AUTOMATIC CREATION OF NUMERICAL MODELS OF SYSTEMS SPECIFIED BY PLA METHOD

Henrikas Pranevičius
Vytautas Germanavičius
Giedrius Tumelis
Kaunas University of Technology
Department of Business Informatics
Studentų 56, Kaunas, Lithuania
E-mail: hepran@if.ktu.lt

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ABSTRACT

The paper discusses the method of constructing numerical models for systems described by Markov processes with discrete states and continuous time. System equations for calculation of stationary probabilities are generated from the system specification described by piece linear aggregates. The approach of the consequence embedding of Markov chains is used for computing stationary probabilities in Markov processes. The paper also presents the description of the software tools that permit to automate the following stages of numerical models: creation of a specification, generation of the system graph from the specification, calculation of stationary probabilities of the system, and calculation of the system characteristics. In addition, it presents an example of a numerical model for the data transmission tract with the adaptive commutation .

INTRODUCTION

The Markov processes are widely used for construction of models of such systems as computer networks and telecommunication systems. Everybody knows that the creation of analytical models requires much effort. Appliance of numerical methods permits to create a model for a wider class of systems.

The book published by Pranevičius and Valakevičius in 1996 describe the system that automates the construction of Markov models. The embedded Markov chains are used for computing stationary probabilities of the Markov process. The performance of the system described in the event language is used for generating the system of Kolmogorov equations.

This paper introduces a system that automates the construction of Markov models while the performance of the system is formally described by the PLA method.

The piece-linear aggregate approach (PLA) was proposed by Buslenko (Buslenko 1971) for the simulation of complex systems. A method for the formal description of PLA based on the notion of controlling sequences was developed by Pranevičius in 1982. The doctoral thesis of H. Pranevičius (Pranevičius 1983) shows how PLA with the semantics of controlling sequences can be applied for the formal specification, validation, and simulation of complex systems. The mathematical model proposed here relies on several implemented simulation systems (Pranevičius et al., 1994) running on machines of various platforms. The PLA formalization approach permits both creation of simulation models and validation of specifications on the base of single specification. The combination of these tasks is very important for analysing the distributed software systems.

The theoretical background of PLA is piece-linear Markov processes. If the duration of operations used in PLA specifications are distributed by exponential law, the piece-linear Markov process becomes linear Markov process involving discrete set of states and continuous time. The presumptions allows to transform a PLA specifications to the specification of Markov processes. This makes it possible to develop a states graph used for calculation of stationary probabilities of Markov processes for the analysed system.

The system proposed in this paper aimed at creation of numerical models differs from the widely known MACOM system (Scittinick and Muller-Clostermann 1990), which is a software tool for the model based on the performance evaluation of communication systems. The models in MACOM are specified by graphical interactive means while the model solution is performed by applying numerical techniques for Markovian models. The MACOM model is a network of sources, sinks, resources and routing elements. In case the requested resource is currently not available, a load unit may utilize an alternative resource, may hold, or get lost.

Section 2 of the paper presents an algorithm for calculating stationary probabilities of Markovian processes; this algorithm applies the method of embedded Markovian chains. Section 3 describes the meta model of aggregate specification. Section 4 introduces the formal specification of the data transfer tract performance and the results of modelling.

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1. GENERATION A SET OF EQUATION AND COMPUTATION OF STATIONARY PROBABILITIES USING SEQUENTIAL EMBEDDING OF MARKOV CHAINS

Stationary probabilities of system are calculated from equation system (1):

$$\sum_{j=1}^{N} q_i \lambda_{ij} = \sum_{j=1}^{N} q_j \lambda_{ji}, \quad i=\overline{1,N}, \qquad (1)$$

where q_i – steady state probabilities of Markov process.

Book (Pranevičius, 1983) shows that if the duration of operations are distributed by exponential law, the piece-linear Markov process used to describe PLA, becomes a Markov process with discrete set of states and continuous time. This makes it possible to develop a system of equations (1) from graph $G=(V,\Lambda)$ of the system described by PLA, here

V - a set of graph nodes (states of system);

Λ - a set of graph edges (transitions between the states of the system).

