Implied volatility based margin calculation on cryptocurrency markets

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KEYWORDS

Crypto Markets; Implied Volatility; Margin

ABSTRACT

The focus of our research is the use of implied volatility in the context of margin calculations on BTC positions. We will compare the standard deviation estimated from historical data to the implied volatility values estimated from ATM option prices. As our main result, we show that the implied volatility is a superior basis for margining purposes. Not only does it perform better in a backtest, it also requires lower average margin levels to provide the same risk profile. Our results also show that the logreturns of BTC cannot be assumed to be normally distributed.

INTRODUCTION

In our paper, we test the performance of implied volatility based margin calculation for Bitcoin (BTC) positions. Our research connects to a large body of already existing literature; it relates to the following areas:

First, the broadest context for our work is provided by the literature on the margining of clearinghouses, more generally the risk management of clearinghouses. Berndsen (2021) provides a good overview on the latter topic, while Murphy, Vasios, and Vause (2016) describes how margins should be defined to meet actual regulations and to cover possible losses coming from counterparty risk. Usually, to open a position on a centralized market, agents need to buy the asset outright or deposit margin (called the initial margin) to signal their ability to settle their obligations and to cover possible future losses incurred in case of failure to pay. The losses and gains from the daily settlement drive the balance of the margin account. Once the level of the margin account falls below the maintenance margin, the trader receives a margin call and is obliged to restore the margin to the required level. If the price movements of the asset decrease the trader's margin to the extent that he will not be able to meet the margin call and his positions have to be closed by the clearinghouse, the losses occurring when closing the position should be covered by the margin account of the trader.

Since the volatility of the given asset is the basis of the margin calculation, measuring and forecasting volatility of financial assets is the second important part of the related literature for our paper. The volatility of logreturns for the asset, i.e. the second moment of the probability distribution, is a risk measure. It quantifies possible deviations of ex-post returns from the expected level. It serves as an input for optimization problems (see e.g. portfolio selection in Markowitz 1952 and Sharpe 1964), risk management models and pricing (see e.g. Black and Scholes 1973). Historical and implied volatility models are already distinguished and compared in the early literature review of Mayhew (1965). There is already strong evidence that returns are not normally distributed (e.g. Fama 1965) and volatility is not constant over time. Several models are applied to assess the empirical characteristics of volatility. Granger and Poon compare the performance of the following three model classes based on their previous work surveying 93 arti-

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cles (Granger and Poon 2002): time-series models (constant historical volatility, autoregressive conditional heteroscedasticity (ARCH) models), stochastic volatility models and implied volatility models. The results of Granger and Poon (2005) show that for forecasting, the implied volatility outperforms time-series models since it incorporates both current information and expectations for the future. Historical ARCH models do not outperform implied volatility models, but they show better results than stochastic volatility forecasts. Granger and Poon (2005) also report that the forecasting power of volatility models can be improved by using data of higher frequency (e.g. 5 minutes-data for developed markets), and short-horizon forecasts are more successful than those for long horizon. Our comparison historical volatility to implied volatility is based on relatively short, 8-hour periods.

Next we provide a short overview of implied volatility estimation. The most important milestone of derivative pricing is the Black–Scholes–Merton model (see Black and Scholes 1973; Merton 1973) where the value of a European call option can be calculated as follows:

$$\mathrm{C}(\mathrm{S}, \mathrm{T}-t) = \mathrm{N}(\mathrm{d}_1)\mathrm{S} - \mathrm{N}(\mathrm{d}_2)\mathrm{K}e^{-r(T-t)} \tag{1}$$

$$\mathrm{d}_1 = \frac{1}{\sigma\sqrt{\mathrm{T}-t}}\left[\ln\left(\frac{S}{K}\right) + (T-t)\left(r + \frac{\sigma^2}{2}\right)\right] \tag{2}$$

$$\mathrm{d}_2 = \frac{1}{\sigma\sqrt{\mathrm{T}-t}}\left[\ln\left(\frac{S}{K}\right) + (T-t)\left(r - \frac{\sigma^2}{2}\right)\right] \tag{3}$$

where

- S: spot price of the underlying product,
- K: strike price,
- σ: standard deviation of the underlying
- r: risk free rate
- T-t: time to expiration
- N: normal cumulative distribution function

