INSPECTION POLICIES FOR FATIGUE-SENSITIVE COMPONENTS IN AIRCRAFT SERVICE

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ABSTRACT

As aircraft structures begin to age (that is, as flight hours accumulate), existing subcritical cracks or new cracks can grow in some high-stress points of the structural components. The usual approach is to inspect the structures periodically at certain intervals. Thus, a catastrophic accident during flight can be avoided. The problem then arises of choosing a sequence of inspection times which avoids both too many inspections, which may be costly, and too few inspections, which may also be costly due to a crack in aircraft structure component not being detected for a long period. In this paper, a simple approach is proposed, where after each inspection (if crack is not detected) we choose the next inspection point so that a crack may occur within an interval between successive inspection times with a given probability. It allows one to find the inspection policies for detection of initial cracks in critical structural components of aircraft under the assumptions that the parameter values of the underlying distributions are unknown; this constraint is often met in practice. Furthermore, obtaining inspection schedules under crack propagation is considered. To illustrate the proposed technique based on ancillary statistics, numerical examples are given.

INTRODUCTION

The total fatigue life of a structural component is the sum of the time to crack initiation and the time of crack propagation until critical crack size. The period of crack initiation is short relative to the crack propagation period. However, for a very small initial defect and low load levels, there may be a significant crack initiation period. In order to avoid a catastrophic accident during flight, the usual approach is to inspect the structures periodically at certain intervals. This paper deals with the case when the fatigue crack can be detected only by actual inspection. Scheduling of inspection times appears in a variety of reliability applications, quality control, medicine, nuclear energy, defence, etc. (Kaio and Osaki 1989; Munford and Shahani 1973; Paliou and Shinozuka 1987; Yang and Trapp 1974). Examples include maintaining continuous production processes (e.g., replacing worn-out tools in automated manufacturing systems), monitoring quality, stand-by systems, alarm systems, planning checkups for ageing aircraft, and so on. In many cases, there are estimable costs associated with the elapsed time between system failure and its detection. For example, if the system is a production system, the costs are associated with the amount of defective products (e.g., products manufactured outside tolerance limits), and the state of the system, good or failed, can only be determined by an inspection (e.g., by checking the quality of machine output). Thus, in such cases, it is important to inspect the unit from time to time in order to determine its condition. However, since these inspections are often costly, inspection times must be chosen so that undetected failure (or late detection) costs and inspection costs are balanced optimally. A commonly used inspection policy with constant intervals between inspections is not optimal relative to a certain cost model; optimum inspection policies have decreasing inspection intervals for aging systems. However, it may be quite difficult to find such an optimum policy. Different authors have produced many interesting and significant results for variations of inspection models The different models developed depend on the assumptions made regarding the time horizon, the amount of information available, the nature of cost functions, the objective of the models, the system's constraints, etc. The different models, for the most part, however, are very similar to a basic model presented by Barlow et al. (1963) and Barlow and Proschan (1965). This basic model is a pure inspection model, i.e., no preventive maintenance is assumed, and the system is not replaced on failure.

In this paper, we investigate this inspection-scheduling problem, and derive optimal schedules over the time span extended only until detection of failure under the main assumption that there is no knowledge concerning the parameters of the distribution function $F(x)$ of the lifetime of the system; this constraint is often met in practice. A simple approach is proposed for situations where it is difficult to quantify the costs associated with inspections and undetected failure, or when these costs vary in time. It allows one to find the inspection policies for detection of initial cracks in critical structural

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components of aircraft under the assumptions that the parameter values of the underlying distributions are unknown. Furthermore, obtaining inspection schedules under crack propagation is considered.

INSPECTION POLICY FOR DETECTION OF INITIAL CRACK OF DETECTABLE SIZE

Suppose an inspection is carried out at time t, and this shows that initial crack (which may be detected) has not yet occurred. We now have to schedule the next inspection. Let X be the random time to crack initiation. Then we schedule the next inspection at time $u>t$, where u satisfies

$$\Pr\{X > u; X > t\} = 1 - \alpha. \quad (1)$$

Equation (1) says that the next inspection is scheduled so that, with probability 1-α, the aircraft structure component is still working and free of initial crack prior to inspection.