Graph nodes are related to the states of the system: $f:V\rightarrow Z$, here Z - a set of the system states. Edges of graph are related to the events that can change the state of the system and the intensity of the events.

Let us say that the description of the system uses the set of PLA $A=\{A_1, A_2, \dots, A_n\}$. There must be described for each aggregate $\forall A_i \in A$:

1. A set of input signals X_i;
2. A set of output signals Y_i;
3. Each input signal $x\in X_i$ is related with external event $e'(x)$, which belongs to a set of external events E'_i.
4. A set internal events E''_i.
5. Controlling sequence, which describes the intensity of internal events E''_i of the aggregate A_i.
6. The discrete component $\nu_i(t)$ of the state of the aggregate A_i.
7. The continuous component $z_{\nu i}(t)=\bigcup_{j=1}^{m} w(e''_j,t)$ of the state of the aggregate A_i, here $w(e''_j,t)$ is a continuous variable related to the external event e''_j, and m – the number of internal events in the set E''_i.
8. The state of aggregate $z_i(t)$ is divided into two components: the discrete component $\nu_i(t)$ and the continuous component $z_{\nu i}(t)$: $z_i(t)=(\nu_i, z_{\nu i})$
9. Each internal and external event has two operators: H and G. Operator H changes values of the continuous and discrete variables, while operator G forms the output signals Y_i.

Under the process of composition of aggregates, the continuous components $z_\nu=\bigcup_{i=1}^{n} z_{\nu i}$ and the discrete components $\nu=\bigcup_{i=1}^{n} \nu_i$ oof all aggregates are combined into one vector $z(t)=(\nu, z_\nu)$. The set of vectors $z(t)$ is marked Z. All the internal events of all aggregates are combined into one set $E=\bigcup_{i=1}^{n} E''_i$. Operators H and G are combined by the scheme of connections of aggregates into a new set of operators $\widetilde{H}$. Operators in the set $\widetilde{H}$ after the event $e\in E$ changes its state from one state $z\in Z$ to a new set of states: $\widetilde{H}: Z\times E\rightarrow \boldsymbol{P}(Z)$. It happens because of the output signals that can be transferred to different inputs of aggregates with different probabilities.

The algorithm for creation of the graph :

1. The following information is taken from formal specification:
   1.1. The set of internal events E;
   1.2. The initial state of system $z(0)$;
   1.3. The set of states of the system Z, which initially has only one member - $z(0)$: $Z=\{z(0)\}$.
   1.4. A set of analysed states of the system Z_{rew}, which initially is empty $Z_{rew}=\emptyset$
2. Take a non-analysed states $z\in Z/Z_{rew}$.
   2.1. For each possible to occur event in the state z - $\forall e\in E|w(e)\neq 0$
      2.1.1. Calculation of set of states to which system can pass: $Z^{+}=\widetilde{H}(z,e)$
      2.1.2. For each $\forall z^{+}\in Z^{+}$
         2.1.2.1 put z^{+} to Z, $Z'=Z\cup\{z^{+}\}$
         2.1.2.2 Add to the set of transitions Λ the transition from the current state z to z^{+}: $\Lambda'=\Lambda\cup\{(z\rightarrow z^{+})\}$.
   2.2. Put z to $Z_{išn}$: $Z'_{isn.}=Z_{isn}\cup\{z\}$.
3. Create set of graph nodes V from the set of states of the system Z: $V=\{z:Z\bullet f^{-1}(z)\}$.

Computation of the stationary probabilities p_i, ($i=\overline{1,N}$, here N is the number of states) of states of the system involves two stages:

1. the stage of embedding the Markov chains
2. the stage of computing the steady-stage probabilities (Pranevičius, Valakevičius 1994).

The algorithm takes the following form:

Let $\lambda_N = \left(\lambda_{ij}^{(N)}\right)_{N \times N}$ be the matrix of transition rates for a Markov chain on the set of states $S = \{s_1, \ldots, s_N\}$.

