ON AN INVENTORY CONTROL MODEL WITH RANDOM LEAD TIME AND RANDOM DEMAND

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ABSTRACT

A single-product inventory control model with different distributions of demand for goods and random lead time is considered. The proposed model uses the regenerative approach. It gives possibility to find optimal values of reorder point and order quantity. Criterion of optimization is minimum of average expenses for goods holding, ordering and losses from deficit per a time unit. Suggested model will be examined in solving practical tasks for Latvian Railway company.

INTRODUCTION

Transport system in the Latvian Republic is an important economic field of the country, which together with the communication industry provides about 21% of the state national gross output and 45% of currency incomes. The Latvian railway as a part of the transport industry of Latvia is a large and complex system and has a number of specific features, which add sufficiently to the efficiency of its performance. Among them we should note the following: a complex, far-flung railways network; large distances between related objects; high dynamics of processes; dependence on random factors; high requirements for reliability and safety of transportation systems; and large financial, labor and material resources. Therefore, even small errors in management and planning the process of transportation result in a considerable decrease of the efficiency of the railway company performance and of the quality of the clients' service and in big financial losses, which, no doubt, negatively affects the development of Latvian economics as a whole.

One of the most important problems for railways' enterprises is to maintain the rolling stock in working conditions. We have to keep up some stock of rather wide range of spare parts and equipment for rolling stock maintenance. If we increase the stock, it makes more expensive the railway transport services; on the other hand it eliminates the losses from the idle rolling stock, expecting reparations and technical services.

It is a quite complicated mathematical task to find the optimal solution for the necessary stock, if you are working in the railways industry. The inventory control management model should take into account the random demand for the spare parts and consider the whole supply chain of the companies, which have influence on spare parts deliveries "manufacturer – supplier – intermediate company – transport company". For the construction the effective inventory control system authors propose to use the modern mathematical tools: inventory control theory, time series analysis, and forecasting methods (Kopytov et al., 2004). In the given paper the inventory control model for one type of product is considered.

Different types of stochastic inventory models are considered in literature (Prabhu 1967; Chopra and Meindl 2001; Ross 1992; Ryzhikov 2001). In practice it is common for inventory manager to answer two basic questions: how much to order and when to order, and there are many different types of inventory control models which provide the decision-maker with a satisfactory solution.

In previous authors' works the main attention was paid to economical aspects of inventory problem. We have to take into account that the sum of costs for goods ordering, holding and losses from deficit should be minimal. In the given paper we suppose inventory model using the same economical criteria. This paper is based on the previous paper (Kopytov and Greenglaz 2004), but here we consider random demand of goods and lead time with distributions for which some constraints are fulfilled.

DESCRIPTION OF THE MODEL

We consider a single-product inventory control model under the following conditions. The demand for goods D_τ for period of time τ after goods delivering is random variable with known distribution $f_{D_\tau}(x)$. We suppose that if m and σ are the mean and standard deviation for X=D_1, then $\tau \cdot m$ and $\sqrt{\tau} \cdot \sigma$ are mean and standard deviation for demand D_τ. We suppose in particular that the distribution of the demand for goods D_τ may be:

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either gamma

$$f_{D_\tau}(x) = \frac{(\lambda x)^{\tau-1}}{\Gamma(\tau)} \lambda \cdot e^{-\lambda x},$$

where $\Gamma(\tau)$ is the gamma-function,
or truncated normal

$$\tilde{f}_{D_\tau}(x) = \begin{cases} 0, & x \le 0; \\ \varphi(x, m\tau, \sigma\sqrt{\tau}) \dfrac{1}{\int_0^\infty \varphi(y, m\tau, \sigma\sqrt{\tau}) dy}, & x > 0, \end{cases}$$

where $\varphi(x; m\tau, \sigma\sqrt{\tau})$ is the density function of normal distribution $N(m\tau, \sigma\sqrt{\tau})$,
and etc. (Andronov et al. 2004).

In the moment of time, when the stock level falls till certain level *r*, a new order is placed. The quantity *r* is called as *reorder point*. The order quantity (order volume) *q* is constant. We suppose that $q \ge r$.

The lead time *L* (time between placing an order and receiving it) is a random variable with known distribution $f_L(t)$ as well. It may be similar to distributions for demand mentioned above. There is the possible situation of deficit, when during lead time *L* the demand D_L exceeds the value of reorder point *r*. We suppose that in case of deficit, the last cannot be covered by expected order, i.e. the shortage will be lost.