Several authors report that the distribution of return has fat tails instead of following normal distribution which allows the so-called volatility smile to occur. (See for a recent detailed overview of the topic: Zulfiqar and Gulzar 2021). When observing the prices of options traded at different exchanges and in different asset classes, traders usually experience a difference between the current market price and the theoretical fair value of the derivative. Implied volatility is the volatility level of the underlying product which, used as input for Black-Scholes-Merton model gives the observed market price as a result of the theoretical model. Since the level of implied volatility reflects the expectations of the market as well, using it as an estimate for future volatility is a widespread and - as cited in the above literature overview - an efficient method.

For margining purposes, there is one further step from volatility estimation to margin calculation. On the basis of the volatility forecasts, a downside risk measure like Value-at-Risk (VaR) or expected shortfall (ES) needs to be calculated (for details see e.g. Jorion 2001). Based on Bams, Blanchard, and Lehnert (2017), in case of VaR calculation, the implied volatility based VaR does not outperform the historical volatility based ones: they arrive at the opposite result to the one we cited earlier in this article. These contradictory, non-obvious results led us to pose our research questions. We compare an implied volatility based margin calculation method to one where the margin level is specified by the actual regulations in the European Union (EMIR 2012, RTS 2013). For simplicity, we will disregard the handling of procyclicality within the initial margin calculation, nor will we apply any add-ons to the calculated margin requirements. Based on the regulation, the margin will be calculated to be sufficient at a 99% significance level, with a 12 months look-back period. As a further simplification, we have modified the required 2-day liquidation period to an 8-hourly liquidation period, which is more in keeping with the global nature of the crypto markets and the prevailing 8-hour margining regime customary there. This margin value will be the basis of comparison to the other margin calculations described later in this paper in the Backtesting section.

The most recent area within the literature is the application and testing of already existing results on the dynamically emerging crypto markets. Hence the focus of our research is how the use of implied volatility can contribute to margin calculation on BTC positions; the most closely related antecedent of our work is the contribution of (Zulfiqar and Gulzar 2021) who are the first to have tested for the volatility smile and implied volatility for BTC options. They use 14-day maturity Bitcoin options of two time periods traded on Deribit Bitcoin Futures and Options Exchange. Root-finding iterative techniques, namely the Newton Raphson and the Bisection methods proved to effectively estimate the implied volatility of BTC options. The results show similarities with that of assets from the commodity class, therefore Zulfiqar and Gulzar classify BTC as a commodity. More generally, all recent works on crypto markets provide a background for our paper. The cryptocurrency markets have shown extreme development over the last years, therefore they received considerable attention as a research topic as well. We can distinguish the following directions in the dynamically evolving research: price discovery, market efficiency and optimal trading strategies (especially hedging) or the use of crypto products in risk management (Alexander et al. 2020; Deng et al. 2019). Pichl and Kaizoji (2017) defines the fundamental importance of Bitcoin and its security aspects also as an existing direction in the literature. The behaviour of volatility on crypto markets is an important question of price discovery and risk management strategies as well. While Pichl and Kaizoji (2017) used a Heterogenous Autoregressive model for Realized Volatility for BTCUSD data, the most widespread methodology is provided by the GARCH models. See e.g. Katsiampa et al. (2017) who compared the forecasting power of several GARCH models for the volatility of BTC. According to them, the AR-CGARCH model achieved the highest goodness-of-fit on their dataset.