1. The stage of embedding the Markov chains:

$$\lambda_{ij}^{(k)} = \lambda_{ij}^{(k+1)} + \frac{\lambda_{i,k+1}^{(k+1)} \cdot \lambda_{k+1,i}^{(k+1)}}{\overline{S}_{k+1}^{(k+1)}}, \qquad i,j = \overline{1,k};\ k = \overline{(N-1),1}$$

$$\overline{S}_{k+1}^{(k+1)} = \sum_{\substack{j=1 \\ j \neq k+1}}^{N} \lambda_{k+1,j}^{(k+1)},$$

2. The stage of computing the stationary probabilities:

$$r_1^{(i)} = 1,$$

$$r_i^{(k+1)} = \begin{cases} r_i^{(k)}, & i = \overline{1,k}, \\ \dfrac{\sum_{j=1}^{N-1} r_j^{(k)} \lambda_{ji}^{(k+1)}}{\overline{S}_{k+1}^{(k+1)}}, & i = k+1; \end{cases} \quad k = \overline{1,(N-1)}$$

$$q_i = \frac{r_i^{(N)}}{\sum_{j=1}^{N} r_j^{(N)}}, \ i = \overline{1,N} \quad .$$

2. THE AGGREGATE SPECIFICATION LANGUAGE

The aggregate specification method is presented in the book (Pranevičius et al., 1994). It presents the meta-model of this specification method described using the UML notation.

It is demonstrated in Figure 1 - the classes' diagram. It describes the structure of data of the linear aggregate: inputs, outputs, discrete and continuous variables of an aggregate.

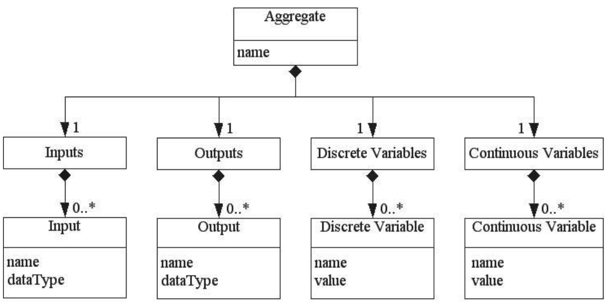

Fig. 1. The Data Structure of an Aggregate.

Continuous variables are used for describing the time moments in which the internal events occur after the operations terminate.

Signals coming through inputs of an aggregate cause external events.

The modelling system consists of aggregates and a connection between them (see Figure 2).

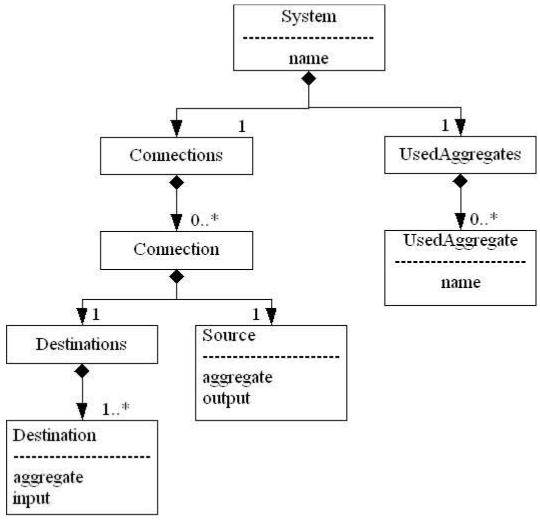

Fig. 2. The Structure of the System of Aggregates.

Aggregates interact in the system. Connections are used for describing their interaction. Signals can be transmitted by connections between the aggregates. The signals' data structure is demonstrated in Figure 3.

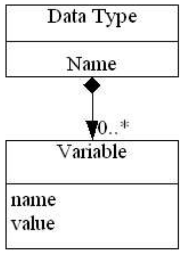

Fig. 3. The Structure of Input and Output Signals.

The connections join the output of one aggregate with one or several inputs of one or several aggregates of the same type.

3. THE SOFTWARE TOOL USED FOR CREATION OF NUMERICAL MODELS

A software tool used for creation of numerical models and performance of the following functions was developed (see Figure 4):

1. The graphical interface (1) is used to form the specification of the system that uses the aggregate method described in the previous chapter „The Meta model of the Aggregate Specification Language". Specification is saved using the XML-based language.

2. The transformation module (2) transforms the aggregate specification described in the XML into the code for generating the graph of states in the system.