Complete Information about $F_\theta(x)$

Let $F_\theta(x)$ be the cumulative distribution function of the time to crack initiation, where θ is a known parameter (in general, vector). Then the inspection times (u_1, u_2, …) can be calculated recursively as follows. It follows from (1) that

$$\frac{\overline{F}_\theta(u_{j+1})}{\overline{F}_\theta(u_j)} = 1 - \alpha, \quad j \ge 0, \quad (2)$$

where $\overline{F}_\theta(u) = 1 - F_\theta(u)$. It can be shown that (2) is equivalent to the equation

$$1 - \frac{\overline{F}_\theta(u_{j+1})}{\overline{F}_\theta(u_j)} = \frac{\overline{F}_\theta(u_j) - \overline{F}_\theta(u_{j+1})}{\overline{F}_\theta(u_j)}$$

$$= \frac{1 - F_\theta(u_j) - [1 - F_\theta(u_{j+1})]}{1 - F_\theta(u_j)}$$

$$= \frac{F_\theta(u_{j+1}) - F_\theta(u_j)}{1 - F_\theta(u_j)} = \alpha, \quad j \ge 0, \quad (3)$$

that is, in other words, the probability that the crack occurs in the time interval (u_j,u_{j+1}) without crack at time u_j is always assumed α.

It follows from (2) that

$$\overline{F}_\theta(u_{j+1}) = (1-\alpha)\overline{F}_\theta(u_j), \quad j \ge 0. \quad (4)$$

With u_0=0, u_1, u_2, … can be calculated recursively from (4). So that:

$$\overline{F}_\theta(u_j) = (1-\alpha)^j, \quad j = 1, 2, 3, \ldots, \quad (5)$$

the time u_j (j=1, 2, 3, ...) is given by

$$u_j = \overline{F}_\theta^{-1}[(1-\alpha)^j], \quad j = 1, 2, 3, \ldots. \quad (6)$$

Let N be the random number of inspections until the initial crack of detectable size occurs. Then

$$\Pr\{N \le j\} = \Pr\{X \le u_j\} = F_\theta(u_j) \quad (7)$$

and

$$E\{N\} = \sum_{j=0}^{\infty} j \Pr\{N = j\}$$

$$= \sum_{j=1}^{\infty} j\,[\Pr\{N > j-1\} - \Pr\{N > j\}]$$

$$= \sum_{j=0}^{\infty} P\{N > j\} = \sum_{j=0}^{\infty} \overline{F}_\theta(u_j) = \alpha^{-1}. \quad (8)$$

For example, if α=0.05 then, from (8), on average 20 inspections will be necessary.

Incomplete Information about $F_\theta(x)$

Let us assume that the parameter θ is unknown, but there is a sample of observations $\mathbf{X}$=(X_1, X_2, …, X_n) from $F_\theta(x)$. Let W=(X, $\mathbf{X}$) be an ancillary statistic (Nechval 1982, 1984; Nechval et al. 2001, 2002) whose distribution does not depend on θ. Then we schedule the next inspection at time $u>t$ on the basis of relation

$$\Pr\{W > w_u; W > w_t\} = \frac{\Pr\{W > w_u\}}{\Pr\{W > w_t\}} = 1 - \alpha, \quad (9)$$

where w_t=$w(t,\mathbf{x})$, w_u=$w(u,\mathbf{x})$.

Example 1

Let $X_{(1)} < X_{(2)} < \ldots < X_{(r)}$ be the first r ordered observations of time to initiation of a fatigue crack of detectable size for identical structural components of aircraft from a sample of size n from a two-parameter Weibull distribution with probability density function

$$f(x;\sigma,\delta) = \begin{cases} \dfrac{\delta}{\sigma}\left(\dfrac{x}{\sigma}\right)^{\delta-1} \exp[-(x/\sigma)^{\delta}], & x \ge 0, \\ 0, & \text{otherwise,} \end{cases} \quad (10)$$

(as the results of fatigue tests conducted on the components), where θ=(σ,δ), the parameters σ and δ (σ>0, δ>0) are unknown, i.e., we deal with Type II censoring.Let us assume that in a fleet of k aircraft there are km of the same individual structure components, operating independently. Suppose an inspection is carried out at time u_j, and this shows that initial crack (which may be detected) has not yet occurred. We now have to schedule the next inspection. Let $Y_{(1)}$ be the minimum time to crack initiation in the above components. In other words, let $Y_{(1)}$ be the smallest observation from an independent second sample of km observations also from the distribution (10). Then the inspection times can be calculated recursively as