SIMULATION DATA

We chose to use the free tier of market data integrator coinapi.com that has downloadable historical data for both underlying and derivative crypto data. This tier allows the daily download of up to 100 klines (open/close/high/low candle data) for up to 100 products (specific option contracts). Given the very large number of option contracts (on 19th January 2022, *coinapi* listed 25,843 options contracts on BTC expiring after 1st January 2021, across all exchanges), we had to settle for a subset of them. *Binance* (binance.com) lists vanilla European options on BTC/USDT with regular weekly and monthly expiration. On 19th January 2022 there were 1,348 contracts on BTC/USDT that had expiration after 1st January 2021. We further reduce this number by considering only those contracts whose strike price was close to being ATM over its liquid (30 day) lifetime. In this context, a contract (strike price K, underlying price S, expiration time T expressed as days) is close to being ATM if:

$$S^{max} = \max_{T-30\leq t\leq T} S_t \tag{4}$$

$$S^{min} = \min_{T-30\leq t\leq T} S_t \tag{5}$$

$$S^{min} - (S^{max} - S^{min}) < K < S^{max} + (S^{max} - S^{min}) \tag{6}$$

This restriction yields 767 contracts for this study.

Each of the contracts has an associated time series of 8-hour kline data (open/high/low/close price, period volume), maximally 100 periods long. Contracts become liquid when they are ATM, otherwise there are often no transactions. There are altogether 7,942 non-empty klines in this data set, which is less than 10% of the maximum possible.

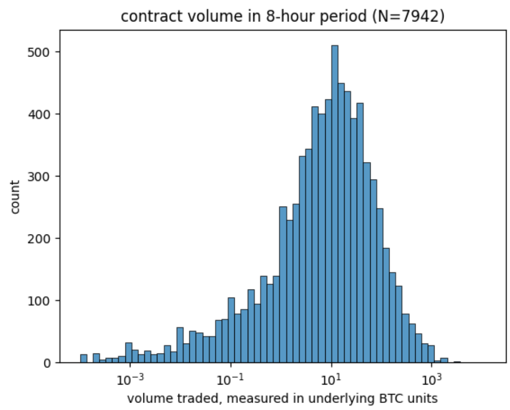

Figure 1: Available volume in BTC options on Binance in 2021

In addition to the *Binance* option data, we also collected information on the underlying security. For *Binance*, the underlying security is the BTC/USDT pair that is very liquid. $S =$

BTC/USDT is the price of BTC in units of USDT, the latter being a stablecoin, i.e. a coin whose value is closely tethered to the fiat USD currency. The data for S is available for all 8-hour periods. We save them between 2020-01-01 and 2022-01-19.

Implied Volatility Calculation

The concept of implied volatility was described above. In our study, we used scipy.optimize.scalar_root (Jones, Oliphant, Peterson, et al. 2001–) to find the implied volatility, which converges in all cases except for prices implying negative time-value for options – these prices were filtered out.

For each 8-hour period, multiple contracts would typically have a non-empty kline: puts, calls at various strike prices and expiration times trade at the same time. In order to estimate a single implied volatility σ_t for each period, we use a weighted average defined below in Equation 7 and 8. In using the Black-Scholes formula to find $\sigma_t^{(i)}$, we assume no dividends and a risk-free rate of 1.85%. This was a typical 2021 US T-Bill rate. The results below are not sensitive to this choice if r is within reasonable bounds, because the time to maturity of the liquid options is so short, typically less than a week.

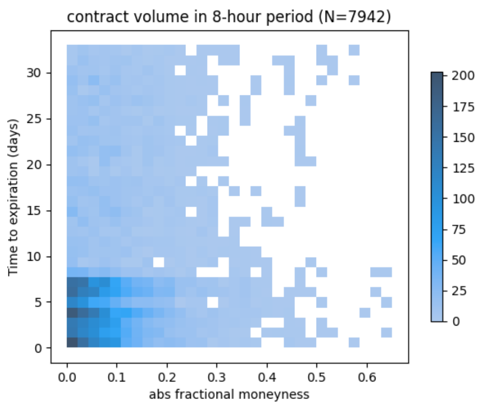

Figure 2: Available volume in BTC options on Binance in 2021 vs time to expiration and moneyness

$$\sigma_t = \frac{\sum_{i \in \mathcal{C}} w_{i,t} \sigma_t^{(i)}}{\sum_{i \in \mathcal{C}} w_{i,t}} \tag{7}$$

$$w_{i,t} = \frac{V_{i,t}}{1 + (S_t - K_i)^2} \tag{8}$$

where

- $\mathcal{C}$ is the set of all contracts,
- $V_{i,t}$ is the volume of contract i in time period t

As seen in Figure 2, the majority of the weight in this average comes from contracts in the last week of trading, with strike prices within 20% of the underlying.