Thus we can determine the quantity of goods *Z* in stock in the time moment immediately after order receiving as function of demand D_L during lead time *L* (see Fig. 1):

$$Z = \begin{cases} r + q - D_L, & D_L < r; \\ q, & D_L \ge r. \end{cases} \tag{1}$$

This expression is basic. It allows expressing different economical indexes of considered process. Let us introduce needed notations.

Ordering cost C_0 is known function of the order quantity *q*, i.e. $C_0 = C_0(q)$. Holding cost is proportional to quantity of goods in stock and holding time with coefficient of proportionality C_H. Losses from deficit are proportional to quantity of deficit with coefficient of proportionality C_{SH}.

Principal aim of the considered model is to define the optimal values of order quantity *q* and reorder point *r*. Criteria of optimization is minimum of average total expenses (costs) per a time unit. We solve this problem using regenerative approach (Ross 1992).

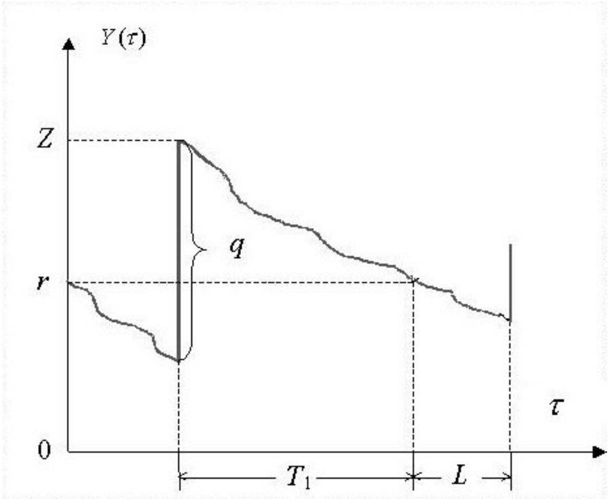

Figure 1: Dynamics of inventory level during one cycle

DISTRIBUTION OF DEMAND DURING LEAD TIME

Let us have $f_L(t)$ - the density function of the lead time *L*. Then the probability that the lead time *L* will be within interval $(\tau, \tau + d\tau)$ is

$$P((L \in (\tau, \tau + d\tau)) = f_L(\tau) d\tau. \tag{2}$$

On the other hand the probability that the demand will take value within interval $(i, i+di)$ by condition that value of lead time will be within interval $(\tau, \tau + d\tau)$ is

$$P(D_L \in (i, i+di) / L \in (\tau, \tau + d\tau)) = f_{D_\tau}(i) di. \tag{3}$$

Then the probability that the demand for lead time *L* will be within interval $(i, i+di)$ is

$$P(D_L \in (i, i+di)) = \int_0^\infty f_{D_\tau}(i) di \cdot f_L(\tau) d\tau. \tag{4}$$

Taking into account that demand has to be nonnegative value we find the distribution function of the demand for the lead time

$$P(0 \le D_L < x) = \int_0^x di \int_0^\infty f_{D_\tau}(i) \cdot f_L(\tau) d\tau. \tag{5}$$

Then the density function of the demand for lead time *L* is

$$f_{D_L}(x) = \int_0^\infty f_{D\tau}(x) \cdot f_L(\tau) d\tau. \tag{6}$$

Using (1) and (6) the density function of the quantity Z of goods in stock at the time after order receiving can be represented as

$$f_Z(x) = f_{D_L}(r+q-x)\,,\ q < x \le q+r. \qquad (7)$$

Really taking into account formula (1) we can write

$$F_Z(x) = P(Z \le x) = P(D_L > r+q-x) = \\ = 1 - P(D_L \le r+q-x) = 1 - F_{D_L}(r+q-x)\,.$$

Taking the derivative by x we have

$$f_Z(x) = f_{D_L}(r+q-x)\,,\ q < x \le q+r.$$

Further

$$F_Z(q) = P(Z \le q) = 1 - F_{D_L}(r) = 1 - \int_q^{q+r} f_Z(x)dx\,.$$

LENGTH OF CYCLE REGENERATION

In view of regenerative approach, time interval between two neighboring order deliveries is called as *regeneration cycle* (Ross 1992).