$$u_{j+1} = \hat{\sigma}\exp(w_{j+1}/\hat{\delta}), \quad j\geq 1, \tag{11}$$

where u_1 is a time of the first inspection, w_{j+1} is determined from

$$\Pr\{W > w_{j+1}; W > w_j, {}^{\bullet}\mathbf{W}^r = {}^{\bullet}\mathbf{w}^r\} = \frac{\Pr\{W > w_{j+1}; {}^{\bullet}\mathbf{w}^r\}}{\Pr\{W > w_j; {}^{\bullet}\mathbf{w}^r\}}$$

$$= \frac{\Pr\left\{\hat{\delta}\ln\left(\frac{Y_{(1)}}{\hat{\sigma}}\right) > w_{j+1}; {}^{\bullet}\mathbf{w}^r\right\}}{\Pr\left\{\hat{\delta}\ln\left(\frac{Y_{(1)}}{\hat{\sigma}}\right) > w_j; {}^{\bullet}\mathbf{w}^r\right\}}$$

$$= \left(\begin{array}{l} \int_0^\infty s^{r-2} e^{s\hat{\delta}\sum_{i=1}^r \ln\left(x_{(i)}/\hat{\sigma}\right)} \\ \times\left(kme^{sw_{j+1}} + \sum_{i=1}^r e^{s\hat{\delta}\ln\left(x_{(i)}/\hat{\sigma}\right)} + (n-r)e^{s\hat{\delta}\ln\left(x_{(r)}/\hat{\sigma}\right)}\right)^{-r} ds \end{array} \right)$$

$$\times \left(\begin{array}{l} \int_0^\infty s^{r-2} e^{s\hat{\delta}\sum_{i=1}^r \ln\left(x_{(i)}/\hat{\sigma}\right)} \\ \times\left(kme^{sw_j} + \sum_{i=1}^r e^{s\hat{\delta}\ln\left(x_{(i)}/\hat{\sigma}\right)} + (n-r)e^{s\hat{\delta}\ln\left(x_{(r)}/\hat{\sigma}\right)}\right)^{-r} ds \end{array} \right)^{-1}$$

$$= 1-\alpha, \quad j\geq 1, \tag{12}$$

where

$$\Pr\{W > w; {}^{\bullet}\mathbf{w}^r\} = \Pr\left\{\hat{\delta}\ln\left(\frac{Y_{(1)}}{\hat{\sigma}}\right) > w; {}^{\bullet}\mathbf{w}^r\right\}$$

$$= \left(\begin{array}{l} \int_0^\infty s^{r-2} e^{s\hat{\delta}\sum_{i=1}^r \ln\left(x_{(i)}/\hat{\sigma}\right)} \\ \times\left(kme^{sw} + \sum_{i=1}^r e^{s\hat{\delta}\ln\left(x_{(i)}/\hat{\sigma}\right)} + (n-r)e^{s\hat{\delta}\ln\left(x_{(r)}/\hat{\sigma}\right)}\right)^{-r} ds \end{array} \right)$$

$$\times \left(\begin{array}{l} \int_0^\infty s^{r-2} e^{s\hat{\delta}\sum_{i=1}^r \ln\left(x_{(i)}/\hat{\sigma}\right)} \\ \times\left(\sum_{i=1}^r e^{s\hat{\delta}\ln\left(x_{(i)}/\hat{\sigma}\right)} + (n-r)e^{s\hat{\delta}\ln\left(x_{(r)}/\hat{\sigma}\right)}\right)^{-r} ds \end{array} \right)^{-1}, \tag{13}$$

$${}^{\bullet}\mathrm{W}^r = ({}^{\bullet}W_1, \ldots, {}^{\bullet}W_r), \quad {}^{\bullet}W_i = \hat{\delta}\ln\left(\frac{X_{(i)}}{\hat{\sigma}}\right), \quad i=1(1)r, \tag{14}$$

$\hat{\sigma}$ and $\hat{\delta}$ are the MLE's of σ and δ, respectively, and can be found from solution of

$$\hat{\sigma} = \left(\frac{\sum_{i=1}^r x_{(i)}^{\hat{\delta}} + (n-r)x_{(r)}^{\hat{\delta}}}{r} \right)^{1/\hat{\delta}}, \tag{15}$$

$$\hat{\delta} = \left[\begin{array}{l} \left(\sum_{i=1}^r x_{(i)}^{\hat{\delta}} \ln x_{(i)} + (n-r)x_{(r)}^{\hat{\delta}} \ln x_{(r)}\right) \\ \times\left(\sum_{i=1}^r x_{(i)}^{\hat{\delta}} + (n-r)x_{(r)}^{\hat{\delta}}\right)^{-1} - \frac{1}{r}\sum_{i=1}^r \ln x_{(i)} \end{array} \right]^{-1}. \tag{16}$$