BACKTESTING

For each 8-hour period in 2021 we calculate the forward 8-hour maximum loss L_{8h}^{max} by using the underlying minimum and maximum over that period. L_{8h}^{max} is the larger of the absolute difference of the next 8-hour maximum (or minimum) of the underlying and the current close price. The goal of a margining policy is to make sure that in any period the maximum loss is covered by the margin posted by the trader at a given significance level. Margin in this simplified simulation will be taken to be a percentage M_t of the trader's net position in the underlying at the end of each period. The trader will be required to post sufficient margin in order to be allowed to increase her position in the next period. In this simulation we examine 3 policies:

1. Realized Trailing Sigma Policy
2. Implied Volatility Policy
3. Normal VaR Policy

Margining regimes will be compared according to their average margin level. Clearly, a CCP will be more competitive in the market if it tends to require smaller margins, while maintaining the same risk profile. We will find, through simulation, that the above methods do differ in how much margin they require on average to reach the same fraction C incidence of insufficient margin. C is usually 1%, based on the 99% significance level requirement of the EMIR 2012.

For a particular margin policy, at time t, the forward incidence rate will be denoted F_t^{fwd}. For each t we determine F_t^{fwd}, and the distribution of $\{F_t^{fwd}|t \in 2021\}$ will be compared to the margin M_t in effect at time t. The total failure rate is $F^{fwd} = E(\{F_t^{fwd} > M_t|t \in 2021\})$.

Realized Trailing Sigma Policy

In this study, the Realized Trailing Sigma Policies involve calculating the population standard deviation (SD) of the realized 8-hour returns of the underlying over a 250-day trailing time interval. The policy parameter m^{hist} is used to multiply the historical sigma to obtain the margin $M_t = m^{\text{hist}}\sigma_t^{trail}$, where σ_t^{trail} is the trailing historical sigma at time t. BTC/USDT data is available for dates prior to 2021, so we are able to use 2020 data to get trailing standard deviation estimates from the beginning of 2021. We look at F^{fwd} as a function of m^{hist}. In this study we do not attempt to cross-validate m^{hist} on an outsample period, mainly due to the scarcity of options data.

Implied Volatility Policy

In contrast to the Realized Trailing Sigma Policy, the Implied Volatility Policy does not rely on a long trailing historical value of the standard deviation of the underlying. Instead, it uses a smoothed function of the implied volatility calculated in the manner described earlier.

For each time t, we compute the implied volatility σ_t^{impl}, and $M_t = m^{\text{impl}}\sigma_t^{\text{impl}}$. We look at F^{fwd} as a function of m^{impl}.

EMA of Maximum Variation Policy

Instead of looking at implied volatilities, we can also consider the recent volatility of the market to suggest an optimal margin. As a short-term estimator of volatility, we use the difference of the minimum and maximum underlying prices (maximum variation) within a kline. An exponential moving average (EMA) of these observations is then taken as the statistic on which to base this policy.

For each time t, we compute the maximum variation σ_t^{var}, and $M_t = m^{\text{var}}\sigma_t^{\text{var}}$. We look at F^{fwd} as a function of m^{var}.

Normal VaR Policy

The Normal VaR Policy is based on the results of the previous two policies, namely the VaR is calculated both from the Realized Trailing Sigma, and from the Implied Volatility as well. In the Normal VaR method, we assume that the periodic profit of a position is normally distributed. If the expected logreturns were normally distributed as $N(0, \sigma)$, the m parameter would be fixed at 2.33 in order that $E(F^{trail}) = 1\%$. In this study, the Realized Trailing Sigma Policy, and the Implied Volatility Policy is the generalization of the VaR policy involving actual calibration of the m parameter on different significance levels (99% and 97%). However the Normal VaR Policy based on the normal assumption is still interesting to consider, since it has been a commonly applied regulatory approach in the past.