3. The graph generator module (3) compiles the graph generator code and generates a graph of states by applying the algorithm described in chapter „Generation a Set of Equation and Computation of Stationary Probabilities Using Sequential Embedding of Markov Chains".

4. The graph validation module (4) validates

correctness of the aggregate specification using the graph of states. The program checks the reachability graph nodes as well as detects dead-locks, and cycles.

5. The module of calculation of stationary probabilities (5) calculates the stationary probabilities of states of the system using the method of embedded Markov chains from the equation system (1). This method is described in chapter „Generation a Set of Equation and Computation of Stationary Probabilities Using Sequential Embedding of Markov Chains“.
6. The module of calculation of characteristics (6) calculates characteristics of the system using the calculated stationary probabilities. The current version of this module calculates the average values of characteristics of the system.

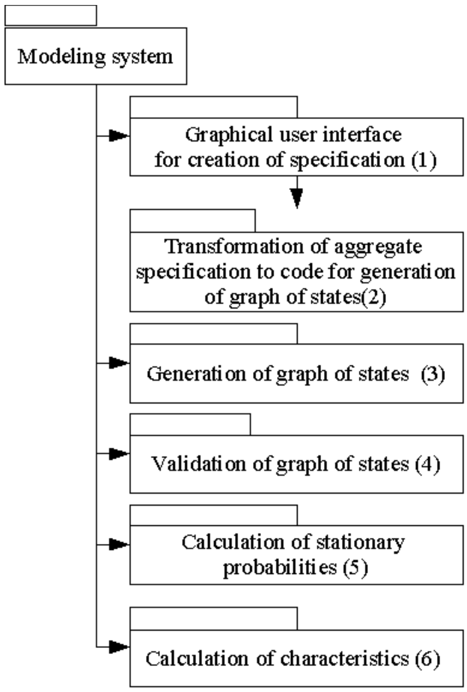

Fig 4. The Structure of the Software Tool for Creation of Numerical Models

4. EXAMPLE

This chapter presents the numerical model of the data transmission track with an adaptive switching method.

The Conceptual Model Of The System

The performance of such a system may be represented as the following queuing system (see Figure 6): the message flow arriving to the track is considered to be distributed by Poisson distribution and the service times are independent and exponentially distributed. Two request flows enter the system: the flow of file sessions with rate λ_1, and the packets flow with rate λ_2.

There are three groups of channels in the track: N_f channels are for the transmission of the file sessions, N_p channels for the transmission of the packets and N_{fp} channels for the transmission either the file sessions or the packets.

The file sessions are served if there are any free channels, otherwise they are rejected. If all the channels are occupied, the message packets wait in the queue Q, the length of which is limited to l.

In case all the channels N_f are occupied when the file session enters the system, it can occupy a free channel in the group N_{fp} only if the number of packets in the queue Q does not exceed l_1 $(0 < l_1 < l)$.

Packets are served by channels of the group N_p, by free channels of the group N_{fp} and during the intervals between the messages in a file sessions transmitted by the channels. A part of idle intervals in the file session is represented by ρ. If a packet finds more than l_2 $(l_1 < l_2)$ requests in the queue, it occupies one channel occupied by file sessions in the group N_{fp}. A file session is served by a channel at the rate μ_1 and a packet is served by a channel at the rate μ_2.

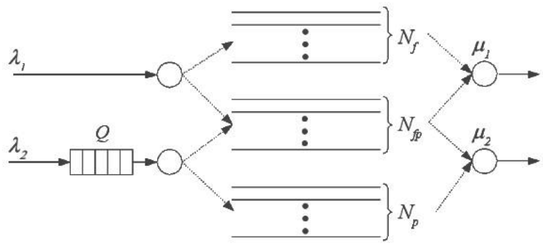

Fig. 5. Presentation of Data Transmission Track by Queuing System

Aggregate Specification Of The System

Aggregate specification is presented by 9 items:

1. The set of input signals $X = \varnothing$.
2. The set of output signals $Y = \varnothing$.
3. The set of external events $E' = \varnothing$.
4. The set of internal events

$$E'' = \{e_1'', e_2'', e_3'', e_4''\},$$

where e_1'' – the arrival of a file session;

e_2'' – the arrival of a packet;

e_3'' – the completion of serving the file session;

e_4'' – the completion of serving the packet.

