THE RISK OF HEDGING

Ágnes Vidovics-Dancs, PhD, CIIA

Corvinus University of Budapest
H-1093, Fővám tér 8, Budapest, Hungary
E-mail: agnes.dancs@uni-corvinus.hu

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ABSTRACT

Hedging financial risk is an essential issue but is far from trivial to implement. There are several hedging assets portfolio managers can select from. However, the choice is not without weight: two portfolios hedged against the same risk factor may have different characteristics depending on this hedging asset. Moreover, hedging against one risk factor may increase the portfolio's sensitivity to other risk factors. That is, a strategy that aims to reduce risk may also increase risk in a paradox way. This should be considered by portfolio managers, risk managers, and regulators as well. The goal of this paper is to raise thoughts on this topic.

INTRODUCTION

Hedging risk is a fundamental motivation in financial markets. Should the investor feel that the portfolio's exposure to a certain risk factor is too high, it can reduce or eliminate this exposure by entering another position inversely related to the same risk factor. However, this restructuring of the original position might modify other characteristics of the portfolio as well.

Hedging for one risk factor might (and almost always will) modify the exposure to other risk factors as well. When we reduce one type of risk, we usually increase another. In this sense, hedging is not necessarily eliminating risk but transforming one kind of risk to another, i.e., balancing among the risk factors. It is clear, for example, that we will undertake the counterparty risk of the partner with whom we created the hedging position. So, we reduce or eliminate market risk but increase counterparty risk. In other situations, we reduce one type of market risk (e.g., foreign exchange risk) but raise a different kind of market risk (e.g., interest rate risk). It might happen that this new risk factor is less important or even irrelevant for certain investors, but in case of institutional investors with portfolio limits, all risk factors should be recognized.

Before showing the trade-off between risk factors, we also illustrate that the choice of the hedging asset may influence not only the newly emerging risk factors but also the efficiency of the neutralization. The basic inspiration of this analysis is an example described in Száz (2009). Throughout the paper, we will use the Black-Scholes-Merton model when analyzing options.

DELTA-HEDGING WITH OPTIONS

In our example, the basic portfolio consists of 100 pieces of Apple stocks. We assume Apple will not pay dividends in the next 6 months. The price of the shares (S) is 152 USD. The portfolio manager decides that it does not want to undertake Apple's market risk anymore, but instead of liquidating the position (i.e., selling the stocks), it hedges the risk with derivative instruments. We will consider 6-month (T) put options with two possible strike prices (K_1 and K_2), 120 USD and 130 USD. Let the volatility of Apple (σ) be 20% and the risk-free logreturn (r) 5%. These primary input data are summarized in Table 1.

Table 1: The primary data

S	K_1	K_2	σ	r	T
152	120	130	20%	5%	0.5

In what follows, we will need the value (p), the delta (Δ), the gamma (Γ) and the rho (ρ) of the two put options, so we summarize these in Table 2. The fundamentals of the Black-Sholes-Merton option pricing model and the calculation and interpretation of the Greek letters can be found in several textbooks, e.g., Hull (2015).

Table 2: Main characteristics of the options

K	120	130
p	0.24	0.93
Δ	-0.03	-0.09
Γ	0.003	0.007
ρ	-2.21	-7.15

Hedging for the market risk of the underlying product means that the portfolio manager creates a delta-hedged portfolio. So, we create a portfolio so that its delta is zero. Since we have two possible put options for this purpose, the zero delta value might be achieved with numerous

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combinations. More precisely, we should solve Equation (1), where x and y are the amounts of the hedging assets. Since we have only one equation and two unknowns, there are infinite solutions.

$$100 * 1 + x * \Delta_{120} + y * \Delta_{130} = 0 \quad (1)$$

For simplicity, we will focus only on the two extreme solutions of Equation 1, where we do not combine the two derivatives. This means that we will buy only one of them in the appropriate amount and let either x or y be zero. With simple calculations, we can determine that for delta-hedging 100 Apple shares, the amount of put options needed is x=3,637 (K=120) or y=1,136 (K=130), respectively. We know that this strategy is dynamic; that is, we should rearrange the portfolio from time to time so that the delta remains zero. However, here we will only focus on the initial portfolio's characteristics.