Let T be the duration of a cycle. Length of cycle consists of two parts: time between receiving the goods and placing a new order T_1 and lead time L, i.e.

$$T = T_1 + L.$$

The average cycle length

$$E(T) = E(T_1) + E(L)\,.$$

If the quantity of goods at the time moment immediately after order receiving equals y, the demand for goods for the time T_1 will be equal $y-r$. Then the probability that the time T_1 will be less than t under condition $Z = y$ is

$$P(T_1 < t \mid Z = y) = P(D_t \ge y-r) = \\ = 1 - P(D_t < y-r) = 1 - \int_0^{y-r} f_{D_t}(u)du\,. \qquad (8)$$

The parameter y may take the values from q till $r+q$. Taking the integral by variable y we find the probability of event, that time T_1 will be less than t:

$$F_{T_1}(t) = P(T_1 \le t) = \int_q^{r+q}(1 - \int_0^{y-r} f_{D_t}(u)du)dF_Z(y). \qquad (9)$$

Taking the derivative of (9) by t we have the density function of the time T_1

$$f_{T_1}(t) = F'_{T_1}(t) = \left(\int_q^{r+q}(1 - \int_0^{y-r} f_{D_t}(u)du)dF_Z(y)\right)' = \\ = -\left(\int_q^{r+q}\int_0^{y-r}\left(\frac{\partial}{\partial t} f_{D_t}(u)\right)du\,dF_Z(y)\right). \qquad (10)$$

Finally we can calculate the expected value of the time T_1:

$$E(T_1) = -\int_0^{\infty} t\left(\int_q^{r+q}\int_0^{y-r}\left(\frac{\partial}{\partial t} f_{D_t}(u)\right)du\,dF_Z(y)\right)dt \qquad (11)$$

and the expected value of the time T:

$$E(T) = E(T_1) + E(L)\,. \qquad (12)$$

HOLDING COST DURING ONE CYCLE

Calculation process of the holding cost during one cycle is divided in two stages: calculation for time T_1 and calculation for time L. If quantity of goods in stock in the moment immediately after order receiving Z equals y and demand for goods during period of time τ is $D_\tau = x$, then holding cost within the interval $[\tau, \tau + d\tau]$ equals $C_H(y-x)d\tau$. Using this fact and idea from the previous paragraph we will get the following formulas for the average holding cost $E(TC_1)$ for the time from receiving the goods till placing the new order

$$E(TC_1) = C_H\int_0^{\infty} d\tau \int_q^{r+q}\int_0^{y}(y-x)f_{D_\tau}(x)dxdF_Z(y)\,, \qquad (13)$$

and for the average holding cost for the lead time L

$$E(TC_L) = C_H\int_\tau^{\infty}\overline{F}_L(\tau)d\tau\int_0^{\tau}(r-x)f_{D_\tau}(x)dx\,, \qquad (14)$$

where

$$\overline{F}_L(\tau) = P\{L > \tau\} = \int_\tau^{\infty} f_L(t)dt\,.$$

Average holding cost $E(TC_H)$ within cycle is the sum of the corresponding addenda:

$$E(TC_H) = E(TC_1) + E(TC_L)\,. \qquad (15)$$

LOSSES FROM DEFICIT WITHIN LEAD TIME

If within lead time L the demand D_L exceeds the value of reorder point r, then deficit of goods is present. Let us note that according to the conditions of our model deficit may be only during lead time. Let D_L=x and $x > r$, then losses from deficit (shortage cost) are $C_{SH}(x-r)$. So, average shortage cost within lead time is

$$E(TC_{SH}) = C_{SH} \int_r^{\infty} f_{D_L}(x)(x-r)dx. \qquad (16)$$

AVERAGE TOTAL COST DURING TIME UNIT

Criterion of optimization is average total costs (expenses) for goods holding, ordering and losses from deficit per a time unit. Denote this average total cost by $E(AC)$. This average cost $E(AC)$ can be found as average total cost during one cycle divided by average cycle time (Ross 1992):

$$E(AC) = \frac{E(TC)}{E(T)}, \qquad (17)$$

where $E(TC) = C_0(q) + E(TC_H) + E(TC_{SH})$; $E(TC_H)$, $E(TC_{SH})$ and $E(T)$ are defined accordingly by formulas (12), (15) and (16).

For optimal inventory control problem solving we have to minimize expression (17) by two parameters r and q.

CONCLUSIONS

In this paper we have presented one task of inventory control. A single-product model with random demand of goods and random lead time is observed. Principal aim of the proposed model is to define the exact amount of the order (order quantity) and moment of order (reorder point) to achieve the minimum expenses for goods holding, ordering and losses from deficit per a time unit. This task can be solved using the proposed model on the base of methods of optimization realized in special mathematical packages (Dyakonov 2001).

Further guidelines of the current research are the following: using the results of given paper for considering inventory control models with different concrete distributions for demand and lead time (for example exponential distribution, Erlang distribution and others), considering multi-product task. We plan to examine suggested model in solving practical tasks for Latvian Railway company and to compare numerical results received using this model with the analogues results received by simulation.

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