5. The transition rates between the system states:

$$e_1'' \to \lambda_1, \; e_2'' \to \lambda_2, \; e_3'' \to \mu_i, \; e_4'' \to \mu_2 .$$

6. The discrete component of state

$$\nu(t) = \{n_f(t), n_{fp}(t), n(t)\},$$

where $n_f(t)$; – the number of the occupied channels in the group N_f at the moment t;

n_{fp} – the number of the channels occupied by file sessions in the group N_{fp} at the moment t;

n(t) – the length of the packet queue *Q* at the moment *t*.

7. The continuous component of state

$$z_\nu(t)=\{w(e_1'',t),w(e_2'',t),w(e_3'',t),w(e_4'',t)\}.$$

8. Initial state

$$z(t)=\{0,0,0,<\infty,<\infty,\infty,\infty\}.$$

9. Transition operators:

$H(e_1'')$: /The file session has arrived/

$$n_f(t+0)=\begin{cases}n_f(t)+1, \text{if } n_f(t)<n_f,\\ n_f(t), \text{otherwise};\end{cases}$$

$$n_{fp}(t+0)=\begin{cases}n_{fp}(t)+1, \text{if } (n_f(t)=N_f)\wedge(n_{fp}<N_{fp})\wedge(n<l_1),\\ n_{fp}, \text{otherwise};\end{cases}$$

$$n(t+0)=n(t);$$

$$w(e_1'',t+0)=w(e_1'',t);$$

$$w(e_2'',t+0)=w(e_2'',t);$$

$$w(e_3'',t+0)=(n_f(t)+n_{fp}(t))^*\mu_1;$$

$$w(e_4'',t+0)=\begin{cases}n_p(t)+(N_{fp}(t)-n_{fp}(t))+(n_{fp}(t)+n_f(t)\cdot\rho)\cdot\mu_2,\\ \qquad\text{if } n(t)>0\\ 0, \text{otherwise};\end{cases}$$

$H(e_2'')$:

$$n_f(t+0)=n_f(t);$$

$$n_{fp}(t+0)=\begin{cases}n_{fp}(t)-1, \text{if } (n_{fp}>0)\wedge(n<l_2),\\ n_{fp}(t), \text{otherwise};\end{cases}$$

$$n(t+0)=n(t)+1;$$

$$w(e_1'',t+0)=w(e_1'',t);$$

$$w(e_2'',t+0)=w(e_2'',t);$$

$$w(e_3'',t+0)=w(e_3,t);$$

$$w(e_4'',t+0)=\begin{cases}n_p(t)+(N_{fp}(t)-n_{fp}(t))+(n_{fp}(t)+n_f(t)\cdot\rho)\cdot\mu_2,\\ \qquad\text{if } n(t)>0\\ 0, \text{otherwise};\end{cases}$$

$H(e_3'')$:

$$n_{fp}(t+0)=\begin{cases}n_f(t)-1, \text{if } (n_f>0)\wedge n_{fp}(t)=0,\\ n_f(t), \text{otherwise};\end{cases}$$

$$n_{fp}(t+0)=\begin{cases}n_{fp}(t)-1, \text{if } n_{fp}>0,\\ n_{fp}(t), \text{otherwise};\end{cases}$$

$$n(t+0)=n(t);$$

$$w(e_1'',t+0)=w(e_1'',t);$$

$$w(e_2'',t+0)=w(e_2'',t);$$

$$w(e_3'',t+0)=(n_f(t)+n_{fp}(t))^*\mu_1;$$

$$w(e_4'',t+0)=\begin{cases}n_p(t)+(N_{fp}(t)-n_{fp}(t))+(n_{fp}(t)+n_f(t)\cdot\rho)\cdot\mu_2,\\ \qquad\text{if } n(t)>0\\ 0, \text{otherwise};\end{cases}$$