For instance, consider the data of fatigue tests on a particular type of structural components of aircraft IL-86: $x_{(1)}$=5, $x_{(2)}$=6.25, $x_{(3)}$=7.5, $x_{(4)}$=7.9, $x_{(5)}$=8.1 (in number of 10^4 flight hours). It follows from (15) and (16), where r=n=5, that the maximum likelihood estimates of unknown parameters σ and δ are $\hat{\sigma} = 7.42603$ and $\hat{\delta} = 7.9081$, respectively. Thus, using (11) with u_1=4.1730 (×10^4 flight hours) (the time of the first inspection), we have obtained the following inspection time sequence (see Table 1).

Table 1: The Inspection Time Sequence

w_j	Time u_j (×10^4 flight hours)	Interval (flight hours) $u_{j+1}-u_j$
$w_1 = -4.55761$	u_1= 4.173000	–
$w_2 = -3.85897$	u_2= 4.558595	3855.95
$w_3 = -3.44800$	u_3= 4.801761	2431.66
$w_4 = -3.15474$	u_4= 4.983170	1814.09
$w_5 = -2.92590$	u_5= 5.129477	1463.07
$w_6 = -2.73805$	u_6= 5.252782	1233.05
$w_7 = -2.57851$	u_7= 5.359829	1070.47
⋮	⋮	⋮

It will be noted that some authors prefer to describe the time to crack initiation by the lognormal distribution, although there is not much experimental evidence to choose one or the other.

INSPECTION POLICY UNDER CRACK PROPAGATION

Fracture mechanics theory is used to determine the length of a propagating crack under random stress. This theory can predict crack size as a function of time. For the purposes of this study, it is assumed that a crack grows according to the Paris and Erdogan (1963) equation:

$$\frac{da(x)}{dx} = Q[a(x)]^{B}, \tag{17}$$

where $a(x)$ is the crack size of a fastener hole at x flight-hours, and Q and B are material parameters. This model has been used successfully to describe the observed propagation of a dominant crack in many experiments and the values Q & B have been established for a wide range of materials. Integrating eqn (17) from the initial crack length $a(\tau_0)$ at the time of crack initiation $x=\tau_0$ up to the current crack length $a(\tau)$ at time $x=\tau$, one obtains the relation between the crack size, $a(\tau)$, at any service time τ and the initial crack size, $a(\tau_0)$, as follows

$$a(\tau) = \frac{a(\tau_0)}{\left[1-[a(\tau_0)]^{B-1}[B-1]Q[\tau-\tau_0]\right]^{1/(B-1)}}. \tag{18}$$

For the special case in which B=1, it can easily be shown that

$$a(\tau) = a(\tau_0)\exp(Q[\tau-\tau_0]). \tag{19}$$

Available in-service inspection data for various types of aircraft indicate that the Weibull or lognormal distribution provides a reasonable fit for B and Q in both cases. In this paper, for the sake of simplicity but without loss of generality, only a special case in which B=1 is considered. This suggests, by taking logarithm, the following model

$$\ln[a(\tau)] = \ln[a(\tau_0)] + Q[\tau-\tau_0], \tag{20}$$

where Q follows a Weibull distribution, the cumulative distribution function of which is given by

$$F_\theta(q) = \begin{cases} 1-\exp[-(q/\sigma)^{\delta}], & q \ge 0, \\ 0, & \text{otherwise,} \end{cases} \tag{21}$$

with unknown parametric vector $\theta=(\sigma,\delta)$. Let $a^\bullet$ be the operational limit crack size for the degradation path, which is permitted for the initial crack to grow and reach $a^\bullet$ at time $x=T^\bullet$, then we can write

$$\ln(a^\bullet)=\ln[a(\tau_0)]+Q[T^\bullet-\tau_0], \tag{22}$$

where

$$T^\bullet-\tau_0 = [\ln(a^\bullet)-\ln[a(\tau_0)]]/Q = \frac{\ln[a^\bullet/a(\tau_0)]}{Q} \tag{23}$$

represents a time permitted for the initial crack to grow and reach the operational limit crack size $a^\bullet$. The distribution function of $T^\bullet-\tau_0$ is given by