It is well known that delta-hedging with options is efficient only if the changes in the price of the underlying asset are small enough. Hedging for more significant changes requires creating a delta-gamma-hedged portfolio, i.e., solving Equations (1) and (2) simultaneously.

$$100 * 0 + x * \Gamma_{120} + y * \Gamma_{130} = 0 \quad (2)$$

Now we have two equations and two unknowns, and the solution is unique. In our example, we can compute that delta-gamma-hedging is possible by selling 13,549 of the first put option and buying 5,368 of the second option. In what follows, we will call the uncovered portfolio "A" and the hedged portfolios B, C, and D, respectively. Table 3 contains the main characteristics of these portfolios.

Table 3: The portfolios

Name		A	B	C	D
Composition	Apple	100	100	100	100
	Put (K=120)	0	3,637	0	-13,549
	Put (K=130)	0	0	1,136	5,368
Delta-hedged		no	yes	yes	yes
Gamma-hedged		no	no	no	yes

Hereafter, we will illustrate two important features of these hedging strategies. First, we will show that the delta-hedged portfolios have different sensitivities for the Apple-price, *depending on the choice of the hedging asset*. Second, we will show that hedging the market risk of Apple will open a new market risk, the sensitivity to the risk-free interest rate.

The efficiency of the hedge

In our first illustration, we analyze the portfolios' sensitivity if Apple's market price changes. We consider these price movements to happen ceteris paribus, all other factors remain the same (even the time does not pass). Table 4 collects how the portfolios' value will change (in percentage) due to different price movements (*dS*, measured in USD). It is not surprising that portfolio D is the less sensitive, since it is not only delta-hedged but also gamma-hedged. What is important to notice is that portfolios B and C are not evenly sensitive, even though both have zero deltas. Hence, it is not enough to know that delta-hedging with options is not a perfect hedge strategy (in the sense that it is only first-order hedging), but the risk manager has to be aware that the choice of the hedging asset influences this imperfection.

Table 4: Sensitivity to the Apple-price

dS (USD)	**A**	**B**	**C**	**D**
-10	-6.58%	4.63%	3.26%	-1.57%
-9	-5.92%	3.63%	2.58%	-1.10%
-8	-5.26%	2.77%	2.00%	-0.74%
-7	-4.61%	2.05%	1.49%	-0.48%
-6	-3.95%	1.46%	1.07%	-0.29%
-5	-3.29%	0.98%	0.73%	-0.16%
-4	-2.63%	0.61%	0.46%	-0.08%
-3	-1.97%	0.33%	0.25%	-0.03%
-2	-1.32%	0.14%	0.11%	-0.01%
-1	-0.66%	0.03%	0.03%	0.00%
0	0.00%	0.00%	0.00%	0.00%
1	0.66%	0.03%	0.03%	0.00%
2	1.32%	0.13%	0.10%	0.01%
3	1.97%	0.27%	0.22%	0.02%
4	2.63%	0.47%	0.38%	0.06%
5	3.29%	0.71%	0.58%	0.10%
6	3.95%	1.00%	0.81%	0.17%
7	4.61%	1.31%	1.08%	0.26%
8	5.26%	1.67%	1.38%	0.37%
9	5.92%	2.05%	1.71%	0.51%
10	6.58%	2.46%	2.06%	0.67%

Here we showed only two possible put options, but of course, many put and call options (with various strike prices and maturities) are available on the market, and delta-hedging with these numerous possible derivatives will all lead to different final sensitivities.

Increasing interest rate risk by delta-hedging

The uncovered portfolio itself (the Apple shares) is not exposed to the risk that the risk-free interest rate may change. If we neutralize the Apple-price risk with options, the value of the hedged portfolio becomes sensitive to the interest rate. Table 5 illustrates this by showing the percentage change of the portfolios' value with respect to +/- 150 basis point change in the interest rate.

As we can see, the value of all the delta-hedged portfolios is negatively related to the interest rate. Should the interest rate rise, the portfolios will lose value and vice versa. The inverse relationship between the interest rate changes and the portfolios is due to the negative rho factors of the put options. The position's rho is negative for portfolio D as well (it is not trivial because we have long and short put options at the same time).