$H(e_4'')$:

$$n_f(t+0)=n_f(t);$$

$$n_{fp}(t+0)=n_{fp}(t);$$

$$n(t+0)=\begin{cases}n(t)-1, \text{if } n(t)>0,\\ 0, \text{otherwise};\end{cases}$$

$$w(e_1'',t+0)=w(e_1'',t);$$

$$w(e_2'',t+0)=w(e_2'',t);$$

$$w(e_3'',t+0)=w(e_3,t);$$

$$w(e_4'',t+0)=\begin{cases}n_p(t)+(N_{fp}(t)-n_{fp}(t))+(n_{fp}(t)+n_f(t)\cdot\rho)\cdot\mu_2,\\ \qquad\text{if } n(t)>0\\ 0, \text{otherwise};\end{cases}$$

Results Of The Modelling

The experiments were carried out with the following data, see Table 1:

Table 1. System's parameters

N_f	N_{fp}	N_p	l	l_1	l_2	μ_1	μ_2	
10	9	1	10	3	8	0,0055	6	0,5

A fragment of the state graph of the data transmission track is presented in Figure 6, when *N_f=1, N_{fp}=2, l=5.*

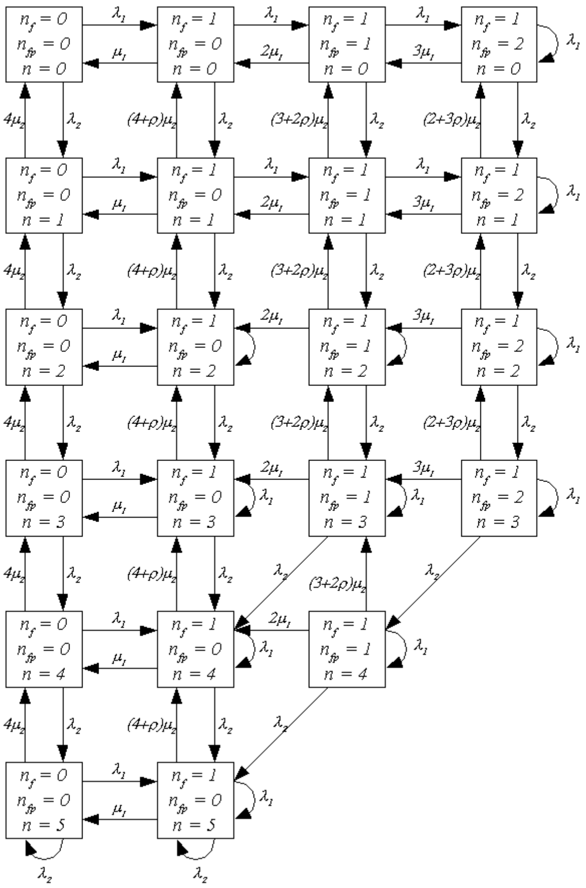

Fig. 6. The State Graph for the Analysed Data Transmission System

Following formulas are used for calculation of the characteristics of the described system:

$$p = \sum_{n=0}^{l} p\left(n_f = N_f, n_{fp} = N_{fp}, n\right) + \\ + \sum_{n_{fp} < N_{fp}} \sum_{n = l_1 + l_2}^{l} p\left(n_f = N_f, n_{nf}, n\right),$$

where p is the probability of rejection of a file session;

$$\bar{n}_f = \sum_{n_f=0}^{N_f} \sum_{n_{fp}=0}^{N_{fp}} \sum_{n=0}^{l} \left(n_f + n_{fp}\right) p\left(n_f, n_{fp}, n\right),$$

where $\overline{n_f}$ is the average number of the channels occupied by file sessions;

$$\bar{n} = \sum_{n_f=0}^{N_f} \sum_{n_{fp}=0}^{N_{fp}} \sum_{n=0}^{l} n p\left(n_f, n_{fp}, n\right),$$

where $\bar{n}$ is the average length of a packet queue;

$$\bar{t} = \frac{\bar{n} + 1}{\left(N_f + N_{fp} + N_p - \bar{n} + \bar{n} \cdot \rho\right) \cdot \mu_2},$$

where $\bar{t}$ is average time of the packet being in the system. The results of modeling are presented in Table 2.