SIMULATION RESULTS

Figure 3 shows characteristics of the underlying returns over the past 2 years. What we see is that the 1-year trailing standard deviation of the 8-hour returns is a poor proxy for the possible maximum loss values of a position, as it is approximately constant. Maximum loss L is defined for each period using the kline data: $L_t = \max_{X\in\{high,low\}}(|S_{t-1}^{close} - S^X|)$

Option data is only available from February of 2021. Figure 4 compares the historical standard deviation of returns to standard deviations implied by options prices. The smoothed implied volatilities in Figure 4 are given by the 5-day rolling maximum of the exponential moving average (halflife = 1 day) of the implied volatility. It is visually clear that the implied volatilities follow the evolution of the actual maximum loss quite well. The 1 month EMA of the maximum variation performs similarly to the implied volatility: it is high when the realized maximum loss is high.

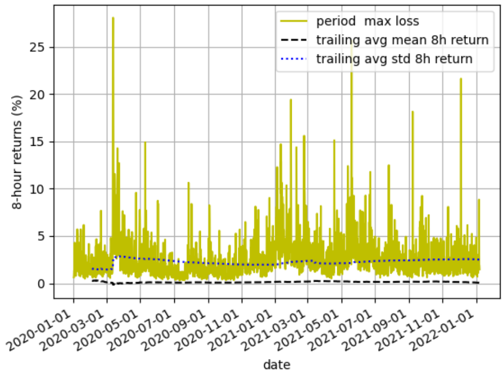

Figure 3: characterization of BTC/USDT 8hr returns

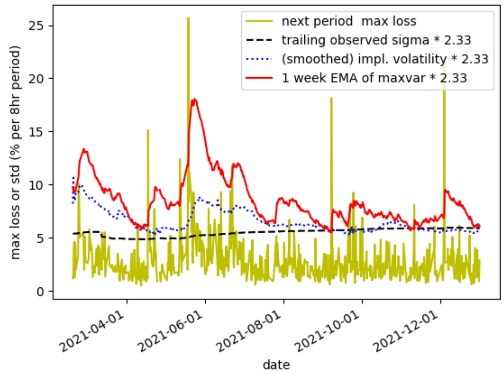

Figure 4: Historical and implied standard deviations compared to next period maximum loss. See text for motivation of the factor 2.33

Comparing Policies

An Implied Volatility Policy and a Realized Trailing Sigma Policy are both defined by a multiplier m^{policy}. In this study we look at what multiplier is necessary to achieve a certain failure rate in 2021. In fact we try a number of multipliers, and for each multiplier we look at what the simulated failure rate is, as well as what the average margin rate is. Due to the scarcity of options data, we are effectively doing an in-sample optimization of m^{policy}. A lower margin rate for a given failure rate signals a better margining policy.

If returns were normally distributed, the multiplier required for a particular desired failure rate would be trivially given by the inverse normal CDF: For example a desired 1% failure rate would imply $m^{policy} = 2.33$ and 3% would correspond to $m^{policy} = 1.88$. The average (over t) required margin M^{avg} would be given by the markers on Fig 5. It is immediately clear that the returns are not normally distributed. For example, the Realized Trailing Sigma VaR Policy with $m = 2.33$ has a failure rate of more than 8%, instead of the intended 1%.

The Implied Volatility Policy with $m = 2.33$ does much better than the Realized Trailing Sigma Policy with $m = 2.33$, as it only fails in about 3% of the time. It is clear that the normal assumption is faulty, and calibrated m-s are necessary.

As the lines on Figure 5 show, even if we calibrate m for the Realized Trailing Sigma and Implied Volatility policies, the latter does better: at all failure rates, a calibrated Realized Trailing Policy gives a higher average margin, than a calibrated Implied Volatility Policy. Moreover if the calculate the VaR based on the Implied Volatility Policy, that backtest performs significantly better.