Table 5: Sensitivity to the interest rate

dr (bp)	**A**	**B**	**C**	**D**
-150	0.00%	0.80%	0.79%	0.75%
-140	0.00%	0.74%	0.73%	0.70%
-130	0.00%	0.69%	0.68%	0.65%
-120	0.00%	0.63%	0.62%	0.60%
-110	0.00%	0.58%	0.57%	0.55%
-100	0.00%	0.52%	0.52%	0.50%
-90	0.00%	0.47%	0.46%	0.45%
-80	0.00%	0.41%	0.41%	0.40%
-70	0.00%	0.36%	0.36%	0.35%
-60	0.00%	0.31%	0.31%	0.30%
-50	0.00%	0.26%	0.25%	0.25%
-40	0.00%	0.20%	0.20%	0.20%
-30	0.00%	0.15%	0.15%	0.15%
-20	0.00%	0.10%	0.10%	0.10%
-10	0.00%	0.05%	0.05%	0.05%
0	0.00%	0.00%	0.00%	0.00%
10	0.00%	-0.05%	-0.05%	-0.05%
20	0.00%	-0.10%	-0.10%	-0.10%
30	0.00%	-0.15%	-0.15%	-0.15%
40	0.00%	-0.20%	-0.20%	-0.20%
50	0.00%	-0.24%	-0.25%	-0.25%
60	0.00%	-0.29%	-0.29%	-0.30%
70	0.00%	-0.34%	-0.34%	-0.35%
80	0.00%	-0.39%	-0.39%	-0.40%
90	0.00%	-0.43%	-0.44%	-0.45%
100	0.00%	-0.48%	-0.48%	-0.50%
110	0.00%	-0.53%	-0.53%	-0.55%
120	0.00%	-0.57%	-0.58%	-0.60%
130	0.00%	-0.62%	-0.62%	-0.65%
140	0.00%	-0.66%	-0.67%	-0.70%
150	0.00%	-0.71%	-0.71%	-0.75%

Besides the fact that the portfolio became sensitive to this new risk factor, we may also observe that the exposure is not the same for the three delta-hedged portfolios. All in all, by delta-hedging the stock portfolio, we have undertaken a new risk factor. It is of utmost importance to emphasize that our conclusion does not mean delta-hedging is a wrong strategy. It only means that we cannot handle hedged portfolios as totally risk-free and protected against all kinds of risk.

HEDGING THE SELLING PRICE

In this section, we will analyze the same portfolio (100 Apple stocks) but will hedge the position in a different sense. Instead of hedging the portfolio's current market value, we assume that the portfolio manager has a fixed holding period. For example, it plans to hold the shares for 6 months and then close the position. The final selling price is, of course, not known in advance. Should we decide to eliminate this risk, we have different possible hedging assets again. One straightforward solution is to sell the shares in a short forward transaction. In this way, the predetermined selling price will be today's 6-month forward price, which is approximately 156 USD. Another solution is to buy a put option. For more comparability, let the put option's strike price be the same, 156 USD. We will call these new portfolios E and F. Table 6 summarizes the main characteristics of the portfolios analyzed in this section.

Table 6: The portfolios

Name		**A**	**E**	**F**
Composition	Apple	100	100	100
	Forward (K=156)	0	-100	0
	Put (K=156)	0	0	100

Now the quantity of the hedging derivative assets is the same as the uncovered position, and the strategies are static. That is, we do not have to rearrange the portfolios from time to time. This is because we would like to fix the selling price of exactly 100 Apple shares 6 months from now.

Naturally, portfolios E and F do not identically guarantee the final selling price. In the case of portfolio E, the selling price will be 156 USD for sure, even if Apple will be priced higher that time. In the case of portfolio F, the selling price will be *at least* 156 USD. Should the Apple-price be above it, we do not have to exercise the options and can sell the shares on the spot market. The initial cost balances this difference in the two strategies: entering the short forward position does not generate any initial cash flow; entering the long put position requires paying the option's (positive) price. We admit that this difference between the two hedging strategies is essential, but this paper will focus on another question.

Similarly to the previous section, we will show that neutralizing the Apple-price market risk will raise other, previously not undertaken risk factors. Again, we can observe that the uncovered portfolio A is not sensitive to the interest rate, but the hedged portfolios E and F are risky in this sense. Table 7 illustrates this by showing the percentage change of the portfolios' value with respect to +/- 150 basis point change in the interest rate.

Again, we can observe that the sensitivity to this new risk factor depends on the hedging asset. Now the difference is sharper than it was in Table 5, but we have to remember that portfolios E and F guarantee the final price in different senses, so they are less comparable. However, the main message is not about comparing them but identifying the new risk factor that emerged from hedging the original one.

