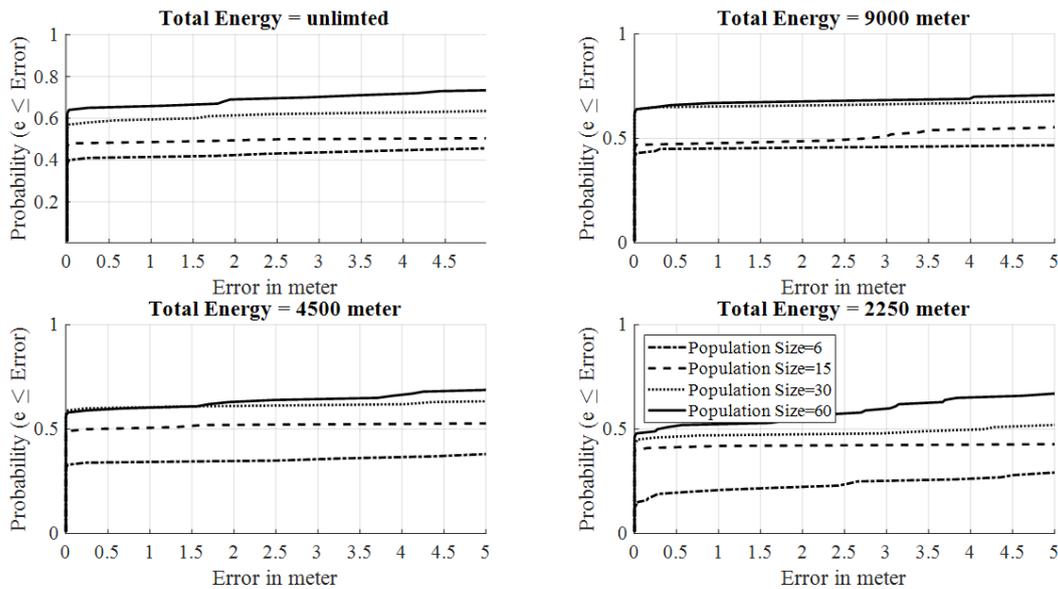


Figure 1: The change in the performance for different population size and different energy capacities

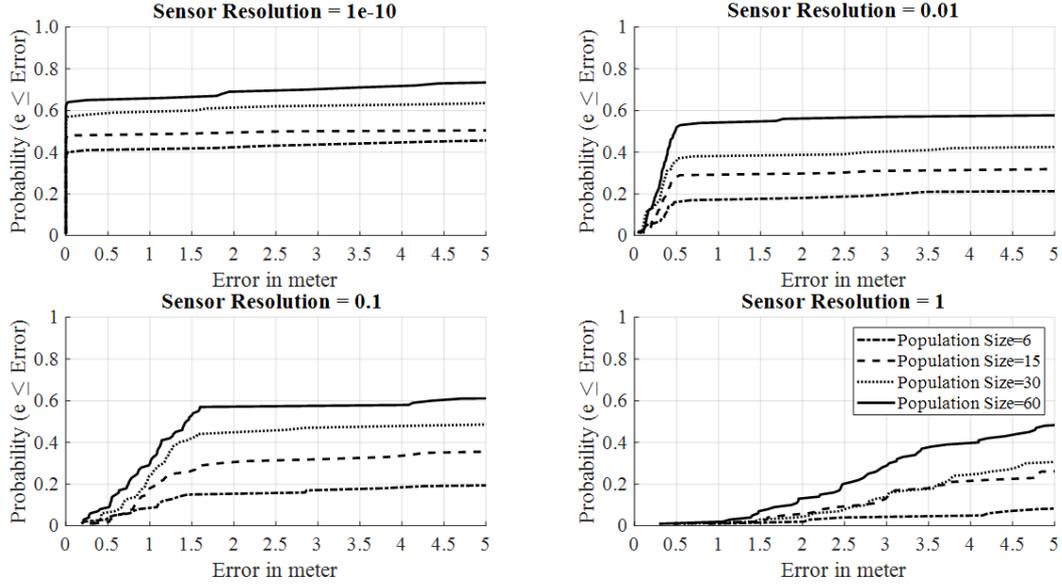


Figure 2: The Influence of the Sensor Resolution on the Performance

### Sensor Resolution

Search algorithms evaluate the quality of a search region for further detailed exploration based on the quality of the samples of the region. The quality of these samples depends on the feature used to trace the target and the quality of the sensor for quantifying this feature. For locating SGDs in this paper, temperature is used as a tracer. The relation between the location of an SGD and the current location depends to some extent on the difference in the temperature. This relation is the objective function of the search algorithm. It is the only mean for differentiating the quality of the sampled locations.