$$G(t^\bullet) = \Pr\{T^\bullet-\tau_0 \le t^\bullet-\tau_0\}$$

$$= \Pr\left\{\frac{\ln[a^\bullet/a(\tau_0)]}{Q} \le t^\bullet-\tau_0\right\} = \Pr\left\{Q \ge \frac{\ln[a^\bullet/a(\tau_0)]}{t^\bullet-\tau_0}\right\}$$

$$= 1-F_\theta\left(\frac{\ln[a^\bullet/a(\tau_0)]}{t^\bullet-\tau_0}\right)$$

$$= \exp\left[-\left(\frac{\ln[a^\bullet/a(\tau_0)]}{\sigma[t^\bullet-\tau_0]}\right)^{\delta}\right], \quad t^\bullet-\tau_0>0. \tag{24}$$

Thus, a random variable $Z(0)\equiv Q=[\ln(a^\bullet/a(\tau_0))]/[T^\bullet-\tau_0]$ follows the Weibull distribution with the cumulative distribution function

$$F_\theta(z(0)) = \begin{cases} 1-\exp[-(z(0)/\sigma)^{\delta}], & z(0)>0, \\ 0, & \text{otherwise,} \end{cases} \tag{25}$$

Let $\mathbf{Z}=(Z_1, Z_2, \ldots, Z_n)$ be a sample of observations of size n from the Weibull distribution with the cumulative distribution function

$$F_\theta(z_i) = \begin{cases} 1-\exp[-(z_i/\sigma)^{\delta}], & z>0, \\ 0, & \text{otherwise,} \end{cases} \tag{26}$$

(as the results of fatigue tests conducted on the identical components), where

$$Z_i = \frac{\ln[a^\bullet/a(\tau_i)]}{T_i^\bullet-\tau_i}, \quad i=1(1)n, \tag{27}$$

the parameters σ and δ ($\sigma>0$, $\delta>0$) are unknown. When there is only one structural component with crack that is still operational at the inspection time u_j, then the next inspection time is u_{j+1}, which is the solution of

$$u_{j+1} = u_j + \frac{\ln[a^\bullet/a(u_j)]}{\hat{\sigma}\exp(w_{j+1}/\hat{\delta})}, \quad j\ge 1, \tag{28}$$

where w_{j+1} is determined from

$$\Pr\{T^\bullet > u_{j+1}; T^\bullet > u_j, {}^\bullet\mathbf{W}^n={}^\bullet\mathbf{w}^n\}$$

$$= \frac{\Pr\{T^\bullet > u_{j+1}; {}^\bullet\mathbf{w}^n\}}{\Pr\{T^\bullet > u_j; {}^\bullet\mathbf{w}^n\}} = \frac{\Pr\left\{\hat{\delta}\ln\left(\frac{Z(j+1)}{\hat{\sigma}}\right) < w_{j+1}; {}^\bullet\mathbf{w}^n\right\}}{\Pr\left\{\hat{\delta}\ln\left(\frac{Z(j)}{\hat{\sigma}}\right) < w_j; {}^\bullet\mathbf{w}^n\right\}}$$

$$= \frac{1-\Pr\{W_{j+1} > w_{j+1}; {}^\bullet\mathbf{w}^n\}}{1-\Pr\{W_j > w_j; {}^\bullet\mathbf{w}^n\}} = 1-\alpha, \qquad (29)$$

$$\Pr\{W_j > w_j; {}^\bullet\mathbf{w}^n\} = \Pr\left\{\hat{\delta}\ln\left(\frac{Z(j)}{\hat{\sigma}}\right) > w_j; {}^\bullet\mathbf{w}^n\right\}$$

$$= \frac{\int_0^\infty s^{n-2} e^{s\hat{\delta}\sum_{i=1}^n \ln(z_i/\hat{\sigma})}\left(e^{sw_j} + \sum_{i=1}^n e^{s\hat{\delta}\ln(z_i/\hat{\sigma})}\right)^{-n} ds}{\int_0^\infty s^{n-2} e^{s\hat{\delta}\sum_{i=1}^n \ln(z_i/\hat{\sigma})}\left(\sum_{i=1}^n e^{s\hat{\delta}\ln(z_i/\hat{\sigma})}\right)^{-n} ds}, \qquad (30)$$

$$Z(j) = \frac{\ln[a^\bullet / a(u_j)]}{T^\bullet - u_j}, \quad W_j = \hat{\delta}\ln\left(\frac{Z(j)}{\hat{\sigma}}\right), \qquad (31)$$

$${}^\bullet\mathrm{W}^n = ({}^\bullet W_1, \ldots, {}^\bullet W_n), \quad {}^\bullet W_i = \hat{\delta}\ln\left(\frac{Z_i}{\hat{\sigma}}\right), \quad i=1(1)n, \qquad (32)$$