Table 2 Results of Modelling

λ_1	λ_2	μ_1	μ_2	p	$\overline{n_f}$	$\bar{n}$	$\bar{t}$
0,0064	7,20	0,0055	6	7,49E-017	1,2	0,02	0,05
0,0192	21,7	0,0055	6	1,49E-008	3,5	0,22	0,07
0,0256	28,9	0,0055	6	1,19E-006	4,6	0,43	0,08
0,0384	43,4	0,0055	6	2,46E-004	6,5	1,25	0,13
0,048	50,6	0,0055	6	1,75E-003	7,4	1,86	0,18

Created numerical model was analysed with several numbers n of states of the system . The following parameters of the system were changed in order to do that: $n_{f.max}$, $n_{fp.max}$, l_1 , l_2 , Q_{max} . 10 graphs of states generated, one of them with more than half million states. The results of the calculations are presented in Table 3.

Table 3: Dependency of the Number of States on the Parameters of the System.

$n_{\_f.max}$	$n_{fp.max}$	l_1	l_2	q_{max}	n
1	1	1	2	3	16
5	5	2	4	5	216
9	9	4	7	9	990
12	12	4	9	12	2158
14	14	4	9	14	3225
16	16	6	11	16	4743
18	18	6	11	18	6460
20	20	6	11	20	8505
20	20	10	15	20	9051
96	96	10	15	96	598393

It was calculated stationary probabilities for every created graphs and evaluated calculation time. The results are presented in Table 4 and Figure 7.

Table 4: Calculation Times of Stationary Probabilities

n	Calculation times in s		
	Gauss elimination algorithm	Algorithm of Embedded Markov chains	Gauss-Seidel iterative algorithm
16	0,03	0,01	0,04
216	0,08	0,15	0,04
990	3,07	3,58	0,14
2158	74,10	26,62	0,32
3225	383,84	74,28	0,25
4743	2091,44	195,09	0,61
6460	...	447,32	0,52
8505	...	972,43	1,02
9051	...	1028.01	1,12
598393			~ 400,00

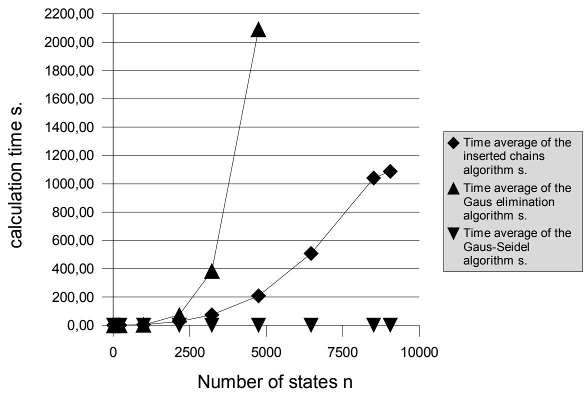

Fig 7. Dependencies of Times of Calculation of Stationary Probabilities

The diagram shows that for solving equation systems the Gauss elimination method is slowest. The method of inserted Markov chains is approximately 10 times quicker and is less complex. The Gauss-Seidel iterative algorithm is much quicker than both Gauss elimination and the inserted Markov chains method.

CONCLUSIONS

From the analysis provided here it becomes clear that the formal methods of PLA, which were used until now only for the imitation modelling and validation of formal specifications, can be used for creation of numerical models, too. It only requires the duration of operations to be distributed by the exponential law.

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AUTHORS BIOGRAPHY

Henrikas Pranevicius, professor, habil. dr., the head of the Business Informatics Department at the Kaunas University of Technology. His research activity is about the use of piece-linear aggregates (PLA) for formal specification, validation and modeling of distributes system. The results of investigations have been successfully applied creating the computerized systems for specification, validation, and simulation/modeling of computer network protocols, logistic systems. H.Pranevičius has published over 200 scientific papers and 5 monographs.

Vytautas Germanavičius PhD student of the Business Informatics Department at the Kaunas University of Technology (Lithuania) since 2002. The title of his thesis: "Creation of Numerical Markov Models Specified by the Aggregate Approach".

Giedrius Tumelis PhD student of the Business Informatics Department at the Kaunas University of Technology (Lithuania) since 2003. The title of his thesis: "The Complexity of Algorithms' Calculation Stationary Probabilities for Embedded Markov Chain Approach".