The search algorithm uses the readings of the temperature sensor at selected locations as the objective value of these locations. The actual objective function used by the search algorithm depends on the details of the sensor. Sensors can modify the original relation between the SGD location and the temperature. Sensors can introduce some noise in mapping locations to temperature depending on their accuracy. Furthermore, the response time of the sensor can introduce errors in this mapping. Instead of optimising the original, response time can lead to optimising  $f: x \in \mathbb{R}^n \rightarrow f(x - \Delta_r) \in \mathbb{R}$ ,  $\Delta_r$  is the accumulated effect of the response time. The sensor resolution, which defines the smallest measurement a sensor can reliably indicate, can transform the objective function into a staircase function.

In this section, the influence of the sensor resolution on the performance of the CMA-ES is evaluated. The experiments were conducted for sensors with resolutions of  $1e-10$  (an ideal resolution),  $0.01$ ,  $0.1$  and  $1.0$  °C. The sensors are assumed to have no accuracy errors and zero response time.

The experiments were conducted with the assumption that the AUVs estimate their location without errors, navigate with zero navigation error, and have unlimited energy capacity.

Figure 2 shows the results of these experiments. The figures show that the sensor resolution influences both the accuracy and the probability of locating the global optimum. The effect of changing the sensor resolution on the performance is more significant than that of the energy capacity. The graphs show that the algorithm with 60 AUVs is able to locate the global optimum with an acceptable accuracy with resolutions up to  $0.1$  °C. The algorithm shows poor performance with a resolution of  $1$  °C. The results of these experiments are expected due to the impact of the resolution on the ability of the algorithm to differentiate between sampled locations.

### Localisation Errors

To assess the impact of the localisation errors on the CMA-ES performance, another set of experiments was conducted. Gaussian probability distributions with standard deviations of  $\{0.0, 0.1, 0.3, 1.0\}$  were used to model the localisation errors. These distributions can produce errors in the range roughly from  $-3\sigma$  to  $3\sigma$ . For AUVs with zero localisation error, a distribution with  $\sigma = 0$  is assumed.  $\sigma = 0.1$  is assumed for AUVs that rely on GPS for localisation.  $\sigma = 0.3$  and  $\sigma = 1.0$  is assumed for AUVs that use underwater localisation techniques. The experiments were carried out with the assumption that the AUVs navigate with zero navigation error, have unlimited energy capacity and ideal temperature sensors.

The results of the experiments, as shown in Figure 3, demonstrate that the change in the localisation errors affect the accuracy of locating the global optimum. The plots show that these errors do not misguide the search but can decrease its ability to find the exact location of the global optimum. It is worth mentioning that the effect on the accuracy is related to the range of localisation errors. This error can be due to the error in reporting the exact location of the global optimum.