$\hat{\sigma}$ and $\hat{\delta}$ are the MLE's of σ and δ, respectively, and can be found from solution of

$$\hat{\sigma} = \left(\frac{1}{n}\sum_{i=1}^n [z_i]^{\hat{\delta}}\right)^{1/\hat{\delta}}, \qquad (33)$$

$$\hat{\delta} = \left[\left(\sum_{i=1}^n [z_i]^{\hat{\delta}} \ln z_i\right)\left(\sum_{i=1}^n [z_i]^{\hat{\delta}}\right)^{-1} - \frac{1}{n}\sum_{i=1}^n \ln z_i\right]^{-1}. \qquad (34)$$

It will be noted that u_1 is given by

$$u_1 = \tau_0 + \frac{\ln[a^\bullet / a(\tau_0)]}{\hat{\sigma}\exp(w_1/\hat{\delta})}, \qquad (35)$$

where w_1 is determined from

$$\Pr\{T^\bullet > u_1; {}^\bullet\mathbf{w}^n\} = \Pr\left\{\hat{\delta}\ln\left(\frac{Z(1)}{\hat{\sigma}}\right) < w_1; {}^\bullet\mathbf{w}^n\right\}$$

$$= \Pr\{W_1 < w_1; {}^\bullet\mathbf{w}^n\} = 1 - \Pr\{W_1 > w_1; {}^\bullet\mathbf{w}^n\}$$

$$= 1 - \frac{\int_0^\infty s^{n-2} e^{s\hat{\delta}\sum_{i=1}^n \ln(z_i/\hat{\sigma})}\left(e^{sw_1} + \sum_{i=1}^n e^{s\hat{\delta}\ln(z_i/\hat{\sigma})}\right)^{-n} ds}{\int_0^\infty s^{n-2} e^{s\hat{\delta}\sum_{i=1}^n \ln(z_i/\hat{\sigma})}\left(\sum_{i=1}^n e^{s\hat{\delta}\ln(z_i/\hat{\sigma})}\right)^{-n} ds} = 1-\alpha.$$

(36)

Example 2

For instance, consider the data of observations of fatigue cracks on a particular type of structural components in a fleet of 10 aircraft. The data are for a complete sample of size 10 (with n=6).

Table 2: Fleet inspection results for $a(\tau_0)$=0.02 mm initial discontinuity

Aircraft	u_1 ($\times 10^4$)	$a(u_1)$ (mm)	$a^\bullet$ (mm)	u_2 ($\times 10^4$)
1	0.9	No crack	10	2.484
2	1.1	No crack	10	2.711
3	1.2	1.00	10	1.805
4	1.3	No crack	10	2.960
5	1.5	1.30	10	2.072
6	1.6	1.50	10	2.148
7	2.2	No crack	10	4.480
8	2.4	1.20	10	3.244
9	2.7	2.00	10	3.417
10	2.9	1.00	10	4.001

Goodness-of-Fit Testing

We assess the statistical significance of departures from the Weibull model by performing empirical distribution function goodness-of-fit test. We use the S statistic (Kapur and Lamberson 1977). For this example,

$$S=0.2396 < S_{n=5;\,1-\alpha=0.95}=0.73. \qquad (37)$$

Thus, there is not evidence to rule out the Weibull model. The maximum likelihood estimates of the parameters σ and δ are respectively $\hat{\sigma} = 2.505386$ and

$\hat{\delta} = 3.666068$. Using the relationship (28), the (example) inspection results can be extrapolated from the expected initial crack size to the time of the next inspection when the operational limit (or critical) crack depth is not exceeded with confidence level $1-\alpha$=0.95, through the detected crack size, as presented in Table 1.

CONCLUSIONS

In this paper, we present innovative statistical models for decision-making in aircraft service. The results of computer simulations confirm the validity of the theoretical predictions of performance of the suggested models. The authors hope that this work will stimulate further investigation using the approach on specific applications to see whether obtained results with it are feasible for realistic applications.

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