Table 7: Sensitivity to the interest rate

dr (bp)	A	E	F
-150	0.00%	0.75%	0.38%
-140	0.00%	0.70%	0.36%
-130	0.00%	0.65%	0.33%
-120	0.00%	0.60%	0.31%
-110	0.00%	0.55%	0.28%
-100	0.00%	0.50%	0.25%
-90	0.00%	0.45%	0.23%
-80	0.00%	0.40%	0.20%
-70	0.00%	0.35%	0.18%
-60	0.00%	0.30%	0.15%
-50	0.00%	0.25%	0.13%
-40	0.00%	0.20%	0.10%
-30	0.00%	0.15%	0.08%
-20	0.00%	0.10%	0.05%
-10	0.00%	0.05%	0.03%
0	0.00%	0.00%	0.00%
10	0.00%	-0.05%	-0.02%
20	0.00%	-0.10%	-0.05%
30	0.00%	-0.15%	-0.07%
40	0.00%	-0.20%	-0.10%
50	0.00%	-0.25%	-0.12%
60	0.00%	-0.30%	-0.15%
70	0.00%	-0.35%	-0.17%
80	0.00%	-0.40%	-0.20%
90	0.00%	-0.45%	-0.22%
100	0.00%	-0.50%	-0.25%
110	0.00%	-0.55%	-0.27%
120	0.00%	-0.60%	-0.29%
130	0.00%	-0.65%	-0.32%
140	0.00%	-0.70%	-0.34%
150	0.00%	-0.75%	-0.37%

Another "risk factor" worth discussing is the time that inevitably passes. The value of short forward and long put positions will change as we approach the maturity. This is a new sensitivity since the price of the Apple shares will not change *ceteris paribus* only because time progresses. Table 8 illustrates this by showing the percentage change of the portfolios' value as 1-5 months pass (all other factors being unchanged).

Apart from the fact that hedging for the final selling price made the portfolio sensitive to time, we can also observe that portfolios E and F react inversely as maturity approaches.

Table 8: Sensitivity to the time

dt (months)	A	E	F
1	0.00%	0.42%	-0.25%
2	0.00%	0.84%	-0.55%
3	0.00%	1.26%	-0.91%
4	0.00%	1.68%	-1.37%
5	0.00%	2.11%	-2.00%

One may argue that the risk factors illustrated above can be ignored if the investor is interested only in the final outcome, i.e. in the portfolio's value at the end of the holding period. In other words, the risk factors do not necessarily generate *realized* losses/profits. This argumentation is true in theory but has several pitfalls in practice. First, most institutional investors are obliged to determine the managed portfolio's value daily. This way, the portfolio's risk factors are essential even if we have a so-called buy-and-hold strategy. Second, the buy-and-hold strategy is not always easy to keep. Apart from psychological factors (it is hard to keep a position in a significant loss), portfolio managers usually face limits they have to hold. Should the value of the financial assets change, limits might be violated, and the manager might be forced to restructure the portfolio.

CONCLUSION

This paper aimed to show that hedging risk is a risky activity. Naturally, the examples presented are not singular. We might cite several other cases where portfolio managers face a trade-off between two or more risk factors. For example, forward hedging for the foreign exchange risk will increase the exposure to domestic and foreign interest rates. On the other hand, neutralizing the interest rate risk of a bond portfolio by modifying its duration with an interest rate swap may increase the cash flow risk of the portfolio (due to the floating leg of the swap). The list is even longer if we account not only for market risk factors but also for credit risk. Whichever type of market risk is hedged, we will undertake the counterparty risk of the new partner. Generally, when talking about risk-free or hedged positions, we should always put the question: hedged against *which risk factor*?

We emphasize that the findings of the paper do not suggest that we should forget about hedging strategies. They only suggest that all the risk factors should be considered. In other words, portfolio managers hold not only a portfolio of financial assets but also a portfolio of financial risk factors. If the new risk factor that emerged by hedging is identified and undertaken with full awareness, then unpleasant surprises can be avoided. This is important not only for portfolio managers but also for regulators since they often intend to limit the risks that certain types of portfolios are allowed to include.

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AUTHOR BIOGRAPHY

Ágnes VIDOVICS-DANCS, PhD, CIIA is an associate professor at the Department of Macrofinance at Corvinus University of Budapest. Currently, she is the Program Director of the Economic and Financial Mathematical Analysis undivided MSc program. Her main research areas are risk management, government debt management in general, and especially sovereign crises and defaults. Formerly, she has been the Chief Risk Officer of a Hungarian asset management company for several years. Her e-mail address is: agnes.dancs@uni-corvinus.hu