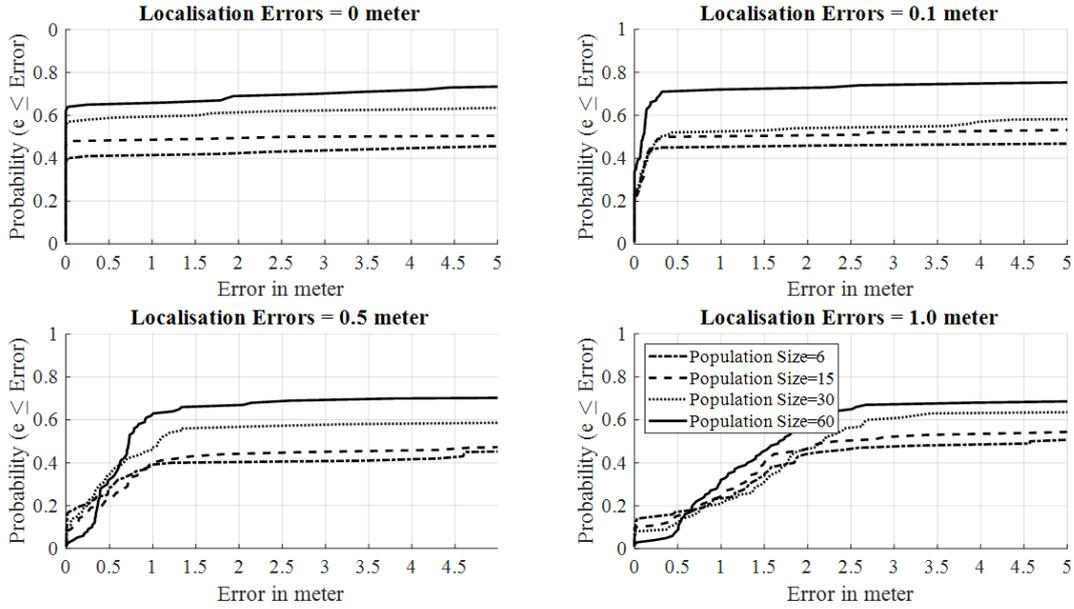


Figure 3: The Impact of Localisation Errors on the Performance

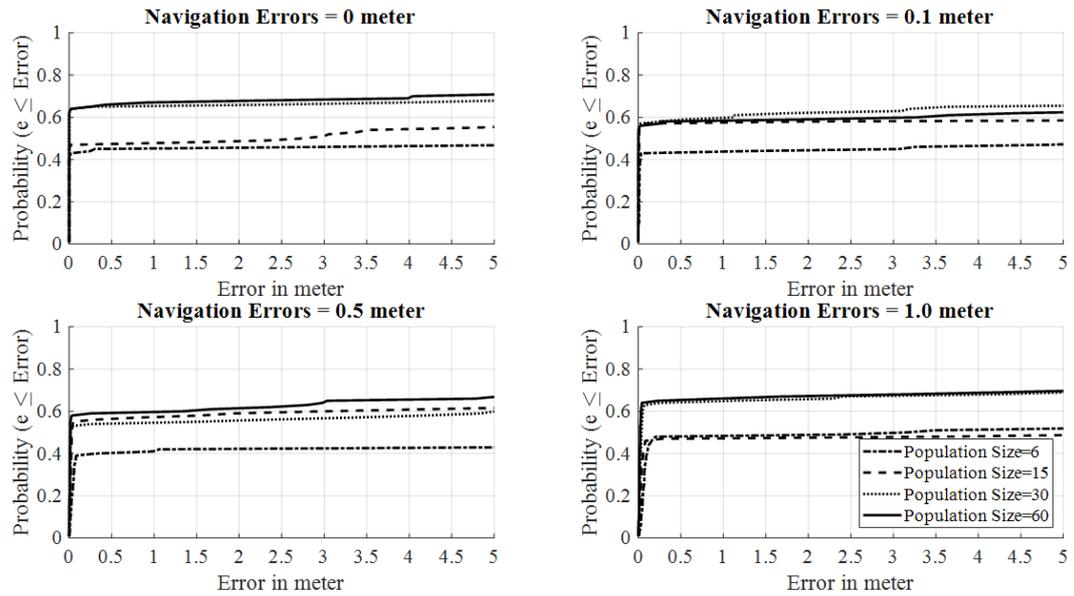


Figure 4: The Navigation Problem and the Performance

On the other hand, the localisation errors have a small impact on the probability of locating the global optimum. The localisation errors in the experiments do not accumulate over time. The localisation process at each sampled location sets upper limits on the localisation error values. The localisation errors can introduce errors in mapping the locations to their quality. However, the CMA-ES algorithm is resilient against these errors.

### Navigation Errors

The last set of experiments were carried out to evaluate the effect of the navigation error on the CMA-ES performance. Gaussian distributions with standard deviation values as those defined for localisation errors are used to model the navigation errors.

These experiments were executed with the assumption that the AUVs can estimate their locations without any error, have unlimited energy capacity and have ideal temperature sensors.

The results of these experiments, depicted in Figure 4, show that the navigation errors have the least significant impact on the performance compared with other constraints. The navigation errors alone can lead the search to sample points other than the target points. The distance between the sampled points and the target points depends on the navigation errors. However, navigation errors do not cause any errors in evaluating the sampled points. It does not introduce any errors in mapping the location to the objective function. Since the CMA-ES is a stochastic algorithm, non-accumulative navigation

errors does not introduce a significant effect on its search behaviour. They can affect slightly the accuracy of locating an SGD as they can guide the search to a location near the exact location in the final stages of the search, as shown in graphs of small population sizes. Moreover, navigation errors can improve the diversity of the population and improve the search results, as shown in Figure 4 for navigation errors with a standard deviation of 0.1.

## CONCLUSION AND FUTURE WORK

Selecting a suitable cooperative search algorithm for a swarm of AUVs necessitate studying its robustness against constrains, which are imposed by the search task. To shed light on the impact of these constrains on the algorithm performance, the performance of CMA-ES as a state-of-the-art algorithm against some of these constrains was evaluated. The performance of the algorithm was studied in locating the global SGD using the water's temperature as a tracer.

The experiments show that using an off the shelf state-of-the-art search algorithm cannot guarantee solving the problem even with a large number of AUVs and with unlimited energy capacities. The experiments also illustrate that using the CMA-ES with a population size, which is recommended in literature (Hansen, 2006), produces a poor performance in locating the global SGD in a search space with two SGDs.

The experiments also demonstrate that the resolution of the sensor has a significant influence on the search behaviour. Sensors with high resolutions empower the discrimination ability of the algorithm between similar solutions. On the other hand, sensors with low resolutions transform the objective function to a kind of staircase function and degrade the algorithm ability of discrimination between sampled solutions.

The localisation errors also have a considerable impact on the performance. The localisation errors can lead to associate the quality of sampled solutions to other solutions. This can lead to errors in evaluating the quality of the sampled solution and can lead the search towards non-optimal solutions. The simulations show that, in most cases, the localisation errors do not prevent the search from locating the global optimum. Meanwhile, they can degrade the algorithm accuracy in reporting the location of the global optimum.

The next step in this research is to investigate possible ways to improve the CMA-ES algorithm performance in locating SGDs. It will include studying the impact of the physical movement of the AUVs and the communication constrains on the algorithm performance. Utilising the  $(1,\lambda)$ -ES with mirrored sampling and sequential selection (Auger, et al., 2011) as a search algorithm for a single AUV search will be also investigated.

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