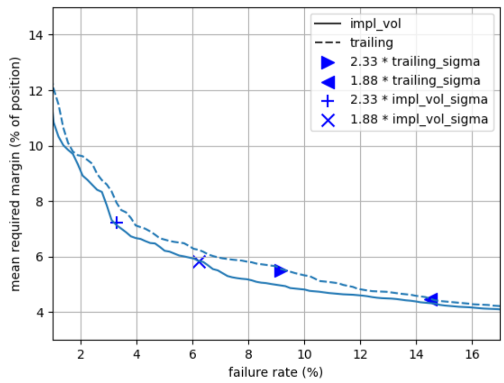

Figure 5: Comparison of Realized Trailing Sigma, Implied Volatility, and EMA of Maximum Variation Policies. For any desired failure rate, the mean margin is obtained by calibrating m, then simulating on 2021

Figure 6 shows a more detailed view of how the Implied Volatility and the EMA of Maximum Variation perform with respect to the baseline Realized Trailing Sigma policy. Over the full range of targeted failure rates, Implied Volatility and EMA policies require lower mean margins, meaning cheaper trading for the investor with the same risk for the exchange.

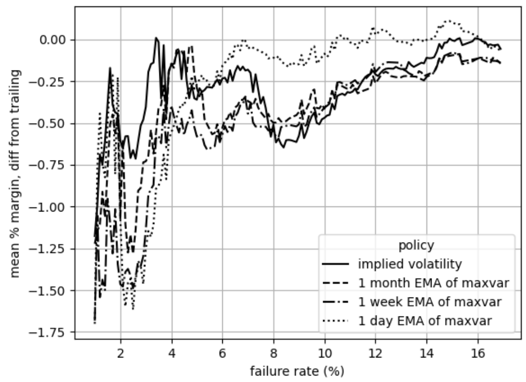

Figure 6: Comparison of Implied Volatility, and EMA of Maximum Variation Policies. We show the mean required margin gain with respect to the Realized Trailing Sigma policy. The greatest gains are to be had for the most relevant (low) failure rates.

CONCLUSIONS

Our paper shows that using implied volatilities in BTC/USDT margining regimes is superior to using historical standard deviations. First of all, implied volatilities predict market volatility much better than historical standard deviations. This allows the CCP to calculate a lower required margin on average, while obtaining the same failure rate. This remains the case whether we assume normally distributed logreturns or not. For normally distributed logreturns, the Implied Volatility Policy gives, over 2021, a 3% failure rate compared to 8% for the Realized Trailing Sigma Policy. If we drop the normality assumption, and calibrate m to obtain a desired failure rate, it is still the case that a calibrated Implied Volatility Policy yields a lower average margin than the Realized Trailing Sigma policy.

Overall, our results are consistent with the commonly held view that BTC/USDT returns follow very fat tailed distributions.

APPENDIX

The raw data as it is downloaded from coinbase consists of csv files for each symbol. For example, for `BINANCEOPTV_OPT_BTC_USDT_211231_48000_P` (vanilla put option on BTC/USDT expiring 2021.12.31, strike 48000), the kline data looks like the sample in . The option and underlying data are in the same format, but it is revealing comparing the two, especially in terms of the volumes.

Option Attribute	Value
time_period_end	2021-12-31T08:00:00.0000000Z
time_period_start	2021-12-31T00:00:00.0000000Z
time_open	2021-12-31T00:38:56.2770000Z
time_close	2021-12-31T03:25:34.6100000Z
price_open	3935.01
price_high	4155.18
price_low	206.67
price_close	989.04
volume_traded	91.1301
trades_count	2500
Underlying Attribute	**Value**
time_period_end	2021-12-31T08:00:00.0000000Z
time_period_start	2021-12-31T00:00:00.0000000Z
time_open	2021-12-31T00:00:00.0000000Z
time_close	2021-12-31T07:59:59.9990000Z
price_open	47120.88
price_high	47550.0
price_low	46825.38
price_close	47191.09
volume_traded	6896.44835
trades_count	248930

Figure 7: Coinapi option data format

We have 7942 relevant options data points, distributed over 579 (8-hour) time periods, 46 expirations (one per week). For each of these time periods, as well as for all of 2020, we have underlying data as well, one kline per 8-hour period.

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